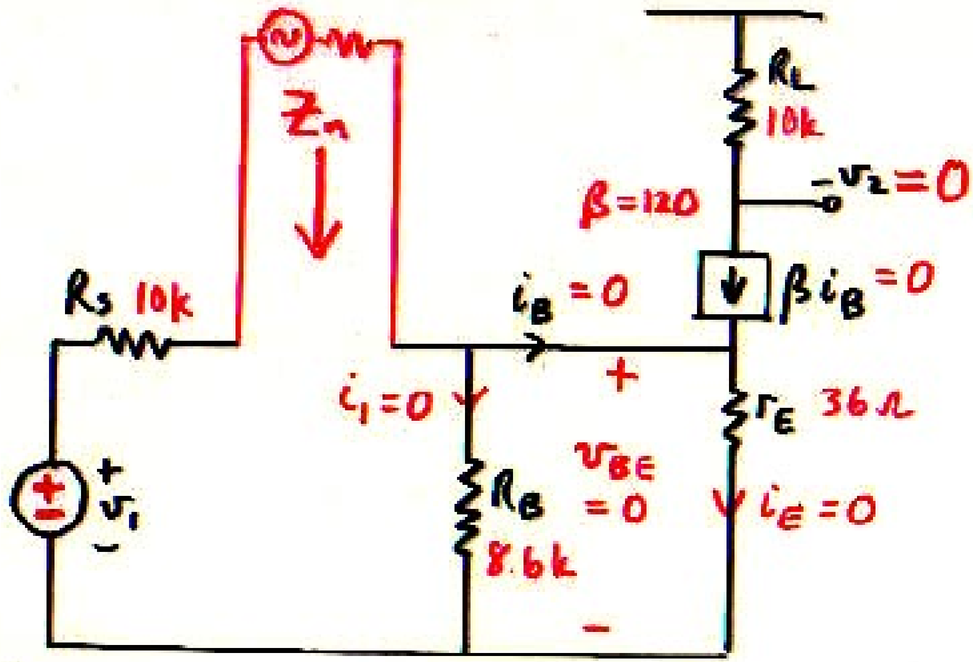
Exercise Solution

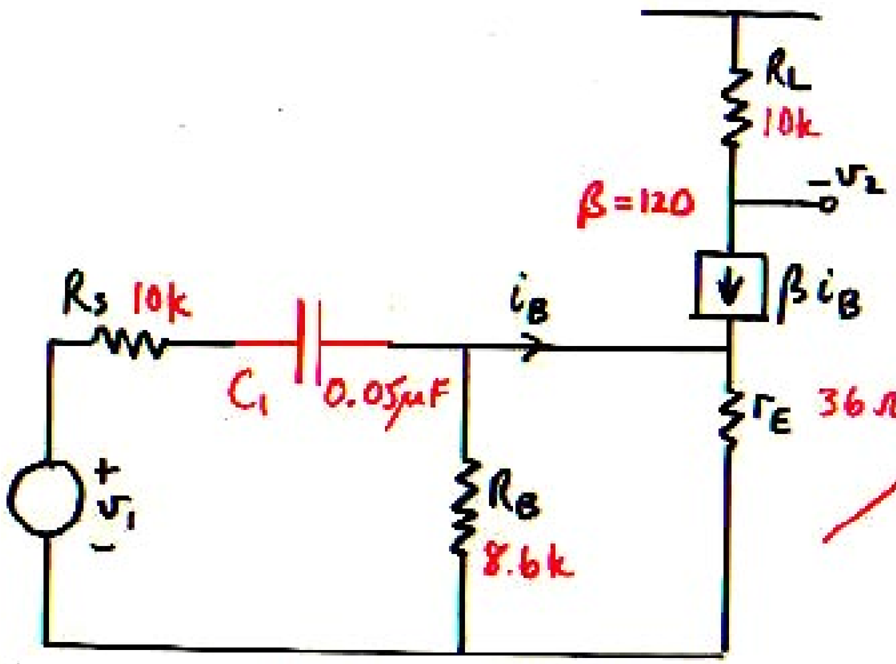


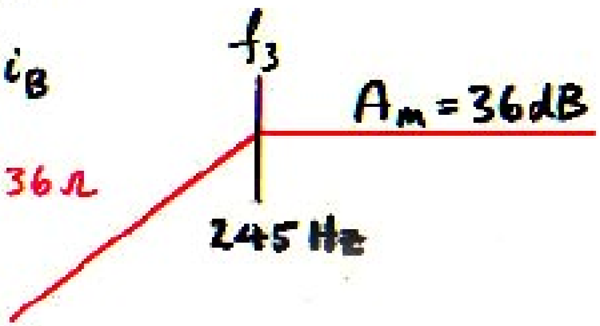
$A_m = 36dB$

Step 2.

$Z_n = R_n = \infty$

Exercise Solution





Hence corrected gain in the presence of $Z \to 1/sC_1$ is

$$A = A_m \frac{1 + \frac{Z}{Z_n}}{1 + \frac{Z}{Z_d}} = A_m \frac{1}{1 + \frac{1}{sC_1 R_d}} = A_m \frac{1}{1 + \frac{\omega_3}{s}}$$

where

$$\omega_3 \equiv \frac{1}{C_1 R_d} \qquad f_3 = \frac{159}{0.05 \times 13} = 245\ \text{Hz}$$

Generalization: Extra Element Theorem — #2

If the reference circuit is purely resistive,
$Z_d = R_d$ and $Z_n = R_n$ are pure resistances.

If, also, the extra element is a pure reactance,
the Extra Element Theorem correction factor gives the corner frequencies directly.

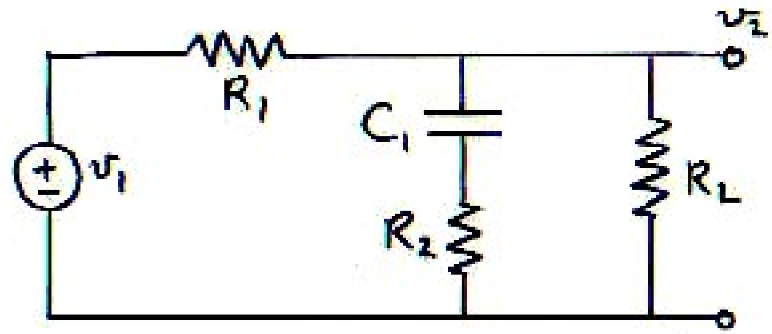
Generalization: Extra Element Theorem — #3

The Extra Element Theorem can profitably be used to divide the analysis of a complicated circuit into successive simpler steps:

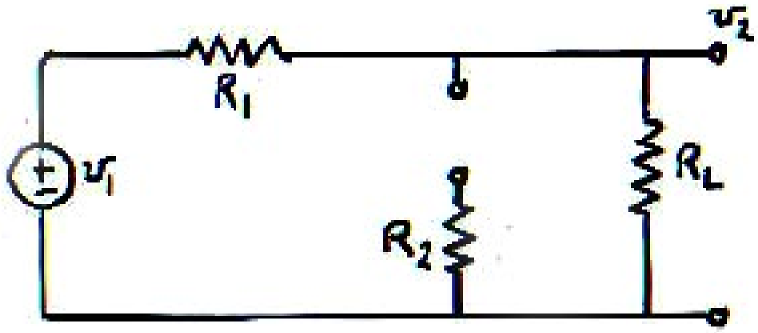
Designate one element as "extra," and the circuit without the element as the "reference circuit." Calculate the gain of the (simpler) reference circuit, then restore the omitted element by the Extra Element Theorem correction factor.

This is a particularly useful approach when the designated "extra" element is a reactance and the reference circuit is purely resistive.

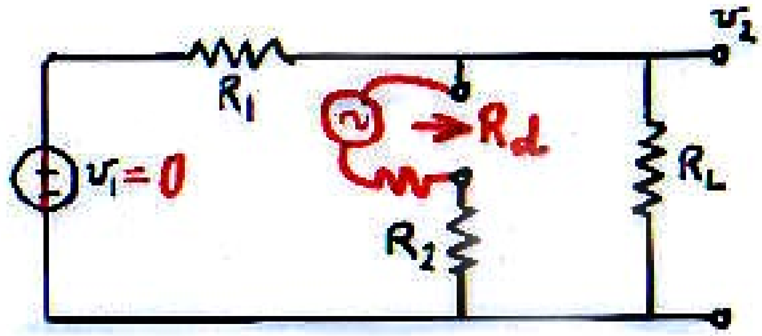
Exercise: Lag-lead network



Find the transfer function $A \equiv v_2/v_1$ by designating C_1 as an "extra" element.

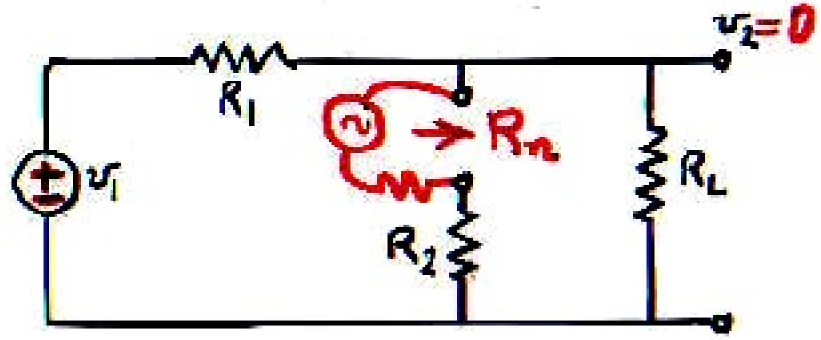


$$\frac{v_2}{v_1} = \frac{R_L}{R_1 + R_L}$$



$$\frac{v_2}{v_1} = \frac{R_L}{R_1 + R_L}$$

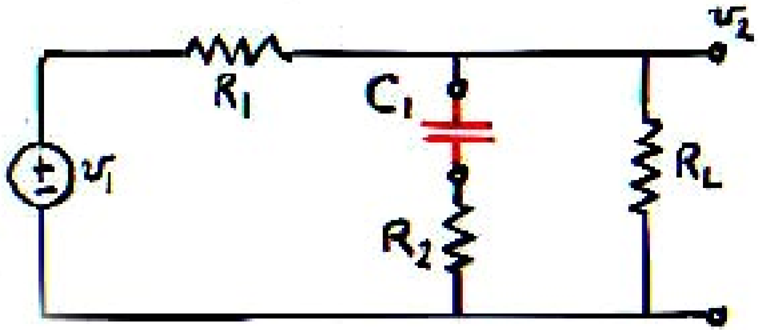
$$R_d = R_2 + R_1 \| R_L$$



$$\frac{v_2}{v_1} = \frac{R_L}{R_1 + R_L}$$

$$R_d = R_2 + R_1 \| R_L$$

$$R_n = R_2$$



$$\frac{v_2}{v_1} = \frac{R_L}{R_1 + R_L}$$

$$R_d = R_2 + R_1 \| R_L$$

$$R_n = R_2$$

$$\frac{v_2}{v_1} = \frac{R_L}{R_1 + R_L} \frac{1 + sC_1 R_2}{1 + sC_1(R_2 + R_1 \| R_L)}$$

The Extra Element Theorem may be used to find an extra element correction factor for <u>any</u> transfer function of a linear circuit. It is necessary merely to identify the "input" and "output" signals; Z_d and Z_n are then calculated as the driving point impedance seen by the extra element with the "input" zero and with the "output" nulled, respectively.

