2

LOW ENTROPY EXPRESSIONS:

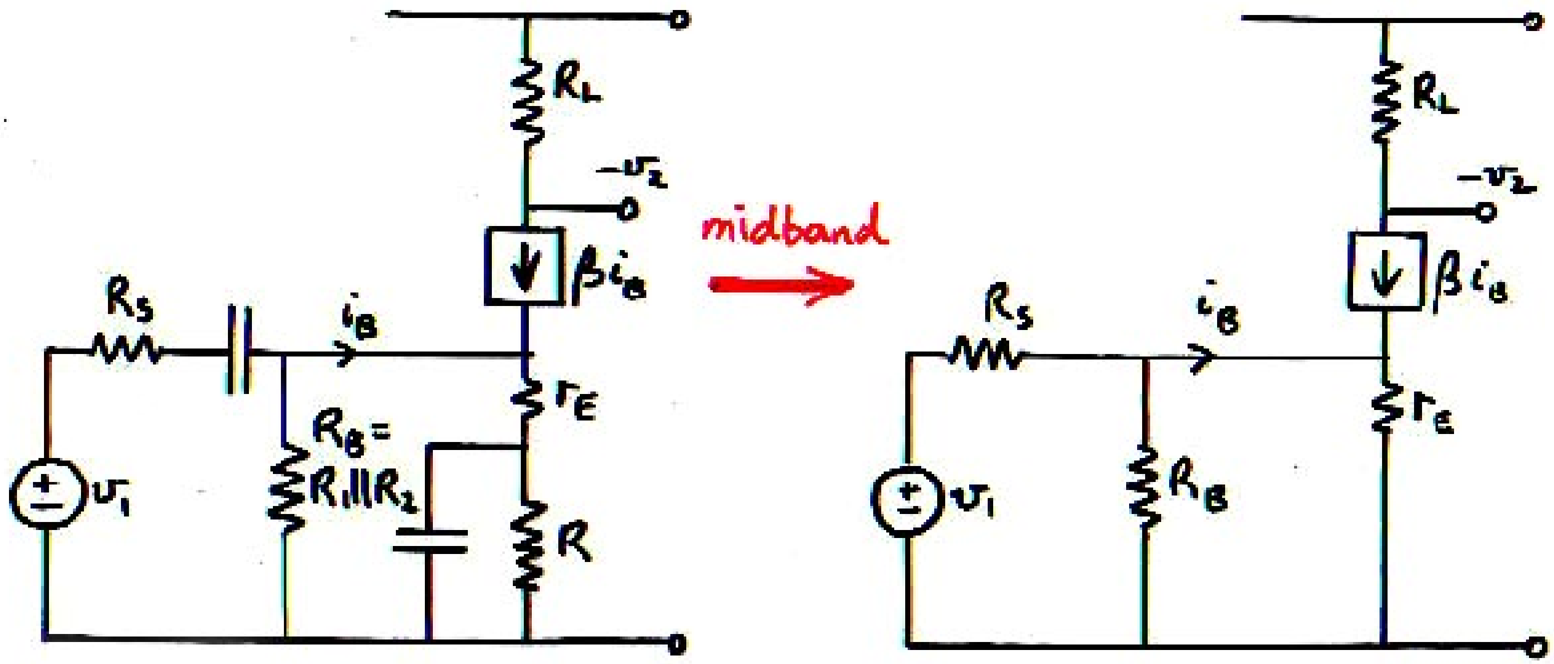
THE KEY TO D-OA

DESIGN-ORIENTED ANALYSIS

Techniques of Design-Oriented Analysis

Lowering the Entropy of an expression
Doing the algebra on the circuit diagram.
Doing the algebra on the graph.
Using inverted poles and zeros.
Using numerical values to justify analytic approximations.
Improved formulas for quadratic roots
The Input/Output Impedance Theorem
The Feedback Theorem
Loop gain by injection of a test signal into the closed loop
Measurement of an unstable loop gain
The Extra Element Theorem (EET)

