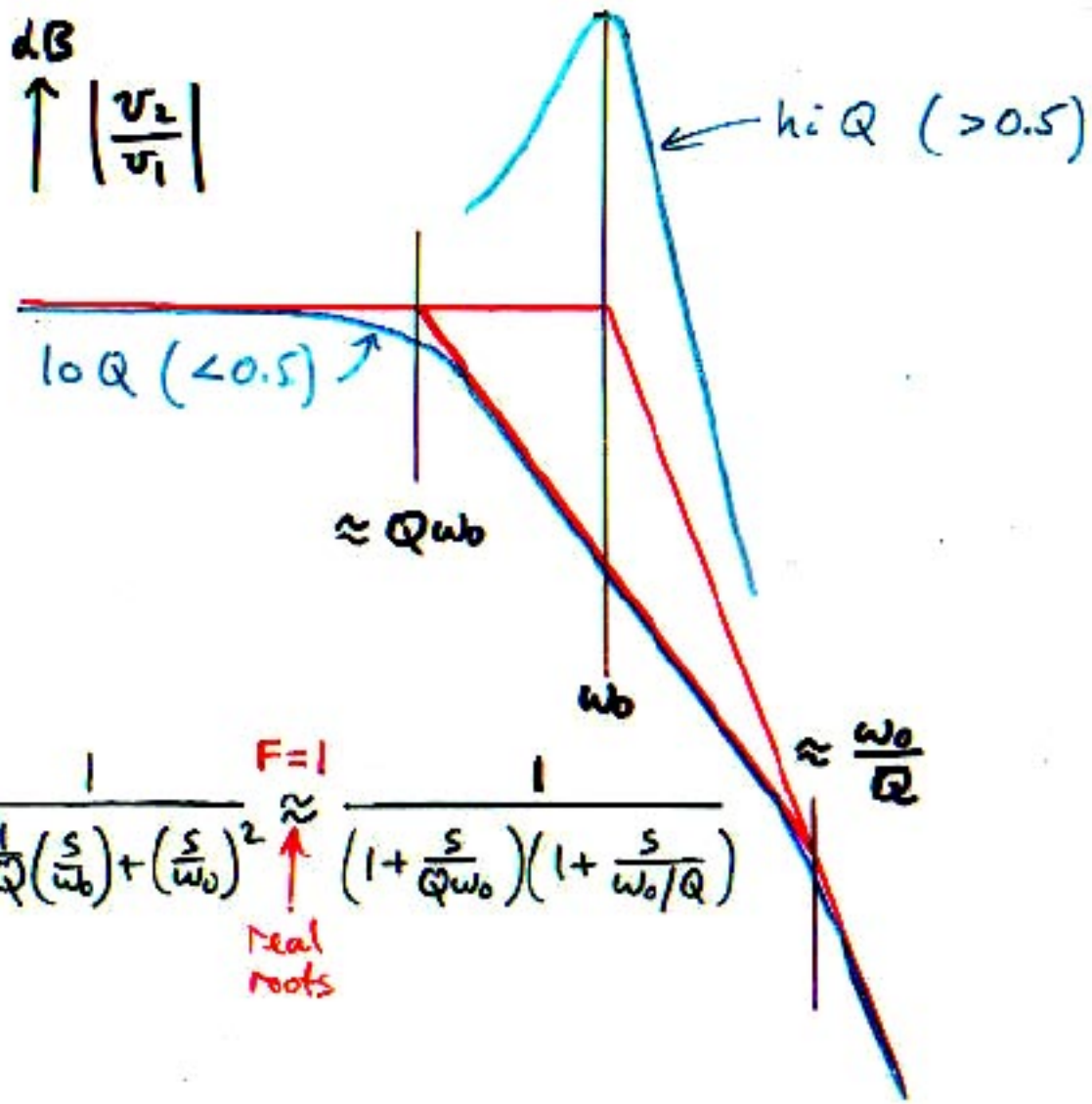
EXERCISE 5.1

SOLUTION

Exercise

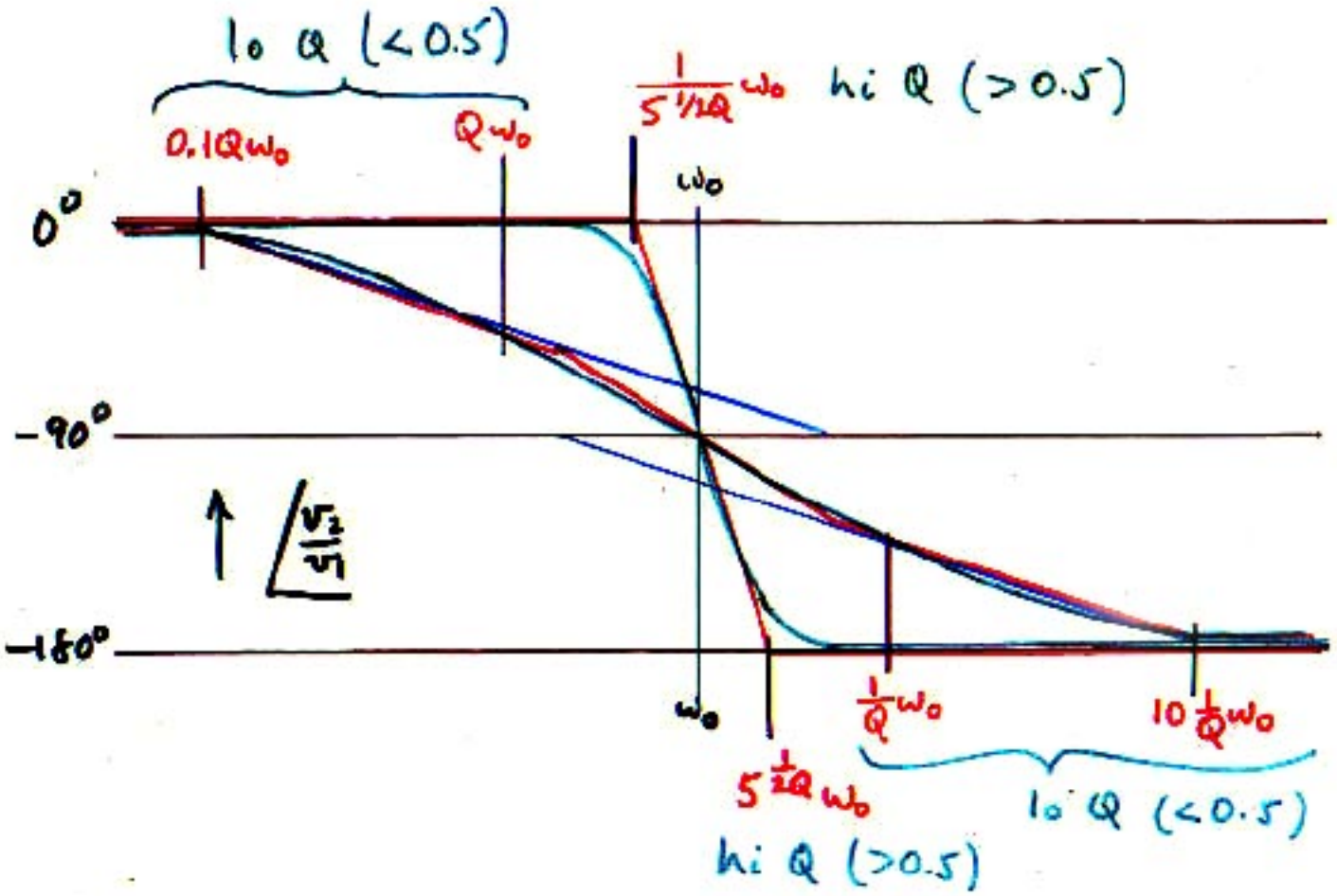
For the two-pole low-pass LC filter, sketch the magnitude and phase asymptotes of Z_i and Z_o for low Q ($<<0.5$). (But take $Q > 0.1$)