Examples of transfer functions:

"output" → current drawn from power supply
"input" → input voltage

$$\frac{\text{current drawn from power supply}}{\text{input voltage}}$$

(a transadmittance)

"output" → output voltage ripple component
"input" → power supply ripple voltage

$$\frac{\text{output voltage ripple component}}{\text{power supply ripple voltage}}$$

(a voltage gain; audio susceptibility of a power supply)

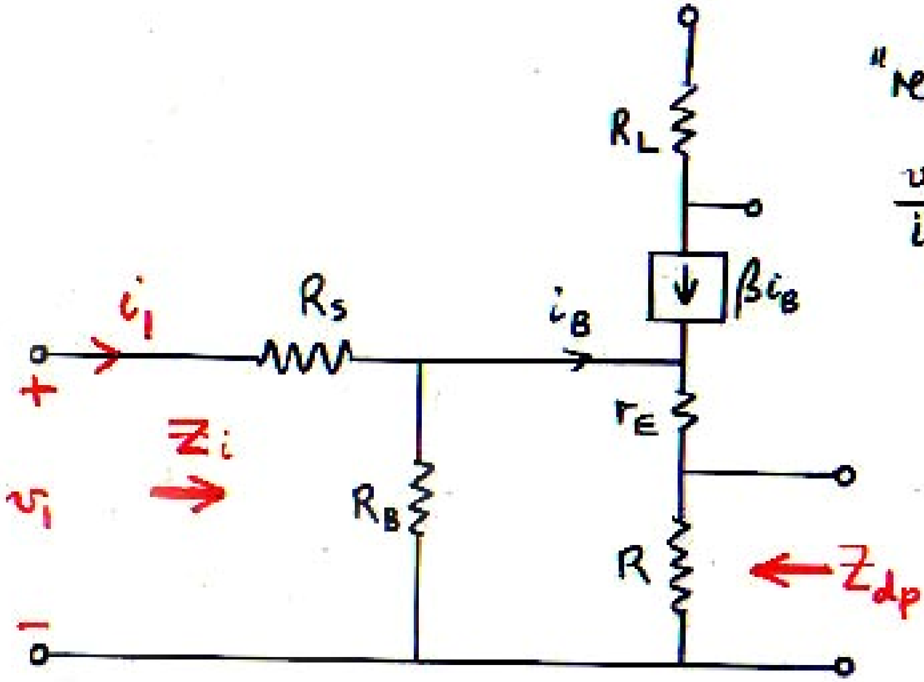
"output" → corresponding driving voltage
"input" → any driving current

$$\frac{\text{corresponding driving voltage}}{\text{any driving current}}$$

(a self-impedance, e.g. input or output impedance)

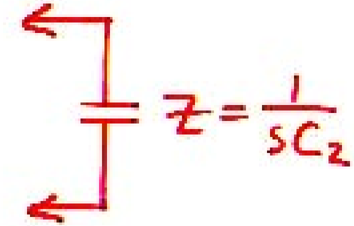
Example: Input impedance Z_i of a CE amplifier stage with emitter bypass capacitance as "extra" element.

"Reference circuit":



"reference transfer function":

$$\left.\frac{v_1}{i_1}\right|_{Z=\infty} = \left.Z_i\right|_{Z=\infty} = R_S + R_B \| (1+\beta)(r_E+R)$$



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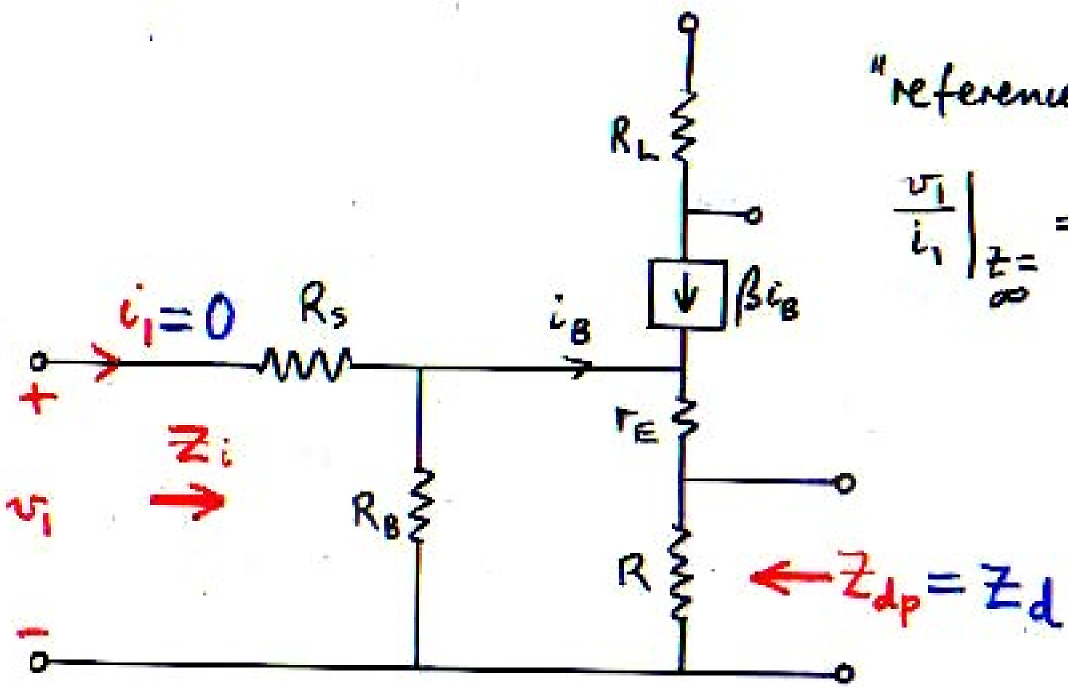

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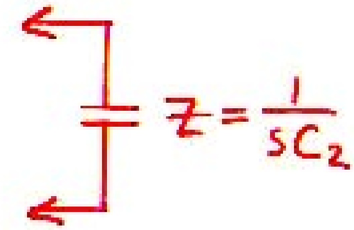

$$Z_d = \left. Z_{dp} \right|_{\text{"input" zero}} = \left. Z_{dp} \right|_{i_1 = 0} = R_d = R \,\|\, \left(r_E + \frac{R_B}{1+\beta} \right)$$

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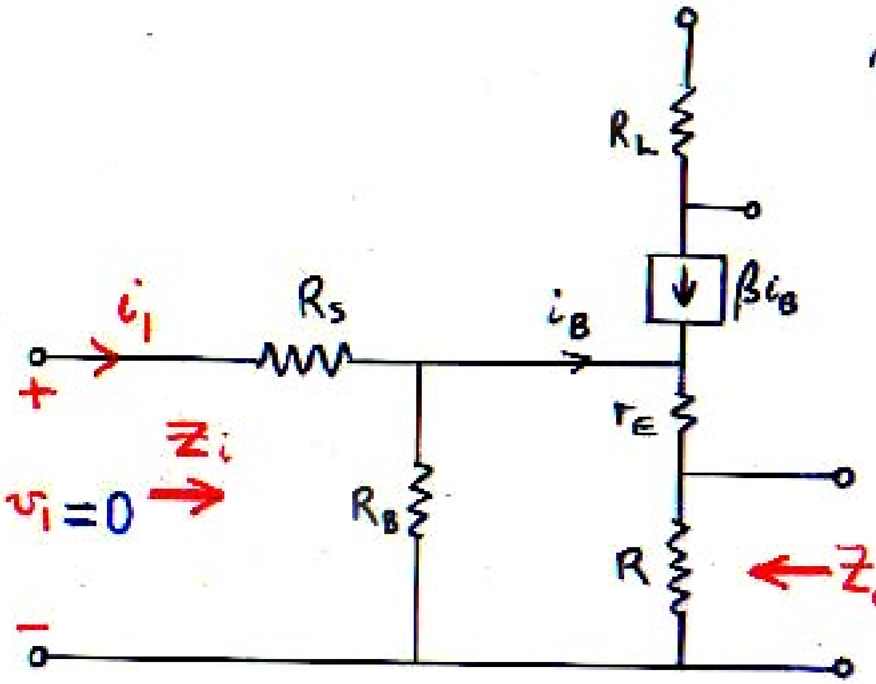