Gain of CE amplifier

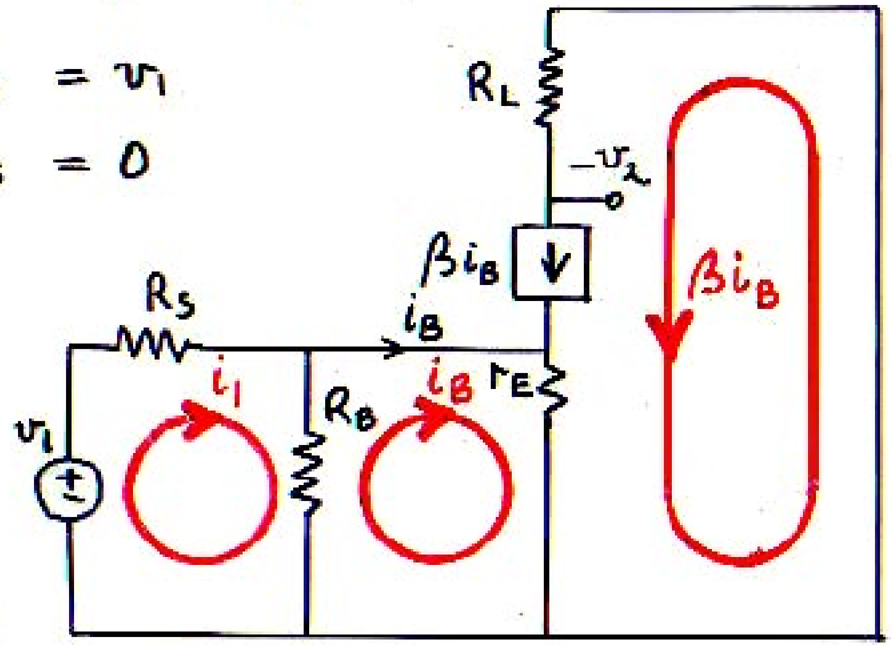


"Midband" means frequencies at which reactive effects are negligible

The "brute-force" method: loop analysis

$$(R_S+R_B)\,i_1 \qquad - R_B\, i_B = v_1$$

$$-R_B\, i_1 + \left[R_B+(1+\beta)r_E\right] i_B = 0$$



$$i_B = \frac{\begin{vmatrix} R_S+R_B & v_1 \\ -R_B & 0 \end{vmatrix}}{\begin{vmatrix} R_S+R_B & -R_B \\ -R_B & R_B+(1+\beta)r_E \end{vmatrix}}$$

$$= \frac{R_B v_1}{(R_S+R_B)\left[R_B+(1+\beta)r_E\right]-R_B^2}$$

$$= \frac{R_B v_1}{R_S R_B+(1+\beta)r_E R_S + R_B^2 + (1+\beta)r_E R_B - R_B^2}$$

Finally, $v_2 = R_L \beta i_B$

which leads to:

$$A_m \equiv \frac{v_2}{v_1} = \frac{\beta R_B R_L}{(1+\beta)r_E R_S + (1+\beta)r_E R_B + R_S R_B}$$

Lower the Entropy of the result:

$$A_m = \frac{\beta R_B R_L}{(1+\beta) r_E R_S + (1+\beta) r_E R_B + R_S R_B}$$

$$= \frac{\beta R_B R_L}{(1+\beta) r_E (R_S + R_B) + R_S R_B}$$

$$= \frac{R_B}{R_S + R_B} \cdot \frac{\beta R_L}{(1+\beta) r_E + R_S \| R_B}$$

$$= \frac{R_B}{R_S + R_B} \cdot \frac{\alpha R_L}{\underset{(1)}{r_E} + \underbrace{(R_S \| R_B)/(1+\beta)}_{(2)}}$$

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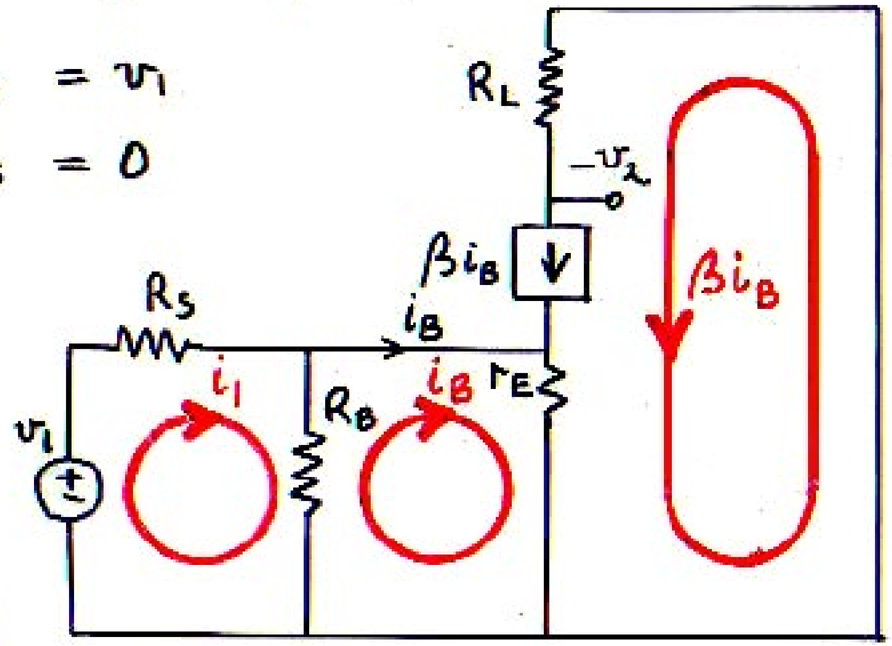

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$$= \frac{R_B}{R_S + R_B} \cdot \frac{\beta R_L}{(1+\beta) r_E + R_S \| R_B}$$

$$= \frac{R_B}{R_S + R_B} \cdot \frac{\alpha R_L}{\underbrace{r_E}_{(1)} + \underbrace{(R_S \| R_B)/(1+\beta)}_{(2)}}$$

The Low Entropy result exposes the following additional information, not apparent from the High Entropy version:

(a) The $R_B/(R_S+R_B)$ factor is identified as a voltage divider;

(b) Resistances appear in series/parallel combinations, so it is clear which ones are dominant;

(c) The relative values of the two terms labeled (1) and (2) determine the sensitivity of the gain A to variations of β.

The additional information makes possible a much better informed choice of element values.

Disadvantages of the "brute-force" method:

1. No direct physical interpretation of the result.
2. Obscures relationships as to how element values affect the result.
3. Difficult to use for design: given A_m (the Specification), how do you choose element values?
4. Purely algebraic derivation increases likelihood of mistakes.

Advantages of the Low-Entropy form of the result:

1. Direct physical interpretation of the result.

2. Clarifies relationships as to how element values affect the result.

3. Easy to use for design: given A_m (the Specification), how do you choose element values?

4. ?