Low-pass 2-pole characteristic:

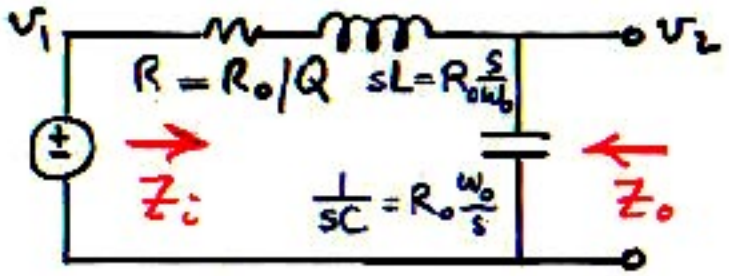


$$\frac{v_2}{v_1} = \frac{1}{1+\frac{1}{Q}\left(\frac{s}{\omega_0}\right)+\left(\frac{s}{\omega_0}\right)^2} \overset{F=1}{\approx} \frac{1}{\left(1+\frac{s}{Q\omega_0}\right)\left(1+\frac{s}{\omega_0/Q}\right)}$$

real roots


lo Q (<0.5)
0.1Qω₀
Qω₀
1/5^{1/2Q} ω₀
hi Q (>0.5)
ω₀
0°
-90°
-180°
5^{1/2}
5^{1/2Q} ω₀
1/Q ω₀
10 1/Q ω₀
lo Q (<0.5)
hi Q (>0.5)

Input and Output Impedances of low-pass filter



$$\omega_0 \equiv \frac{1}{\sqrt{LC}} \qquad R_0 \equiv \sqrt{\frac{L}{C}} \qquad Q = \frac{R_0}{R}$$

$$Z_i = \frac{R_0}{Q} + R_0 \frac{s}{\omega_0} + R_0 \frac{\omega_0}{s}$$

$$= R_0 \frac{1 + \frac{1}{Q}\left(\frac{s}{\omega_0}\right) + \left(\frac{s}{\omega_0}\right)^2}{\left(\frac{s}{\omega_0}\right)}$$

$$Z_o = \frac{\left(\frac{R_0}{Q} + R_0 \frac{s}{\omega_0}\right) R_0 \frac{\omega_0}{s}}{\frac{R_0}{Q} + R_0 \frac{s}{\omega_0} + R_0 \frac{\omega_0}{s}}$$

$$= R_0 \frac{\left(\frac{s}{\omega_0}\right)\left(1 + \frac{\omega_0/Q}{s}\right)}{1 + \frac{1}{Q}\left(\frac{s}{\omega_0}\right) + \left(\frac{s}{\omega_0}\right)^2}$$

Note how the algebra is shortened when the analysis starts with the normalized element values.

$$Z_i = R_o \times \left[1 + \frac{1}{Q}\left(\frac{s}{\omega_o}\right) + \left(\frac{s}{\omega_o}\right)^2\right] \times \left[\frac{1}{\frac{s}{\omega_o}}\right]$$