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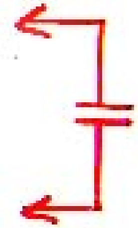
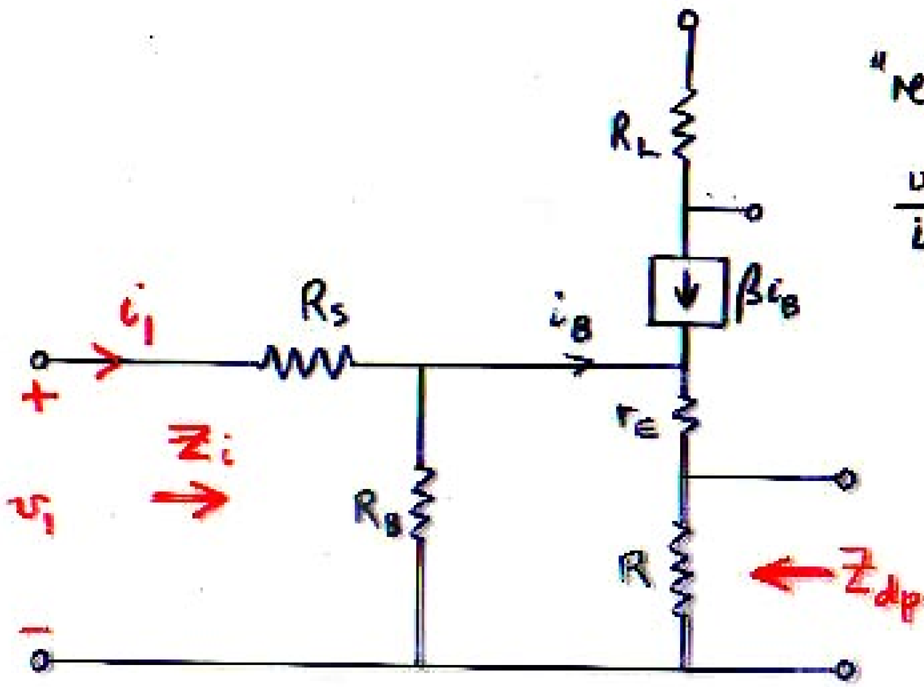


$$Z = \frac{1}{sC_2}$$

$$Z_n = \left. Z_{dp} \right|_{\substack{\text{"output"} \\ \text{nulled}}} = \left. Z_{dp} \right|_{v_1 = 0} = R_n = R \,\Big\|\, \left(r_E + \frac{R_S \| R_B}{1+\beta} \right)$$

Example: Input impedance Z_i of a CE amplifier stage with emitter bypass capacitance as "extra" element.

"Reference circuit":



"reference transfer function":

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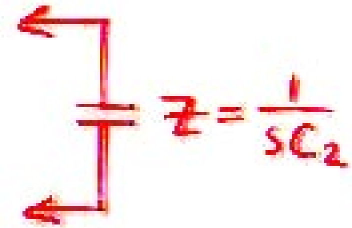
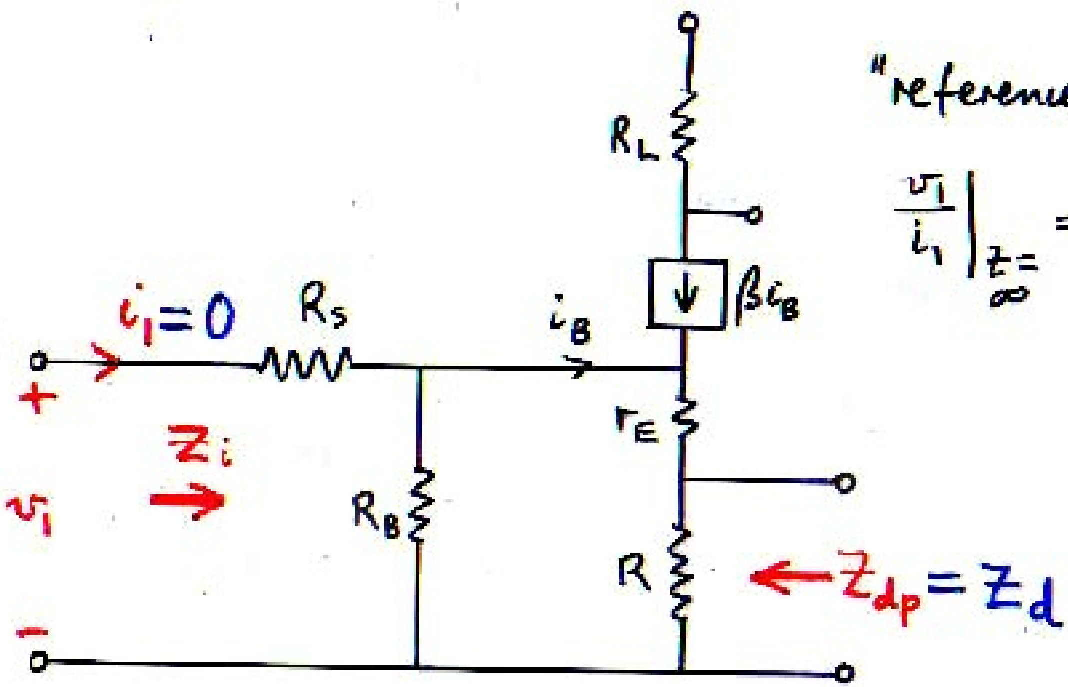


Hence, $$Z_i = \left[R_s + R_B \| (1+\beta)(r_E+R)\right] \frac{1+sC_2R_n}{1+sC_2R_d}$$

Example: Input impedance Z_i of a CE amplifier stage with emitter bypass capacitance as "extra" element.

"Reference circuit":



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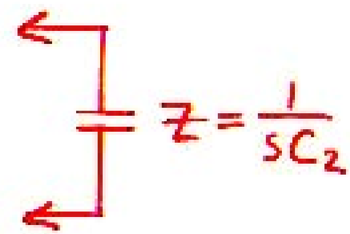
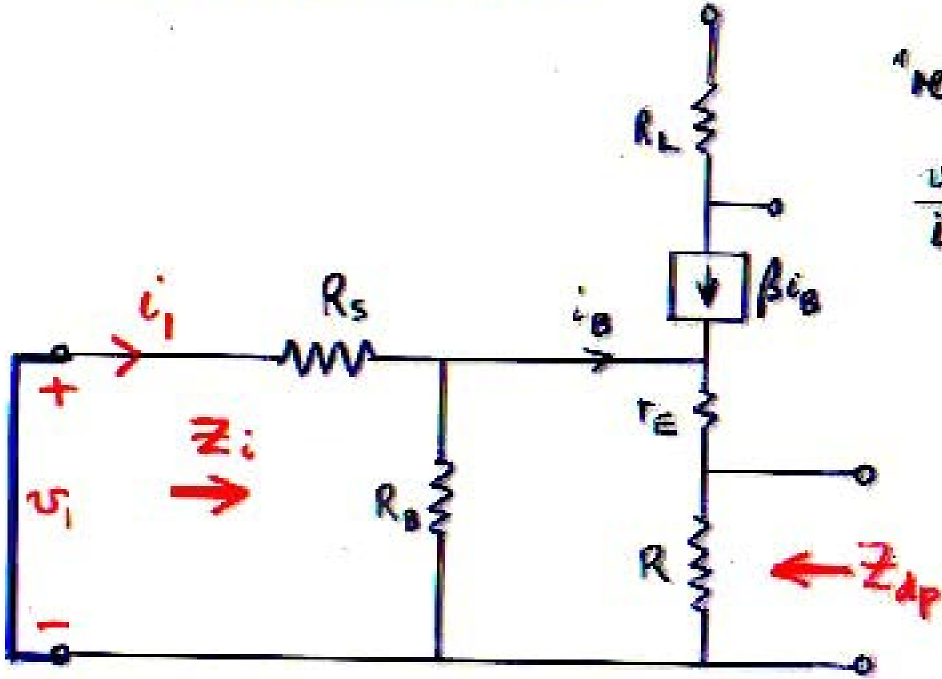


$$Z_d = \left.Z_{dp}\right|_{\text{"input" zero}} = \left.Z_{dp}\right|_{i_1=0} = R_d = R \| \left(r_E + \frac{R_B}{1+\beta}\right)$$

"Reference circuit":



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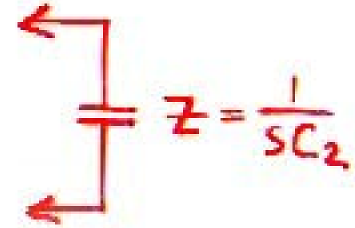
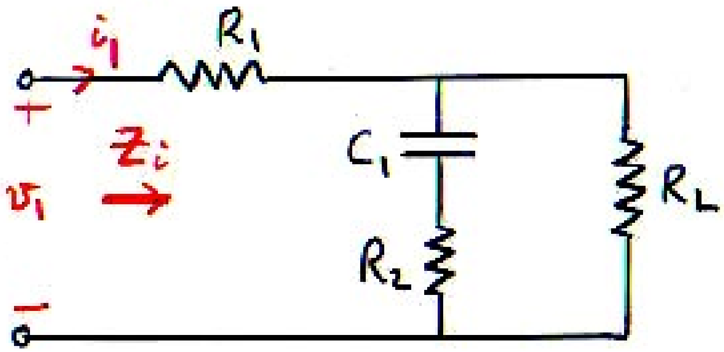


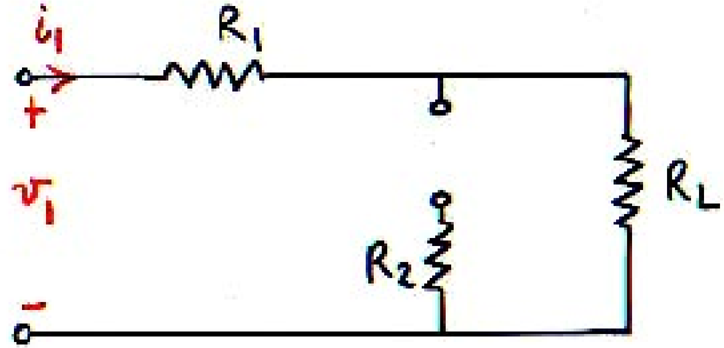
NOTE: In the special case of a self-impedance, nulling the "output" voltage is the same as shorting the "input" current, because the "output" and "input" are at the same node pair.

Example: Lag-lead network



Find the input impedance $Z_i = v_1 / i_1$ by designating C_1 as an "extra" element.