It is easier to keep the Entropy low from the start of the analysis than it is to lower the Entropy once it has increased.

The "brute-force" method: loop analysis

$$(R_s + R_B)\, i_1 \qquad - R_B \cdot i_B = v_1$$

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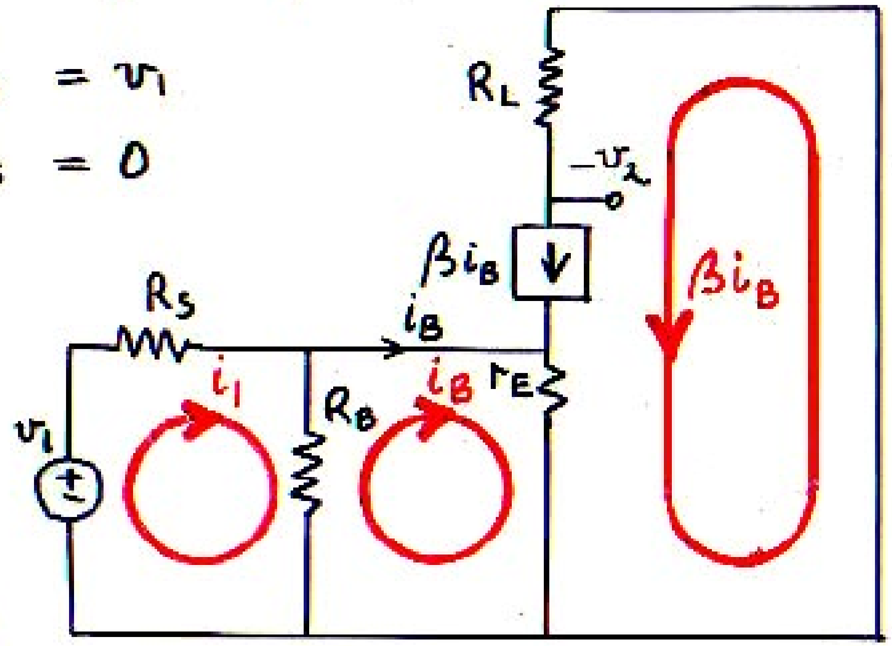

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Finally, $v_2 = R_L \beta i_B$

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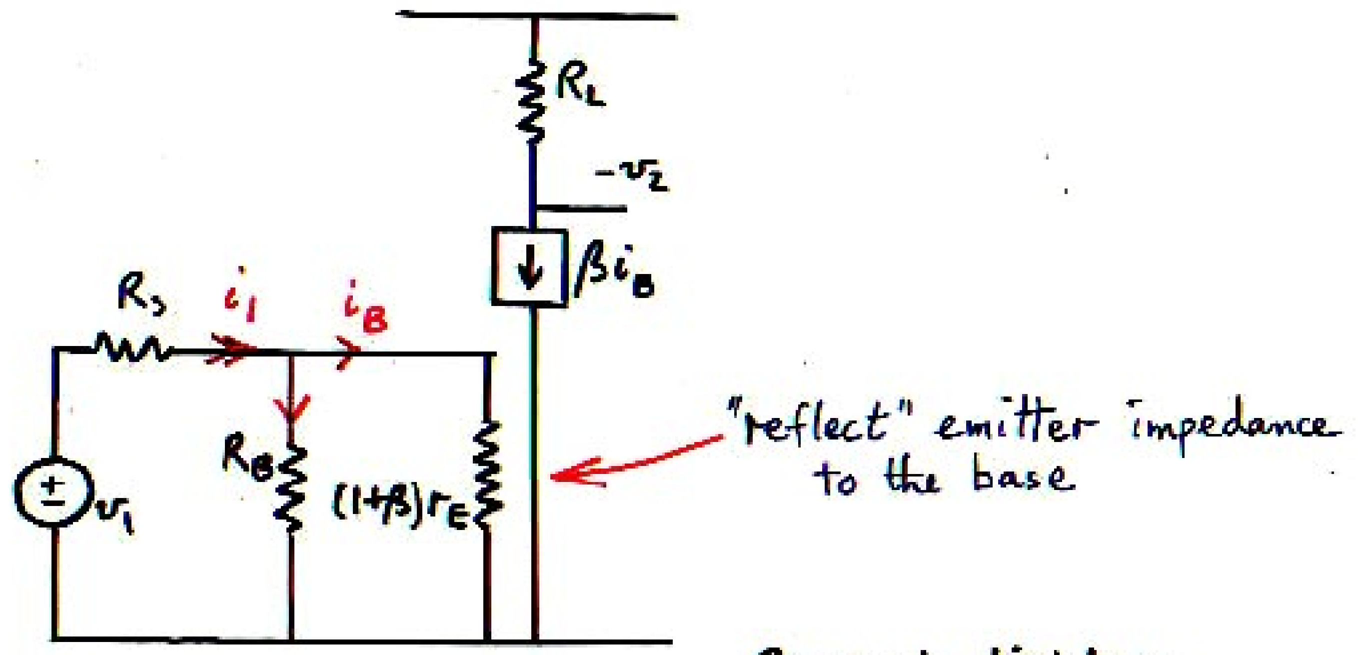
$$A_m \equiv \frac{v_2}{v_1} = \frac{\beta R_B R_L}{(1+\beta) r_E R_s + (1+\beta) r_E R_B + R_s R_B}$$