ref. value

$\frac{1}{H}$

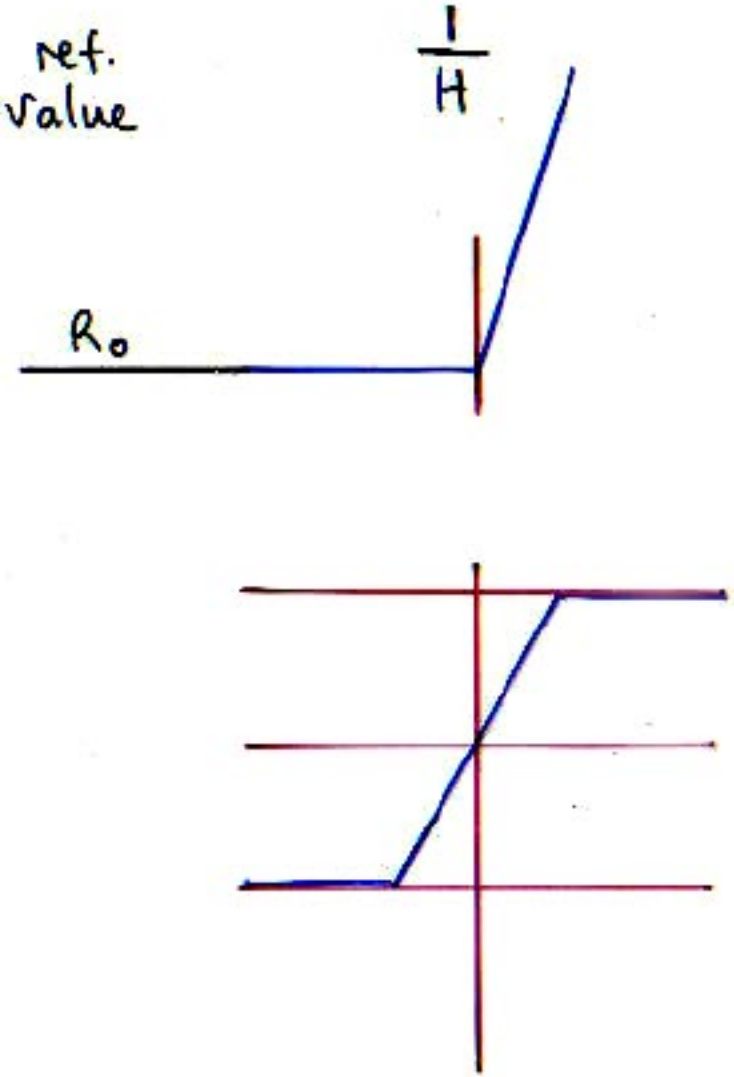
single slope

R_o

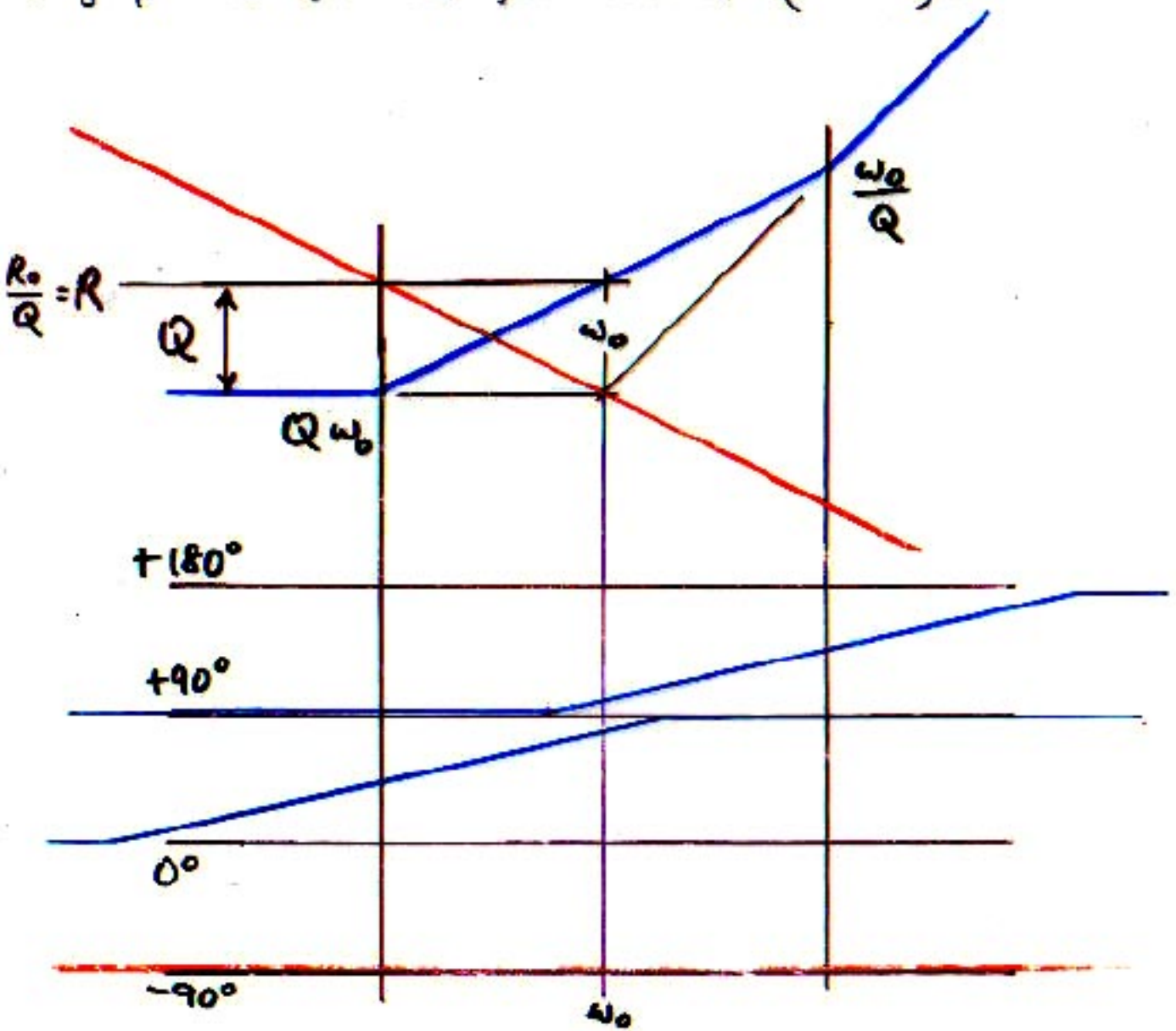
$$Z_i = R_o \times \left[1 + \frac{1}{Q}\left(\frac{s}{\omega_o}\right) + \left(\frac{s}{\omega_o}\right)^2\right] \times \left[\frac{1}{\frac{s}{\omega_o}}\right]$$