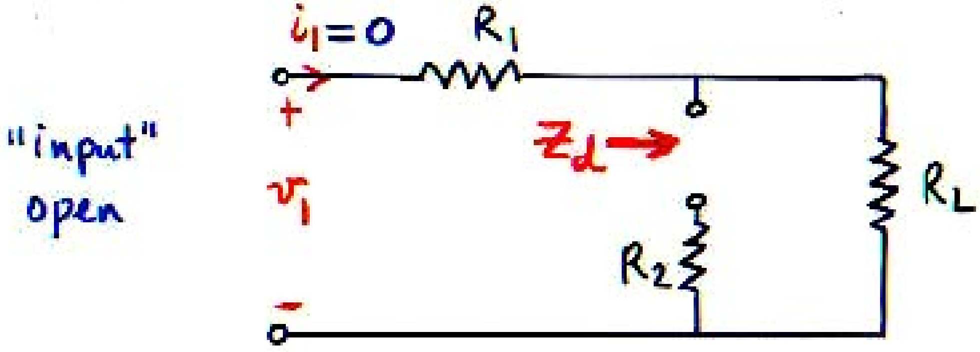
"Reference circuit:"



"Reference" input impedance:

$$Z_i\Big|_{Z=\infty} = \frac{v_1}{i_1}\Big|_{Z=\infty} = R_1 + R_L$$

"Reference circuit:"

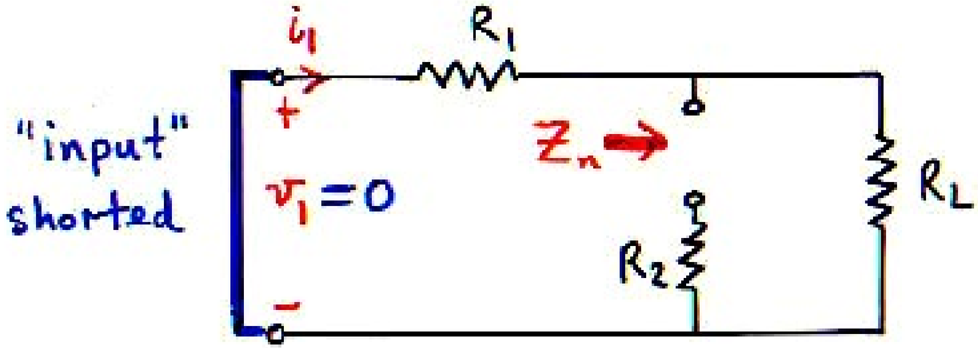


"Reference" input impedance:

$$Z_i\Big|_{Z=\infty} = \frac{v_1}{i_1}\Big|_{Z=\infty} = R_1 + R_L$$

$$Z_d = R_d = R_2 + R_L$$

"Reference circuit:"



"Reference" input impedance:

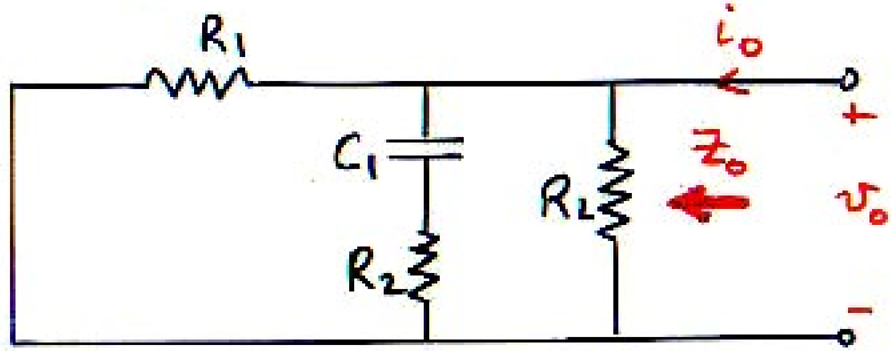
$$Z_i\Big|_{z=\infty} = \frac{v_1}{i_1}\Big|_{z=\infty} = R_1 + R_L$$

$$Z_n = R_n = R_2 + R_1 \| R_L$$

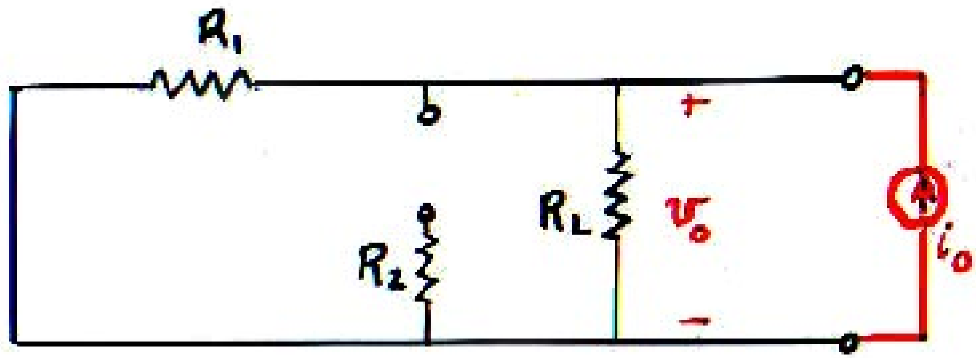
Hence:

$$Z_i = (R_1 + R_L)\,\frac{1 + sC_1R_n}{1 + sC_1R_d}$$

Exercise: Lag-lead network

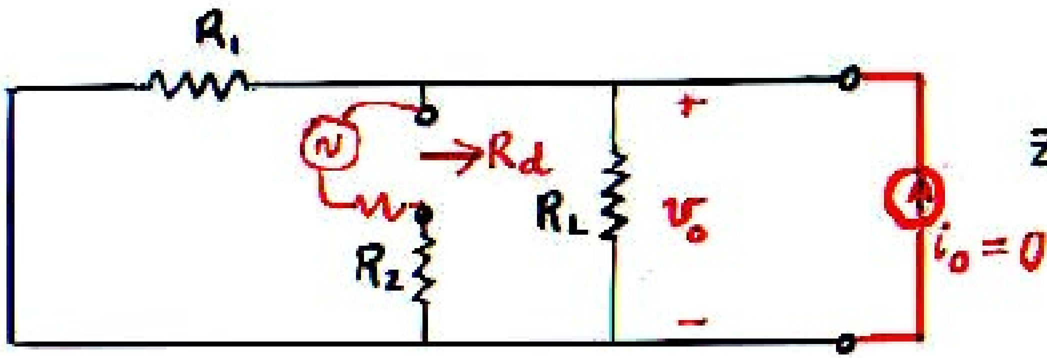


Find the output impedance $Z_o = v_o / i_o$ by designating C_1 as an "extra" element.



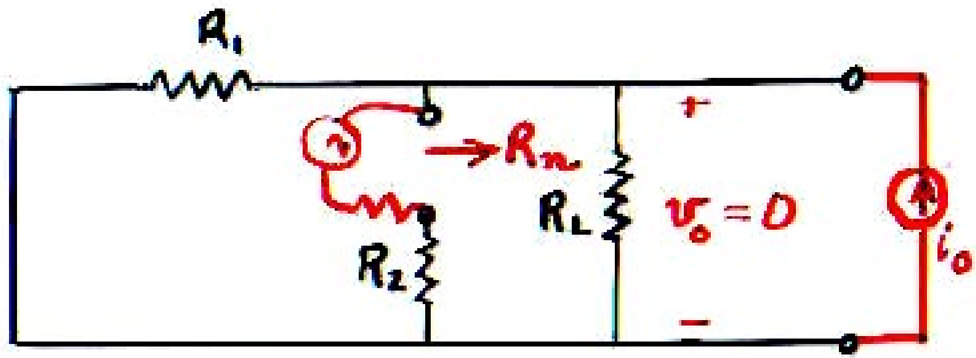
$$Z_o\Big|_{Z=\infty} \equiv \frac{v_o}{i_o}\Big|_{Z=\infty}$$

$$= R_1 \| R_L$$



$$Z_o\Big|_{Z=\infty} \equiv \frac{v_o}{i_o}\Big|_{Z=\infty} = R_1 \| R_L$$

$$R_d = R_2 + R_1 \| R_L$$



$$Z_o\Big|_{Z=\infty} \triangleq \frac{v_o}{i_o}\Big|_{Z=\infty}$$

$$= R_1 \| R_L$$

$$R_d = R_2 + R_1 \| R_L$$

$$R_n = R_2$$

With C_1 replaced:

$$Z_o = R_1 \| R_L \, \frac{1 + sC_1 R_2}{1 + sC_1 (R_2 + R_1 \| R_2)}$$

Generalization: Extra Element Theorem - #4

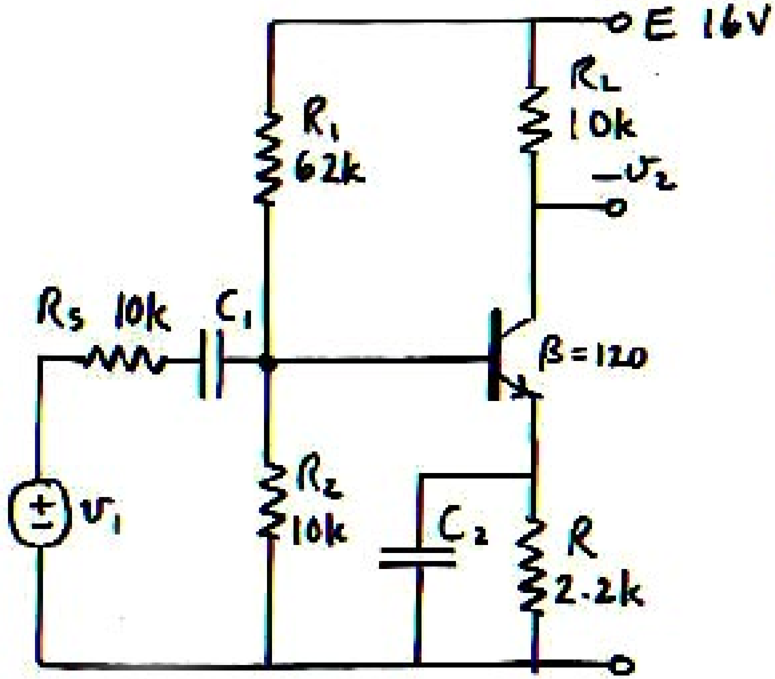
The Extra Element theorem can be used to find an extra element correction factor for *any* transfer function; Z_d and Z_n are then the driving point impedances seen by the extra element with the "input" zero and with the "output" nulled, respectively.