Better method #1:



$$i_1 = \frac{v_1}{R_S + R_B \| (1+\beta) r_E}$$

Current divider:

$$\frac{i_B}{i_1} = \frac{\text{opposite branch impedance}}{\text{sum of branch impedances}}$$

$$v_2 = \beta R_L i_B \qquad = \frac{R_B}{R_B + (1+\beta) r_E}$$

$$A_m \equiv \frac{v_2}{v_1} = \frac{R_B}{R_B + (1+\beta) r_E} \cdot \frac{\beta R_L}{R_S + R_B \| (1+\beta) r_E}$$

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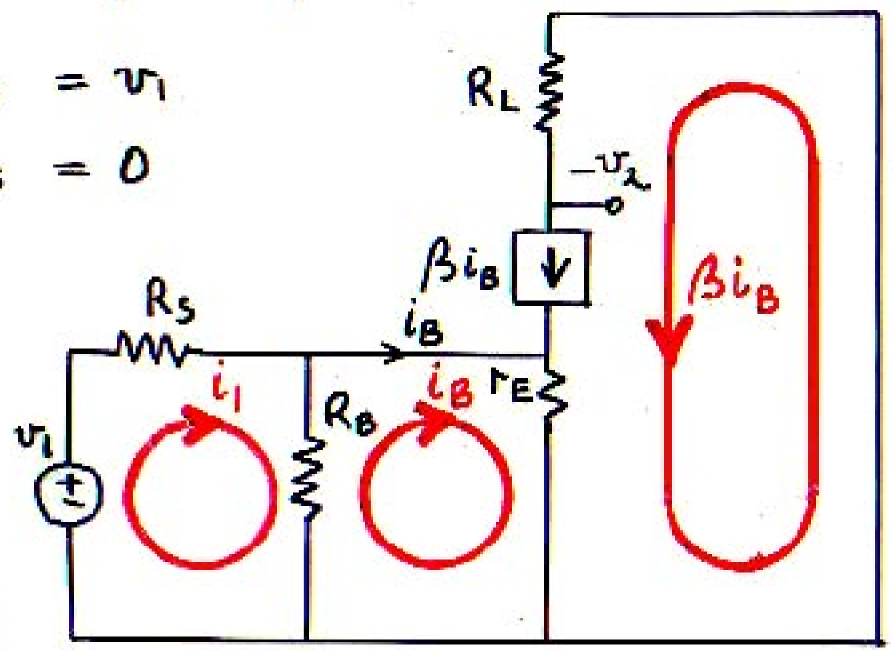

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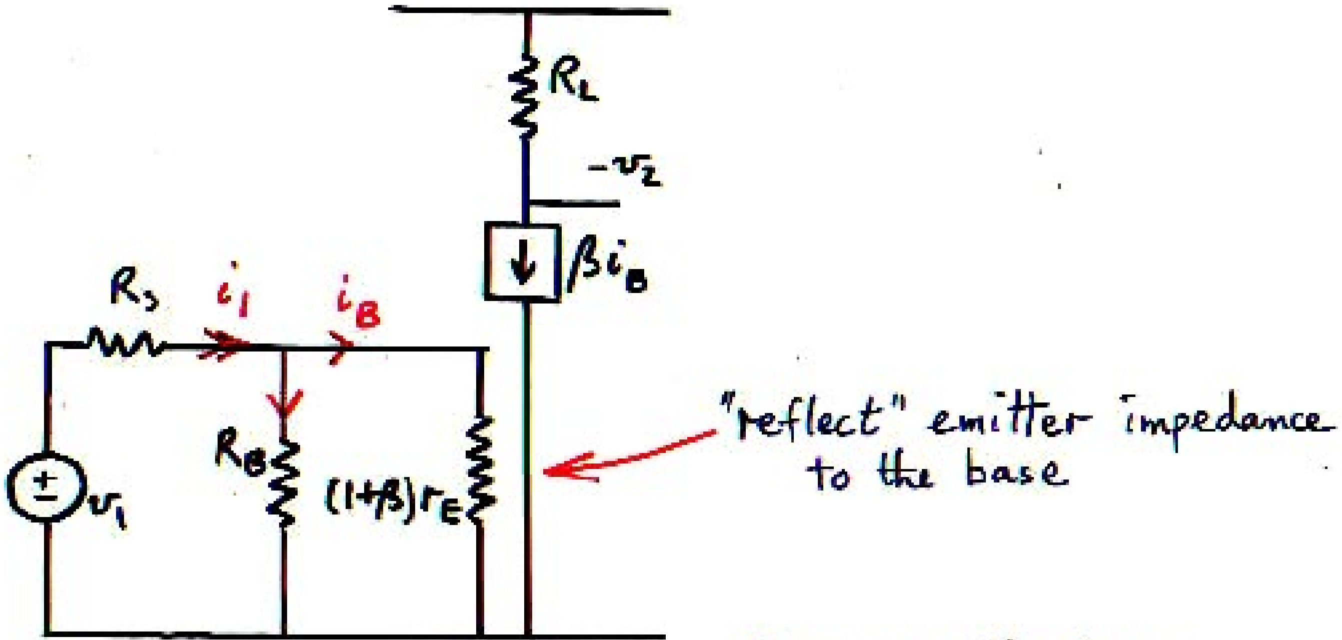