ref. value

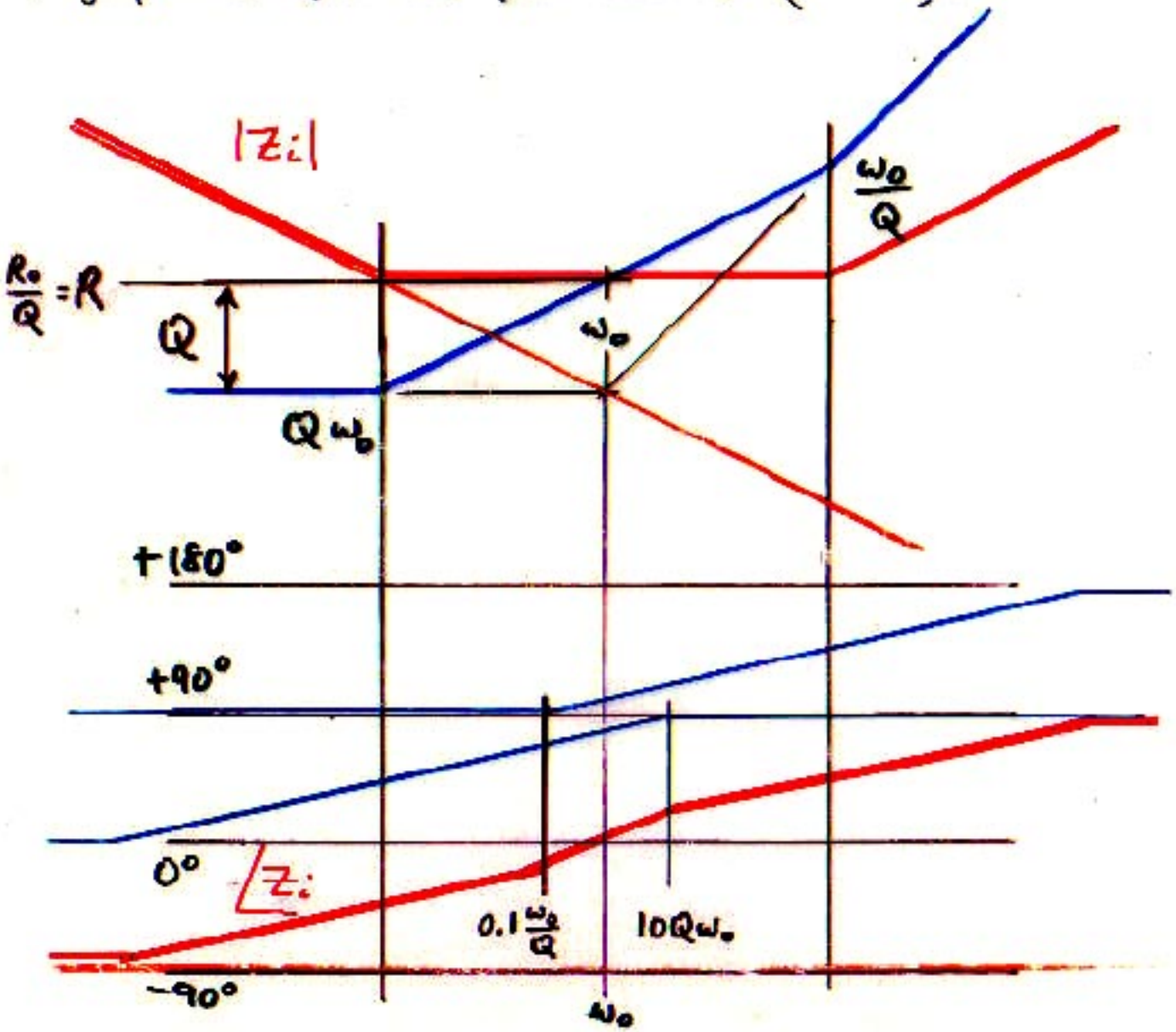
single slope



Asymptotes for Z_i for low Q ($<<0.5$):



Asymptotes for Z_i for low Q ($Q \ll 0.5$):



$$Z_o = R_o \times \left[\frac{\frac{s}{\omega_o}}{1 + \frac{1}{Q}\left(\frac{s}{\omega_o}\right) + \left(\frac{s}{\omega_o}\right)^2}\right] \times \left(1 + \frac{\omega_o/Q}{s}\right)$$