When the transfer function is a self-impedance, such as the input impedance Z_i or the output impedance Z_o, nulling the "output" is the same as shorting the "input," hence

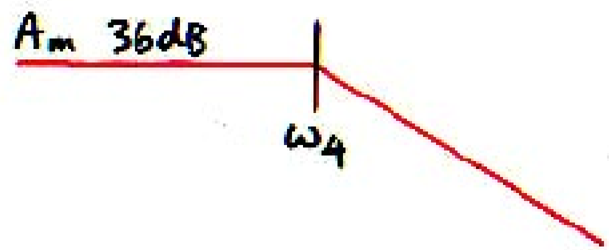
$$Z_d = Z_{dp}\Big|_{\substack{\text{"input"}\\ \text{zero}}} = Z_{dp}\Big|_{\substack{\text{"input"}\\ \text{open}}}$$

$$Z_n = Z_{dp}\Big|_{\substack{\text{"output"}\\ \text{nulled}}} = Z_{dp}\Big|_{\substack{\text{"input"}\\ \text{shorted}}}$$

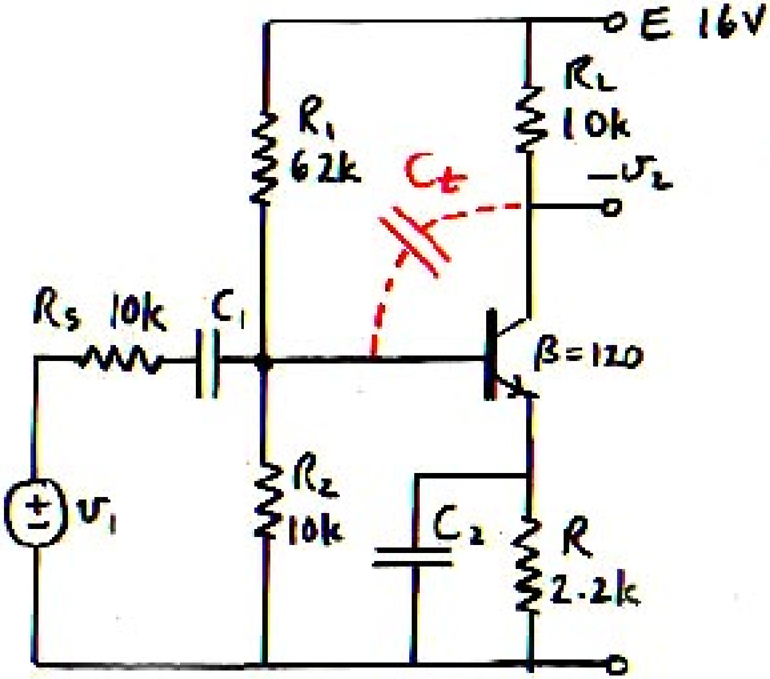
High-frequency properties of CE amplifier



Measurement indicates that there is a high-frequency pole ω_4:



High-frequency properties of CE amplifier



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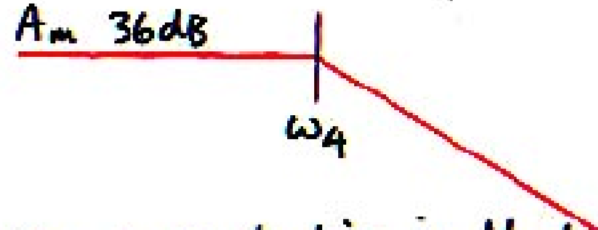
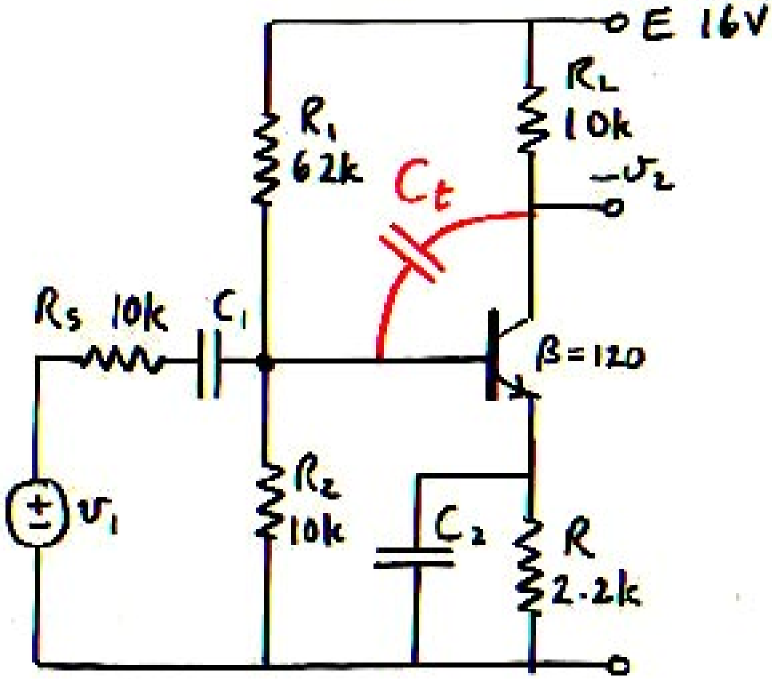


The expectation is that this is caused by the collector-base transition-layer capacitance C_t.

A typical value is $C_t = 5pF$. The resulting corner frequency with $R_L = 10k$ is $159/5 \times 10^{-6} \times 10 = 3.2MHz$. Since the actual corner frequency is much lower, there must be a multiplying effect on C_t resulting from its connection to the transistor base instead of to ground.

High-frequency properties of CE amplifier



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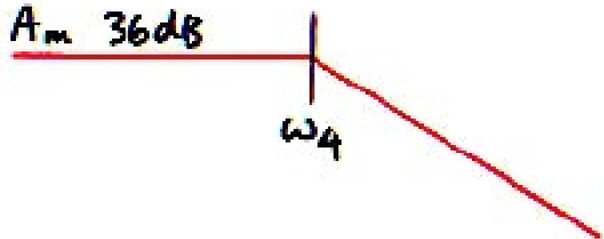
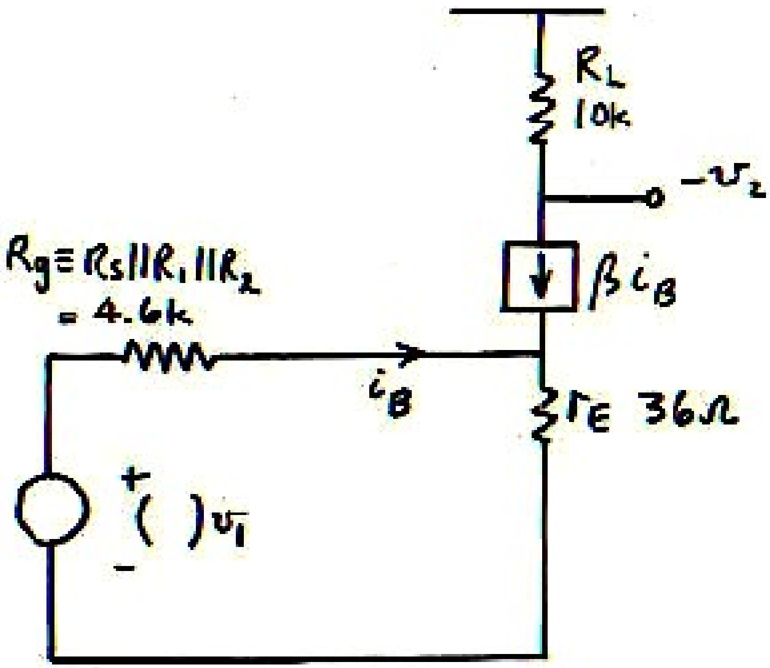
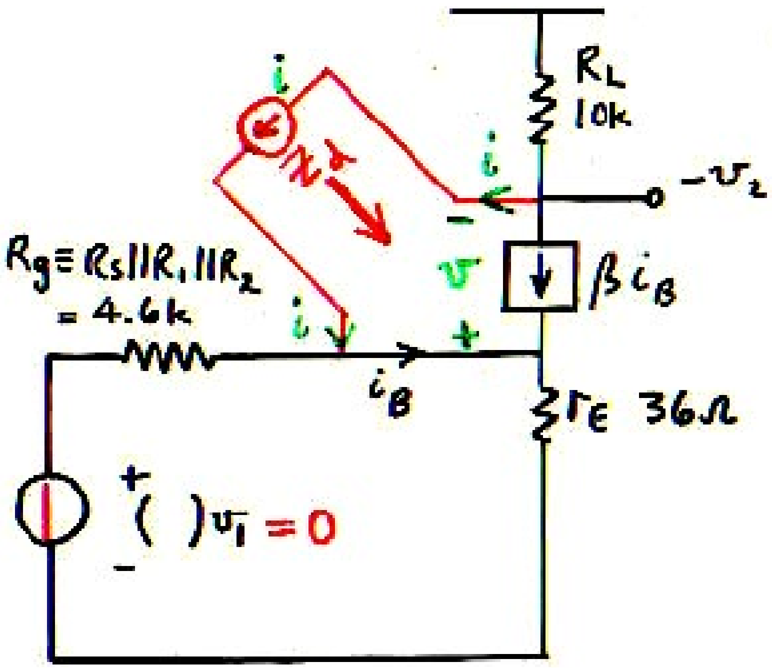


Since the midband gain $A_m = 36\text{dB}$ has already been determined, use the Extra Element Theorem to find the correction factor resulting from inclusion of $Z \to 1/sC_t$.