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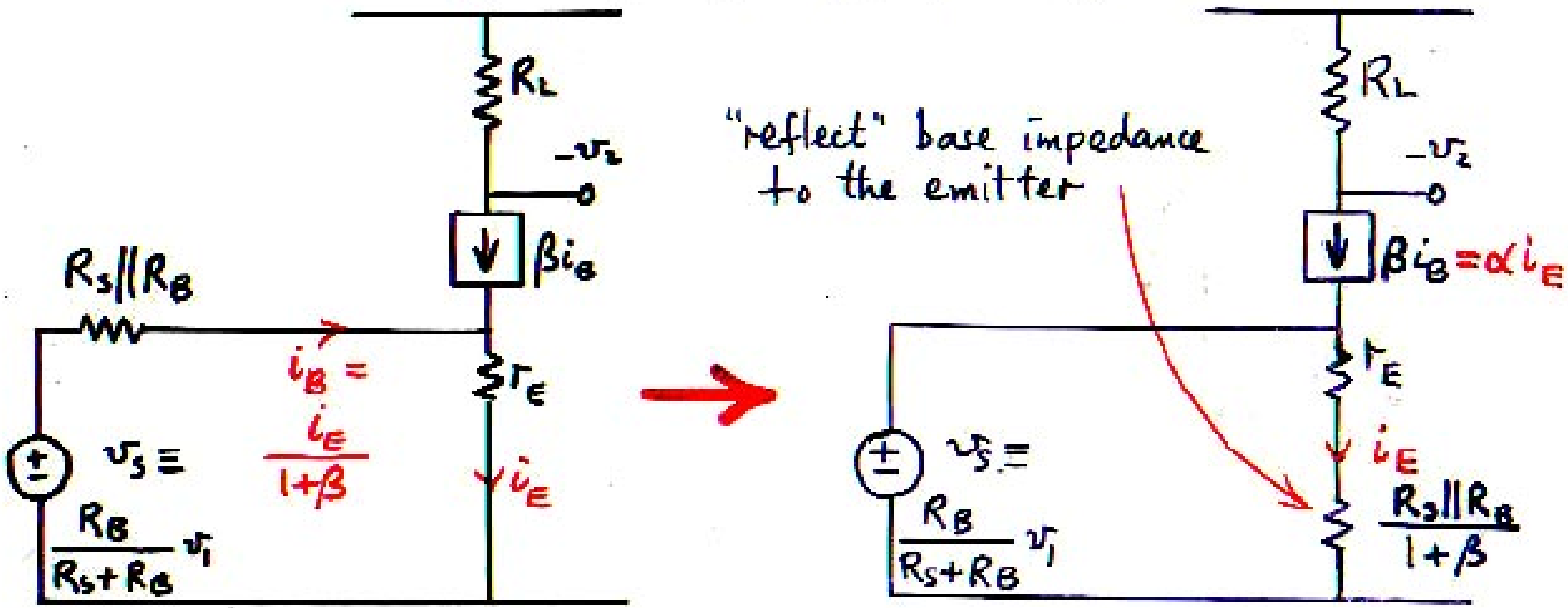
Doing the algebra on the circuit diagram

Better method: Use Thevenin's Theorem at the start



$$i_E = v_S \frac{1}{r_E + (R_S \| R_B)/(1+\beta)} \qquad v_2 = R_L i_C = \alpha R_L i_E$$

$$\frac{v_2}{v_S} = \alpha \frac{R_L}{r_E + (R_S \| R_B)/(1+\beta)} = \alpha \frac{\text{total collector load}}{\text{total emitter load including reflected base impedance}}$$

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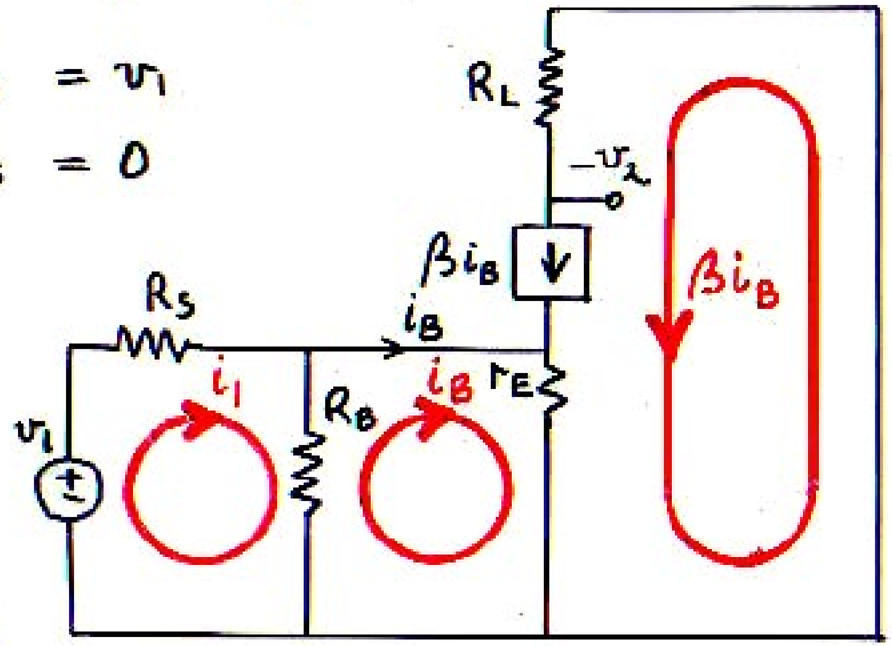