ref. value

$\frac{1}{Z_i}$

inverted zero

R_o

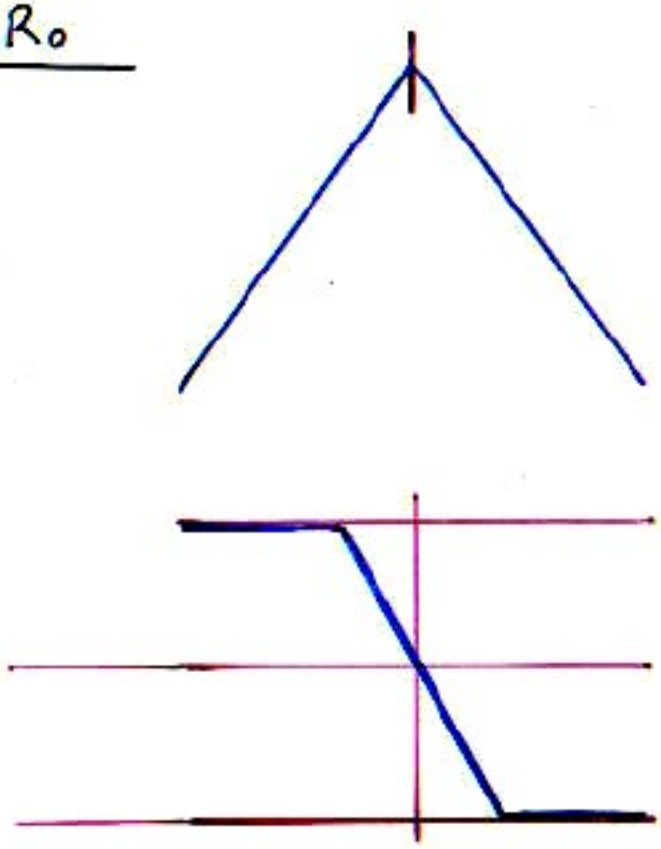
$$Z_o = R_o \times \left[\frac{\frac{s}{\omega_0}}{1 + \frac{1}{Q}\left(\frac{s}{\omega_0}\right) + \left(\frac{s}{\omega_0}\right)^2}\right] \times \left(1 + \frac{\omega_0/Q}{s}\right)$$

ref. Value

$\frac{1}{Z_i}$

inverted zero

R_o

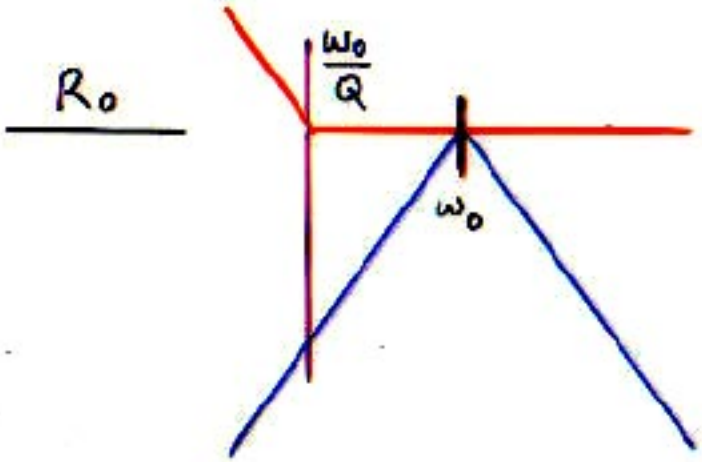


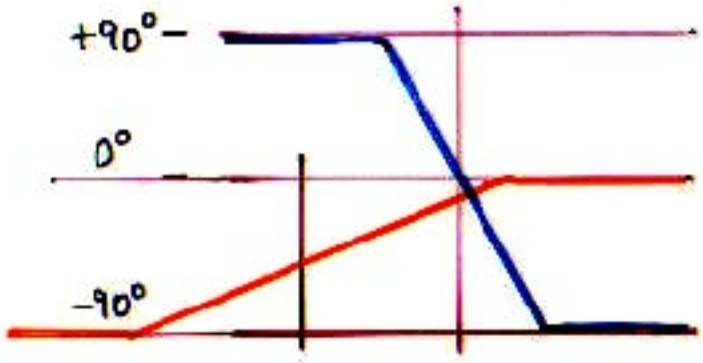
$$Z_o = R_o \times \left[\frac{\frac{s}{\omega_o}}{1 + \frac{1}{Q}\left(\frac{s}{\omega_o}\right) + \left(\frac{s}{\omega_o}\right)^2}\right] \times \left(1 + \frac{\omega_o/Q}{s}\right)$$