Midband model after Thevenin reduction of R_S, R_1, R_2:

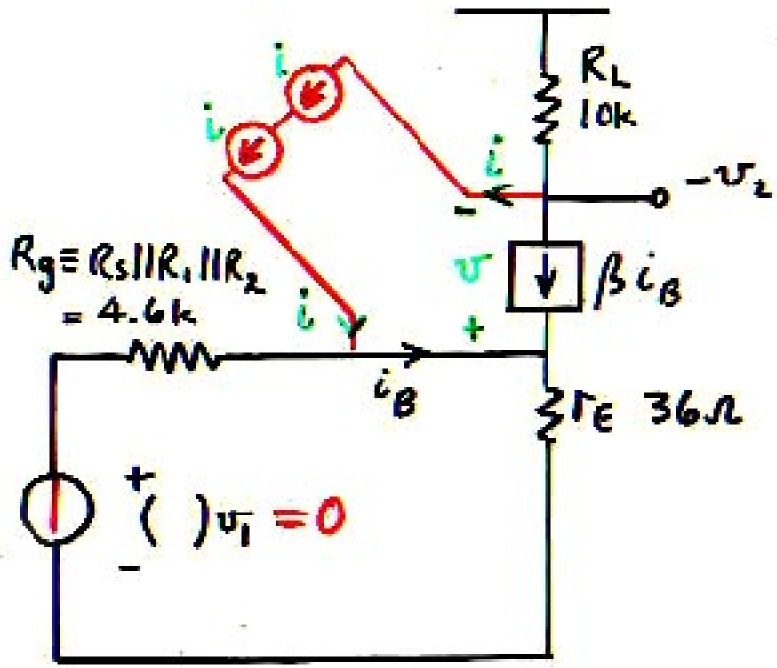


Midband model after Thevenin reduction of R_s, R_1, R_2:



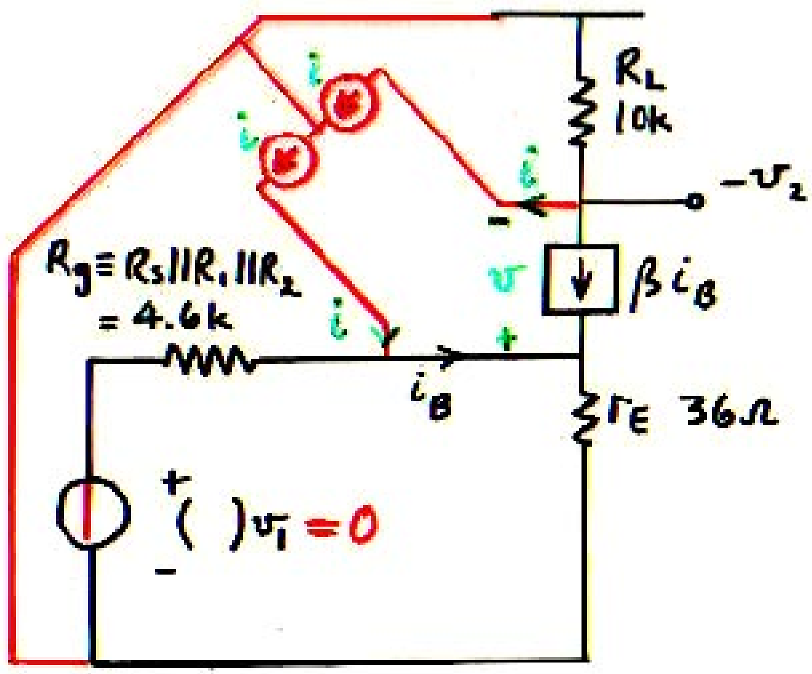
$$Z_d = \left.\frac{v}{i}\right|_{v_1 = 0}$$

Midband model after Thevenin reduction of R_S, R_1, R_2:



The current generator i can be divided into two equal current generators in series.

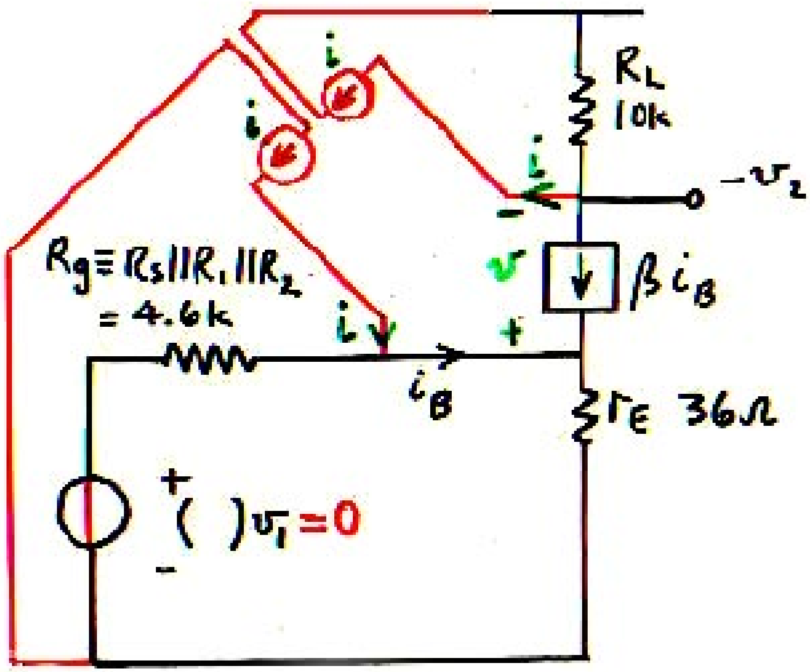
Midband model after Thevenin reduction of R_S, R_1, R_2:



The current generator i can be divided into two equal current generators in series.

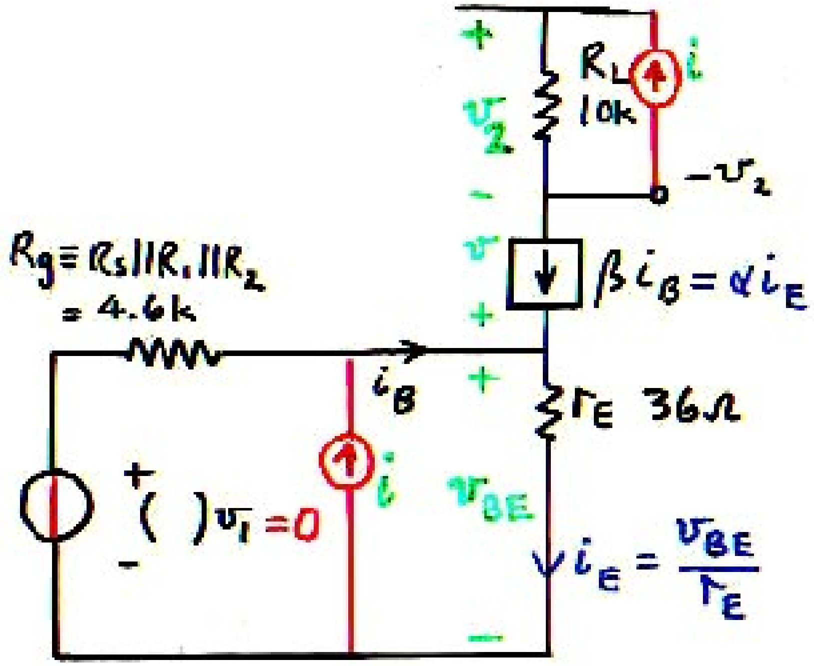
Since the voltage at the junction of the two current generators i is immaterial, the junction can be grounded.

Midband model after Thevenin reduction of R_S, R_1, R_2:



A separate ground can be identified for each current generator i.

Midband model after Thevenin reduction of R_S, R_1, R_2:



Rearranged diagram.

$$Z_d = R_d = \frac{v}{i} = \frac{v_{BE}}{i} + \frac{v_2}{i}$$

$$v_{BE} = \left[R_g \| (1+\beta) r_E\right] i$$

$$v_2 = R_L (\alpha i_E + i) = R_L \left(\frac{\alpha}{r_E} v_{BE} + i\right)$$

$$R_d = \frac{v_{BE}}{i} + R_L \left(\frac{\alpha}{r_E} \frac{v_{BE}}{i} + 1\right)$$

$$R_d = \left(1 + \frac{\alpha R_L}{r_E}\right)\left[R_g \| (1+\beta) r_E\right] + R_L = R_L \left[R_g \| (1+\beta) r_E\right]\left[\frac{1}{R_L} + \frac{\alpha}{r_E} + \frac{1}{R_g \| (1+\beta) r_E}\right]$$

$$= R_L \left[R_g \| (1+\beta) r_E\right]\left[\frac{1}{R_L} + \frac{\beta}{(1+\beta) r_E} + \frac{1}{R_g} + \frac{1}{(1+\beta) r_E}\right]$$

$$= \frac{R_g \| (1+\beta) r_E}{R_g \| r_E \| R_L} R_L = \frac{4.6 \| 4.3}{4.6 \| 0.036 \| 10} R_L = 62 R_L$$

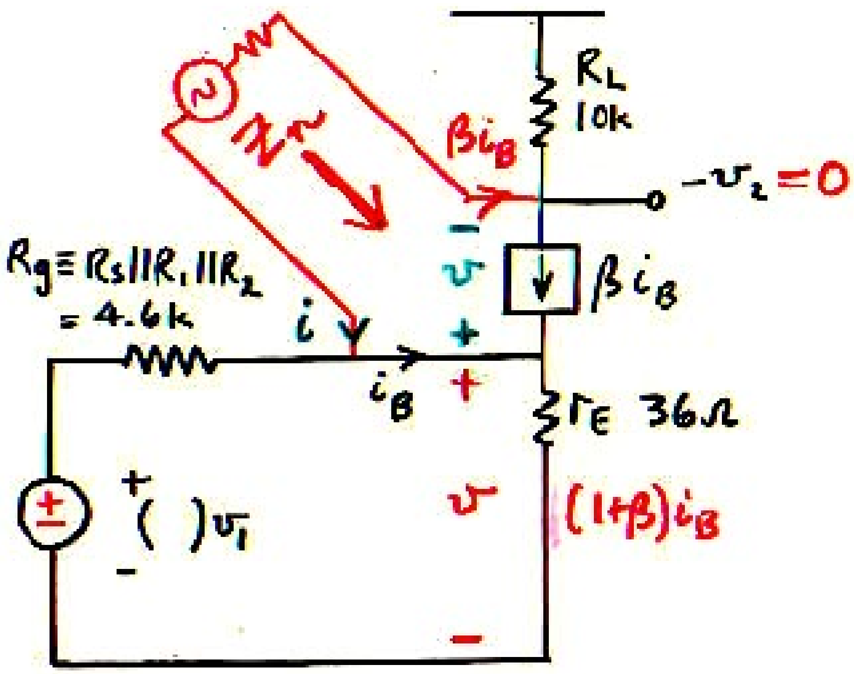
$$= 620k$$

Generalization: Floating Current Generator

A floating current generator can be replaced by two separate, equal, grounded current generators.