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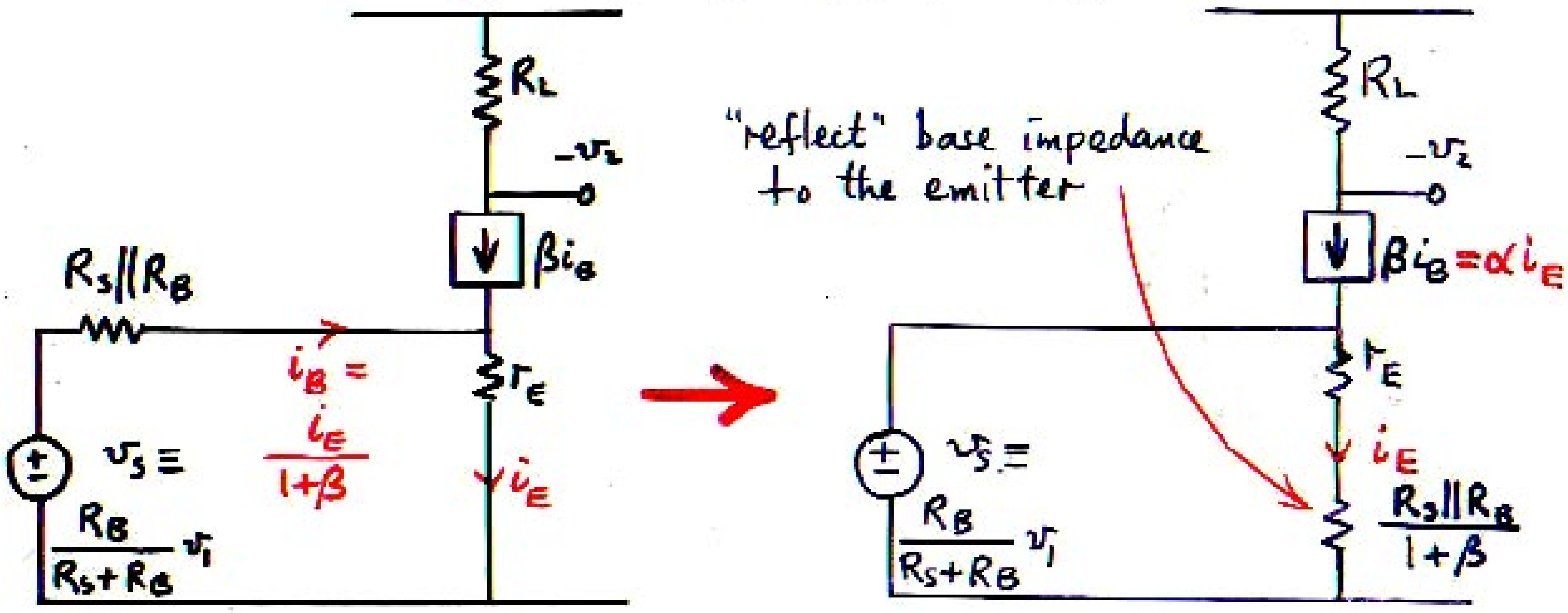

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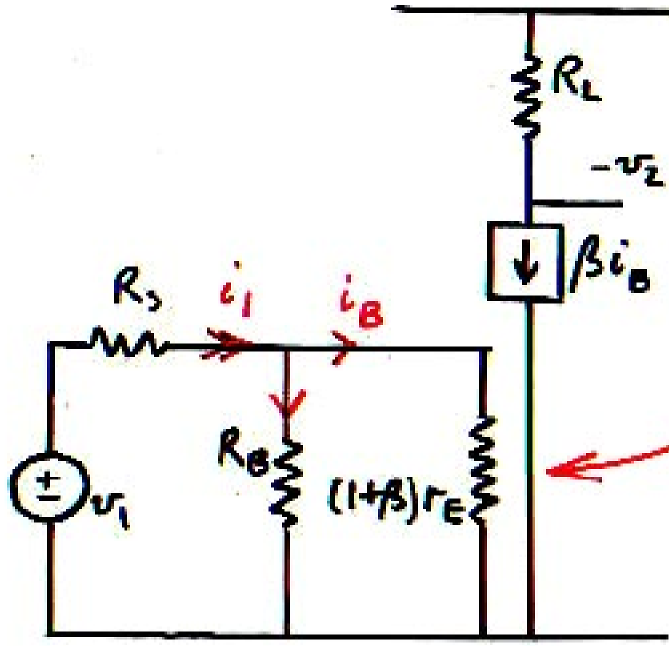