ref. value

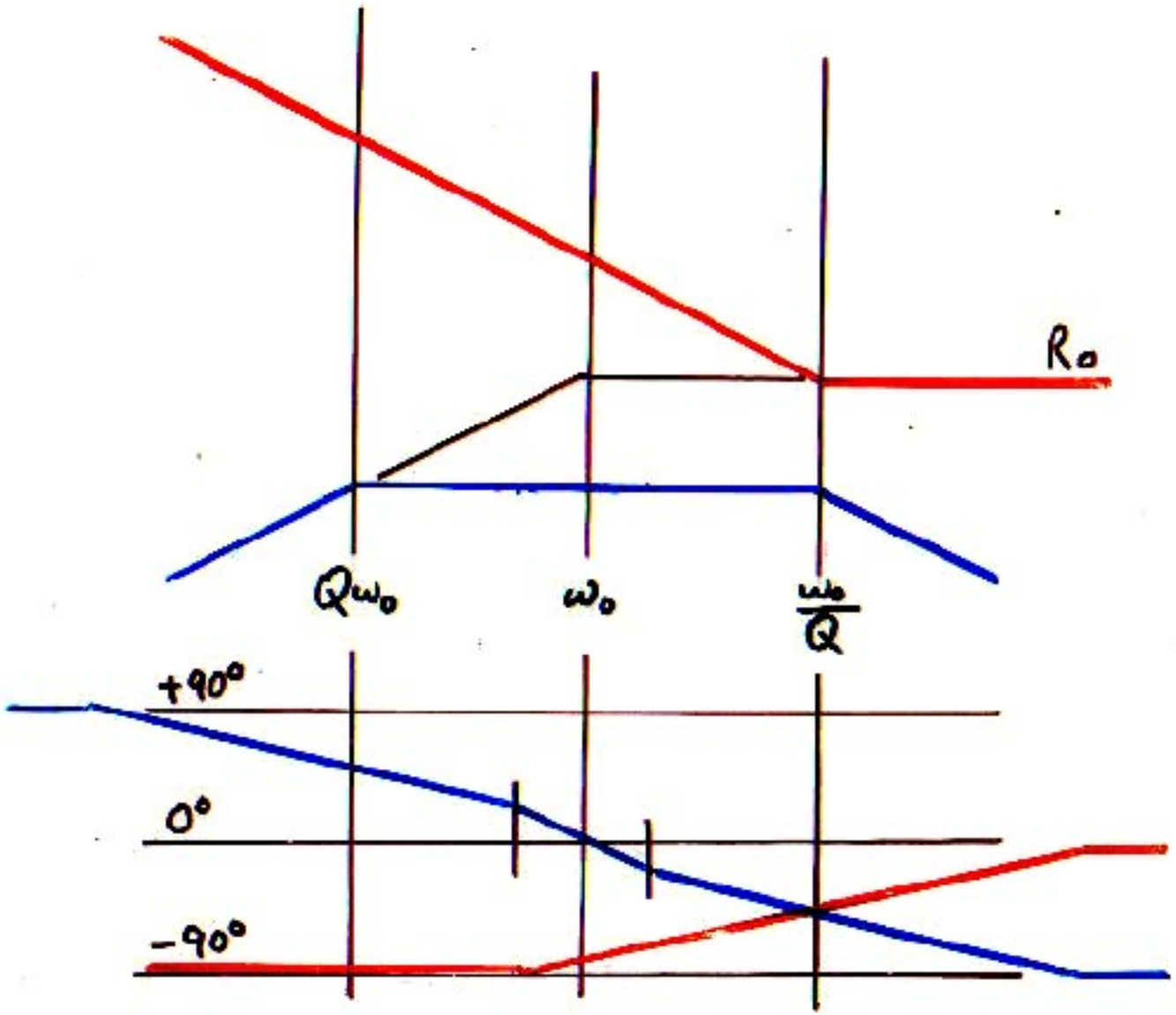
$\frac{1}{Z_i}$

inverted zero





Asymptotes for Z_o for low Q ($<< 0.5$):



Asymptotes for Z_o for low Q ($<< 0.5$):