"reflect" emitter impedance to the base

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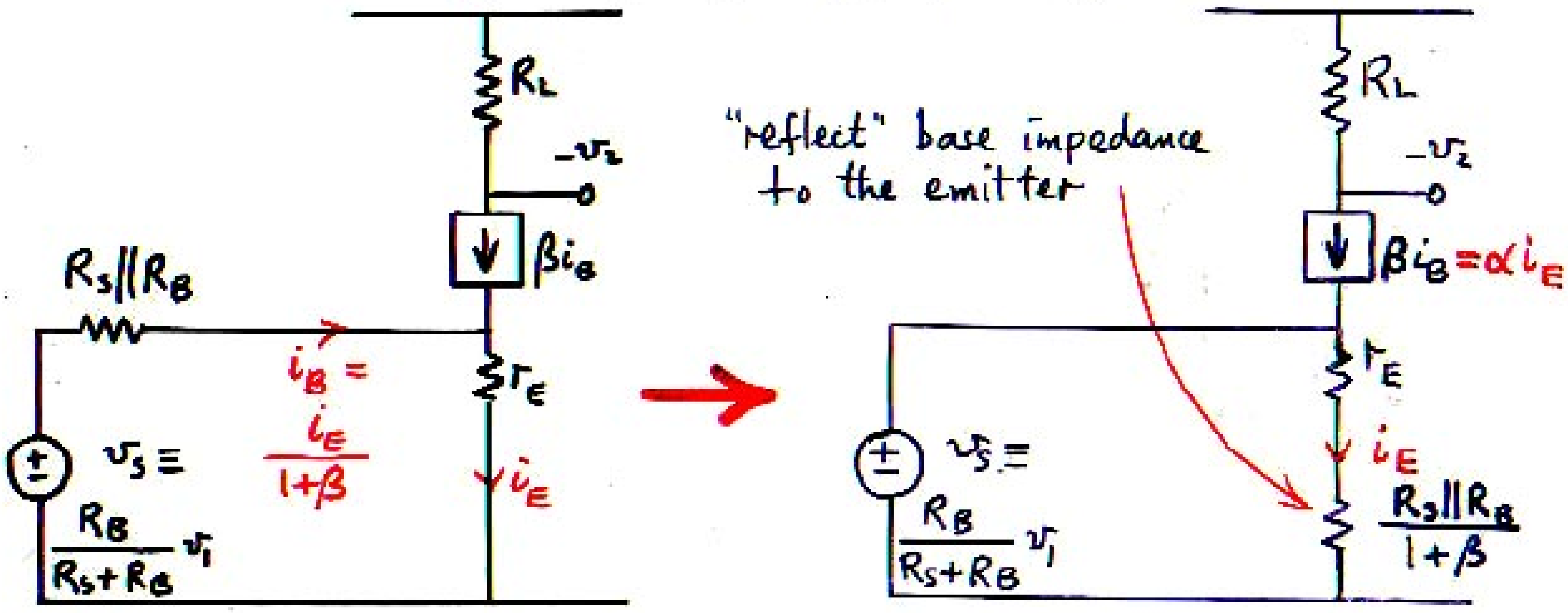

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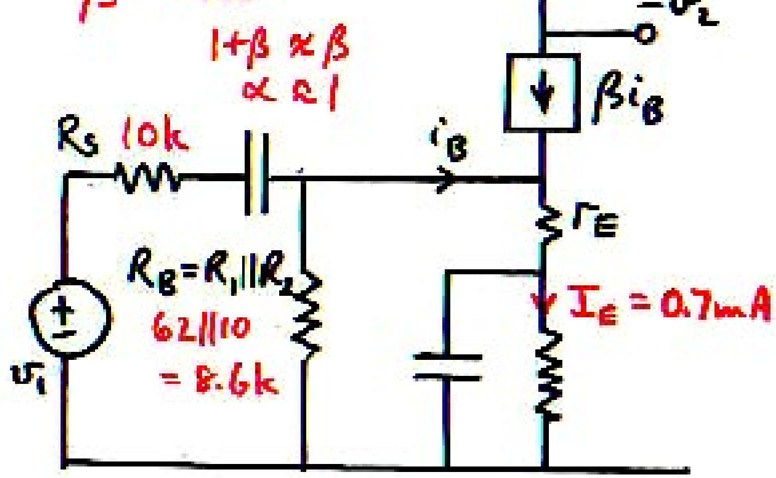
Example: Previous designed circuit

$r_E = \frac{V_t}{I_E} = \frac{25}{0.7} = 36\Omega$

$\beta = 120$



$$A_m = \frac{R_B}{R_s + R_B} \cdot \frac{\alpha R_L}{r_E + (R_s \| R_B)/(1+\beta)}$$

$$= \frac{8.6}{10+8.6} \; \frac{10}{0.036 + \underbrace{(10\|8.6)/120}_{0.039}}$$

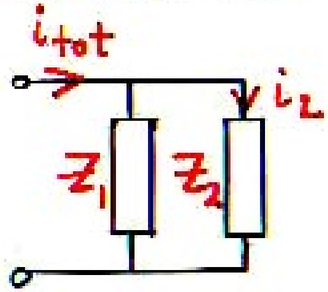
$$= 62 \Rightarrow 36\,dB$$

The results for A_m by the two methods are of course the same, but the element contributions are grouped differently.

Any grouping contains more useful information about the relative contributions of the various elements than does the multiplied-out result obtained by the "brute-force" solution of simultaneous loop or node equations.

Generalization: Current and Voltage Dividers

Current divider

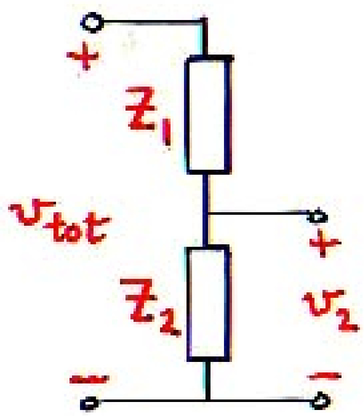


$$\frac{i_2}{i_{tot}} = \frac{Z_1}{Z_1 + Z_2}$$

$$\frac{\text{current in one branch}}{\text{total current}} = \frac{\text{opposite branch impedance}}{\text{sum of branch impedances}}$$

This is the dual of the:

Voltage divider

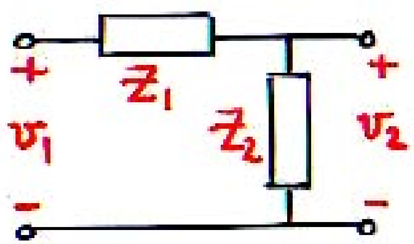


$$\frac{v_2}{v_{tot}} = \frac{Z_2}{Z_1 + Z_2}$$

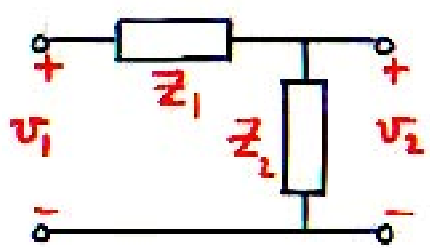
$$\frac{\text{voltage at tap}}{\text{total voltage}} = \frac{\text{tap impedance to ground}}{\text{sum of impedances to ground}}$$

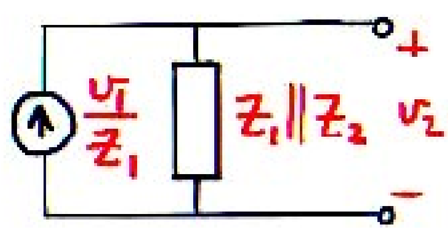
Generalization: Loop and Node Removal

Every time Thevenin's theorem is used, one loop is removed from the circuit:



Every time Norton's theorem is used, one node is removed from the circuit:





Generalization: Loop and node removal by Thevenin and Norton reduction

By successive use of the Thevenin and Norton theorems, a multi-loop, multi-node circuit can be reduced to a simple form from which the analytical results can be written by inspection.

This is an example of the powerful technique of <u>doing the algebra on the circuit diagram</u>

Generalization: Loop and Node Removal

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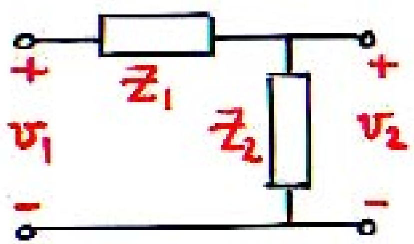

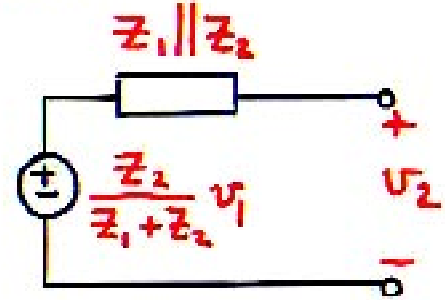

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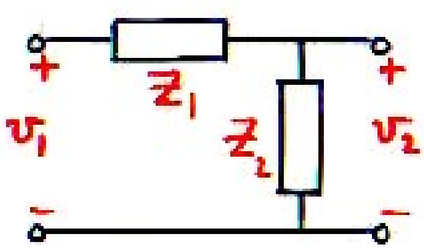
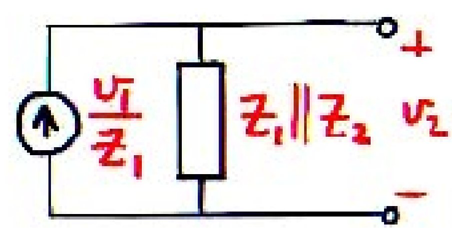
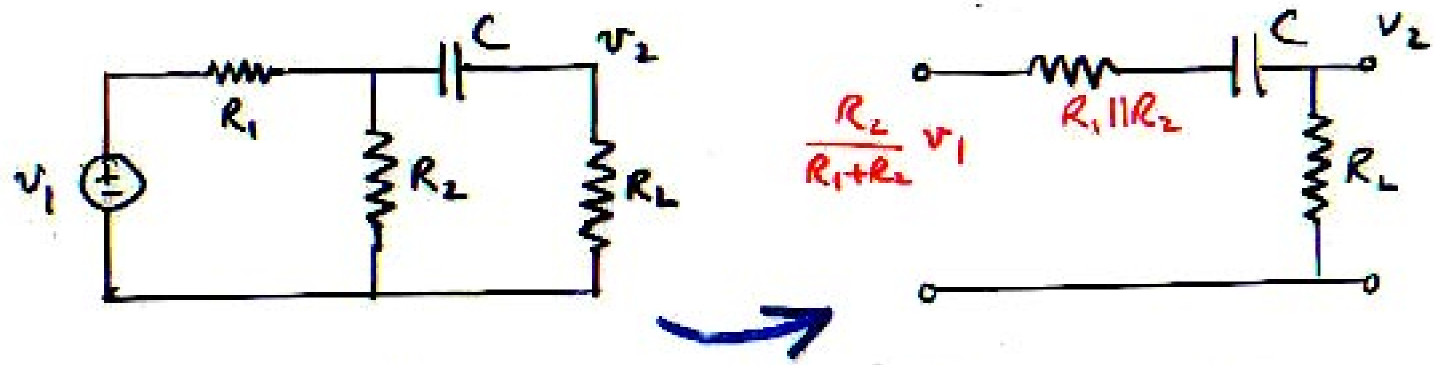




Another example:



This is how <u>element groupings</u> arise <u>naturally</u>, by circuit reduction through successive loop and node removal.

Generalization: Advantages of Doing the Algebra on the Circuit Diagram

1. Simultaneous solution of multiple loop or node equations is replaced by sequential, simple, semigraphical steps.

2. The element values in the successively reduced models automatically appear in usefully grouped combinations (to facilitate tradeoffs).

3. Less likelihood of making algebraic mistakes.

4. Because the physical origin of all terms in the analytic results remain explicit, the results are in optimum form for design: element values can be chosen so that the results meet the specifications.