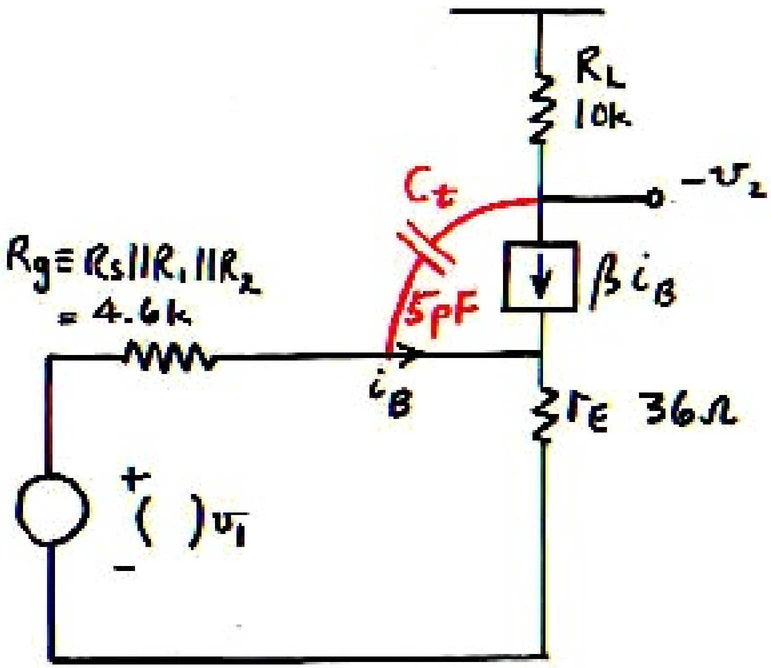
This is a useful technique in "doing the algebra on the circuit diagram."

Midband model after Thevenin reduction of R_S, R_1, R_2:



$$Z_n = R_n = \frac{v}{i} = \frac{(1+\beta) r_E \, i_B}{-\beta i_B} = -\frac{r_E}{\alpha} = -36\Omega$$

Midband model after Thevenin reduction of R_s, R_1, R_2:



Hence corrected gain after inclusion of C_t is

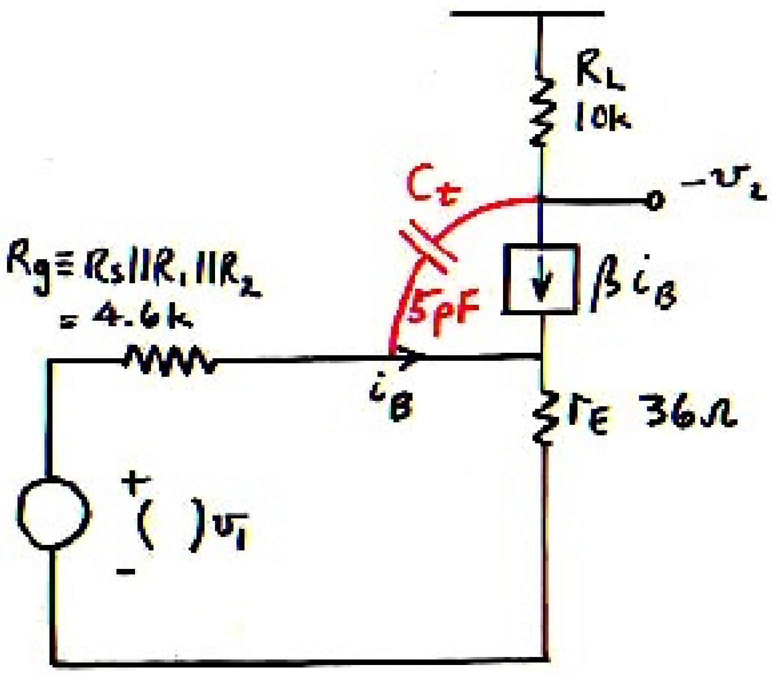
$$A = A_m \frac{1 + \frac{Z_n}{Z}}{1 + \frac{Z_d}{Z}} = A_m \frac{1 + sC_t R_n}{1 + sC_t R_d}$$

$$= A_m \frac{1 - \frac{s}{\omega_5}}{1 + \frac{s}{\omega_4}} \quad \text{where}$$

$$\omega_4 \equiv \frac{1}{C_t R_d} \qquad f_4 = \frac{159}{5 \times 10^{-6} \times 620} = 51\,\text{kHz}$$

$$\omega_5 \equiv \frac{1}{C_t R_n} \qquad f_5 = \frac{159}{5 \times 10^{-6} \times 0.036} = 880\,\text{MHz}$$

Midband model after Thevenin reduction of R_s, R_1, R_2:



Hence corrected gain after inclusion of C_t is

$$A = A_m \frac{1 + \frac{Z_n}{Z}}{1 + \frac{Z_d}{Z}} = A_m \frac{1 + sC_tR_n}{1 + sC_tR_d}$$

$$= A_m \frac{1 - \frac{s}{\omega_5}}{1 + \frac{s}{\omega_4}} \quad \text{where}$$

$$\omega_4 \equiv \frac{1}{C_tR_d} \qquad f_4 = \frac{159}{5\times10^{-6}\times 620} = 51\,\text{kHz}$$

$$\omega_5 \equiv \frac{1}{C_tR_n} \qquad f_5 = \frac{159}{5\times10^{-6}\times 0.036} = 880\,\text{MHz}$$

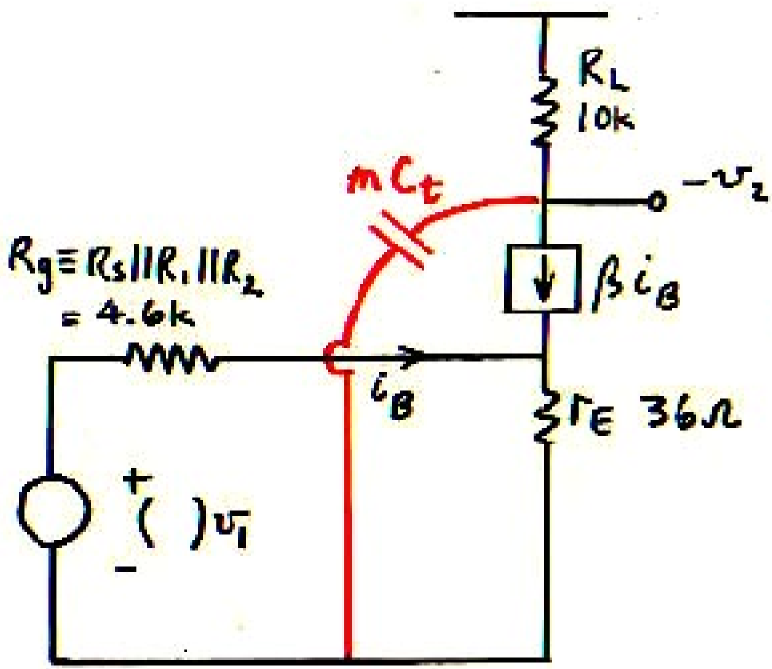
Note that the zero $\omega_5 = \frac{1}{C_tR_n} = \frac{\alpha}{C_tr_E}$ is negative (right half-plane), and is at a very high frequency unless there is substantial external emitter resistance and/or there is substantial external collector-base capacitance (as often exists).

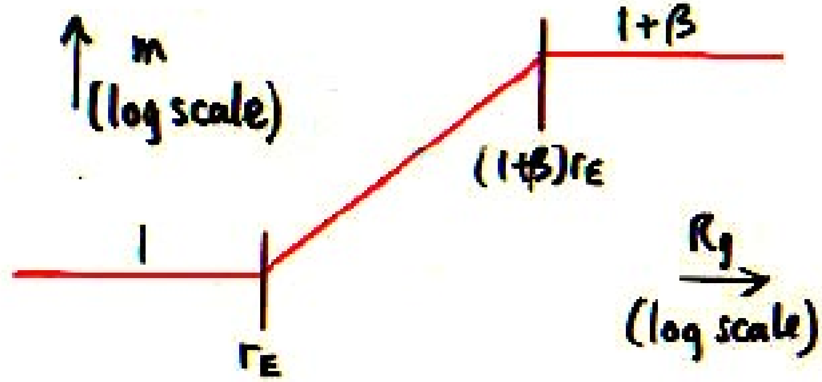
Note that the pole $\omega_4 = \frac{1}{C_t R_d} = \frac{1}{C_t R_L} \frac{R_g \| r_E \| R_L}{R_g \| (1+\beta) r_E}$ is at a much lower frequency than $\omega_t \equiv \frac{1}{C_t R_L}$, and can be ascribed to an effective multiplication of C_t by a factor

$$m \equiv \frac{R_g \| (1+\beta) r_E}{R_g \| r_E \| R_L} = \frac{R_g \| (1+\beta) r_E}{R_g \| r_E} \left(1 + \cancel{\frac{R_g \| r_E}{R_L}}\right) \xrightarrow{R_g \gg (1+\beta) r_E} 1+\beta$$

$$\xrightarrow{R_g \ll r_E} 1$$

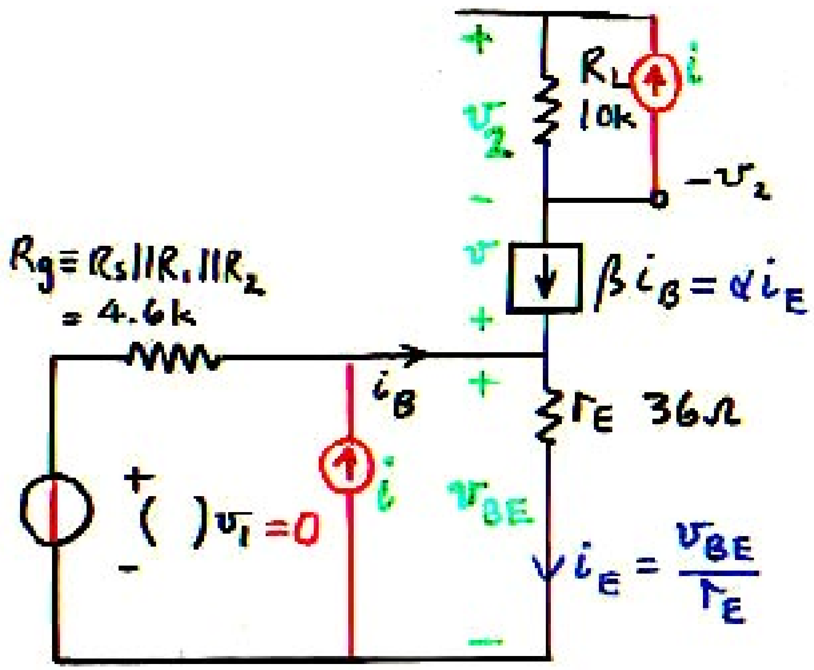
Midband model after Thevenin reduction of R_S, R_1, R_2:





This example: m = 62

Midband model after Thevenin reduction of R_S, R_1, R_2:



Rearranged diagram.

$$Z_d = R_d = \frac{v}{i} = \frac{v_{BE}}{i} + \frac{v_2}{i}$$

$$v_{BE} = \left[R_g \| (1+\beta) r_E\right] i$$

$$v_2 = R_L(\alpha i_E + i) = R_L\left(\frac{\alpha}{r_E} v_{BE} + i\right)$$

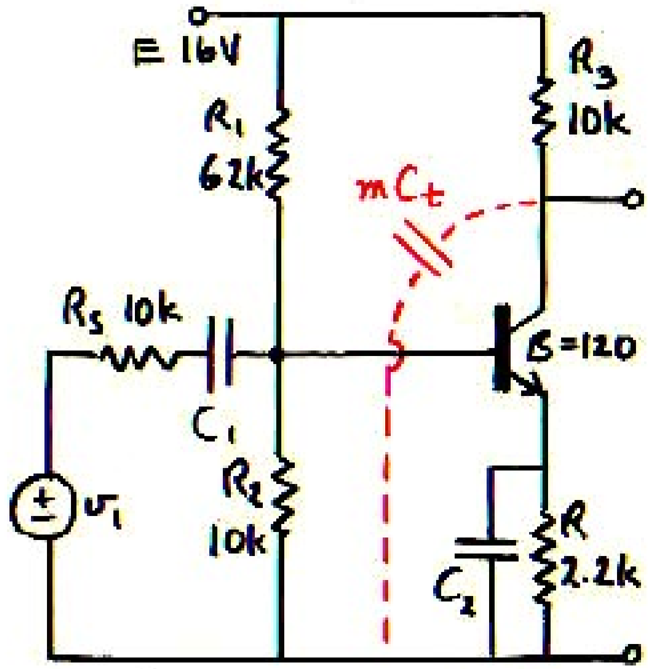
$$R_d = \frac{v_{BE}}{i} + R_L\left(\frac{\alpha}{r_E}\frac{v_{BE}}{i} + 1\right)$$

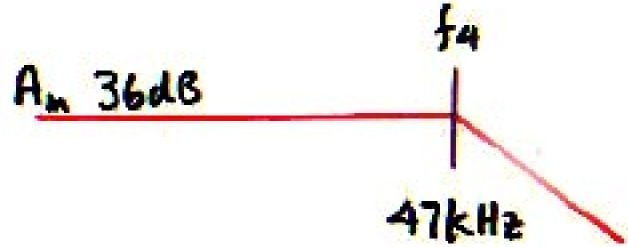
$$R_d = \left(1 + \frac{\alpha R_L}{r_E}\right)\left[R_g \| (1+\beta) r_E\right] + R_L = R_L\left[R_g \| (1+\beta) r_E\right]\left[\frac{1}{R_L} + \frac{\alpha}{r_E} + \frac{1}{R_g \| (1+\beta) r_E}\right]$$

$$= R_L\left[R_g \| (1+\beta) r_E\right]\left[\frac{1}{R_L} + \frac{\beta}{(1+\beta) r_E} + \frac{1}{R_g} + \frac{1}{(1+\beta) r_E}\right]$$

$$= \frac{R_g \| (1+\beta) r_E}{R_g \| r_E \| R_L} R_L = \frac{4.6 \| 4.3}{4.6 \| 0.036 \| 10} R_L = 62 R_L$$

$$= 620k$$





By measurement, $f_4 = 47\text{kHz}$; hence

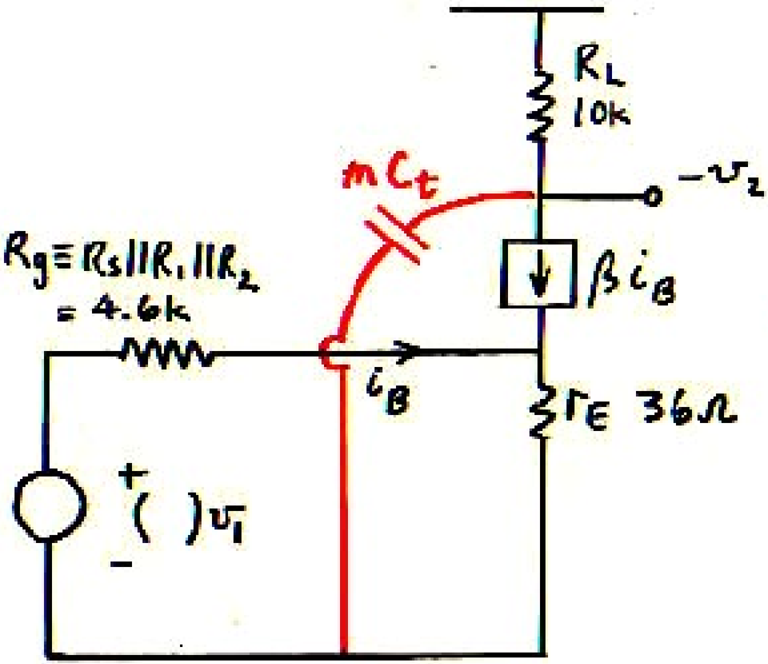
$$mC_t = \frac{159}{0.047 \times 10} = 340\text{pF}$$

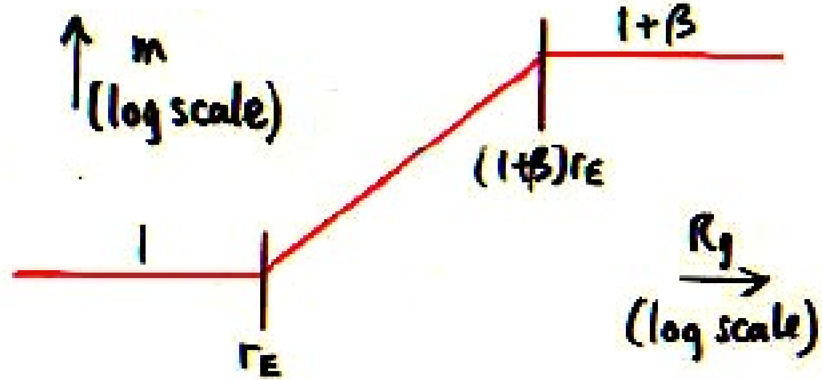
So:

$$C_t = 340/62 = 5.5\text{pF}$$

(Note: 13 pF of 10:1 scope probe is negligible)

Midband model after Thevenin reduction of R_s, R_1, R_2:





This example: $m = 62$

Alternative method for calculation of Z_d

There are two forms of the Extra Element theorem:

$$A = A\big|_{Z=\infty} \frac{1+\frac{Z_n}{Z}}{1+\frac{Z_d}{Z}} = A\big|_{Z=0} \frac{1+\frac{Z}{Z_n}}{1+\frac{Z}{Z_d}}$$

where

$$A\big|_{Z=0} = \frac{Z_n}{Z_d} A\big|_{Z=\infty}$$

Hence in general

$$\frac{A\big|_{Z=0}}{A\big|_{Z=\infty}} = \frac{Z_n}{Z_d}$$

It may be easier to find $A\big|_{Z=0}$, $A\big|_{Z=\infty}$, and Z_n than to find Z_d directly.

Example: Addition of collector-base capacitance C_t to the CE amplifier stage. $A\big|_{Z=\infty}$ and Z_n were easily found:

$$A\big|_{Z=\infty} = A_m = \frac{R_B}{R_S+R_B} \cdot \frac{\beta R_L}{R_g+(1+\beta)r_E} \qquad Z_n = R_n = -\frac{r_E}{\alpha}$$

The Extra Element Theorem as derived applies to the correction factor resulting from an extra <u>shunt</u> element.

There is a corresponding form to find the correction factor resulting from an extra <u>series</u> element:

$$\text{gain}\Big|_{Z} = \underbrace{\text{gain}\Big|_{Z=\infty}}_{\text{reference gain}} \frac{1+\frac{Z_n}{Z}}{1+\frac{Z_d}{Z}} = \text{gain}\Big|_{Z=\infty} \frac{\frac{Z_n}{Z}}{\frac{Z_d}{Z}} \frac{\frac{Z}{Z_n}+1}{\frac{Z}{Z_d}+1}$$

$$= \underbrace{\text{gain}\Big|_{Z=0}}_{\text{reference gain}} \frac{1+\frac{Z}{Z_n}}{1+\frac{Z}{Z_d}} = \underbrace{\frac{Z_n}{Z_d}\,\text{gain}\Big|_{Z=\infty}}_{\text{This must be the gain when } Z=0} \frac{1+\frac{Z}{Z_n}}{1+\frac{Z}{Z_d}}$$

Alternative method for calculation of Z_d

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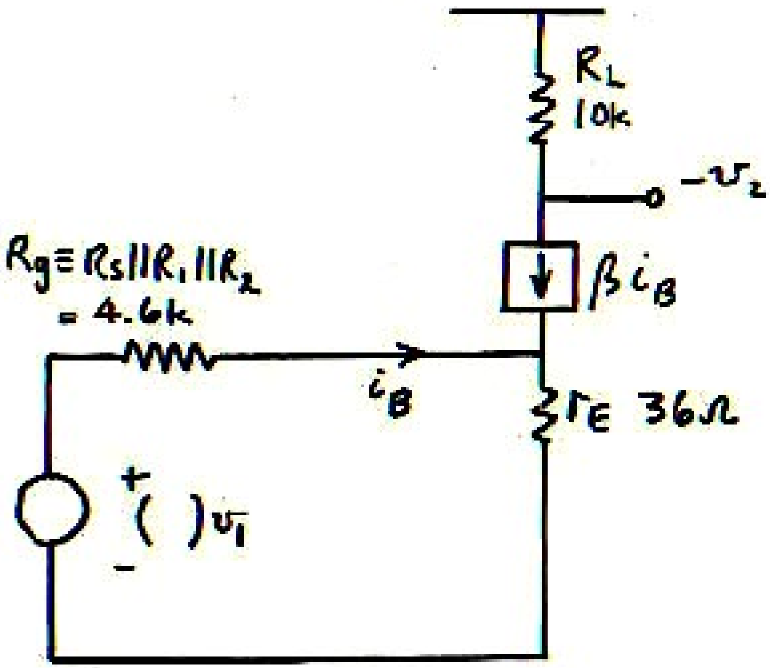
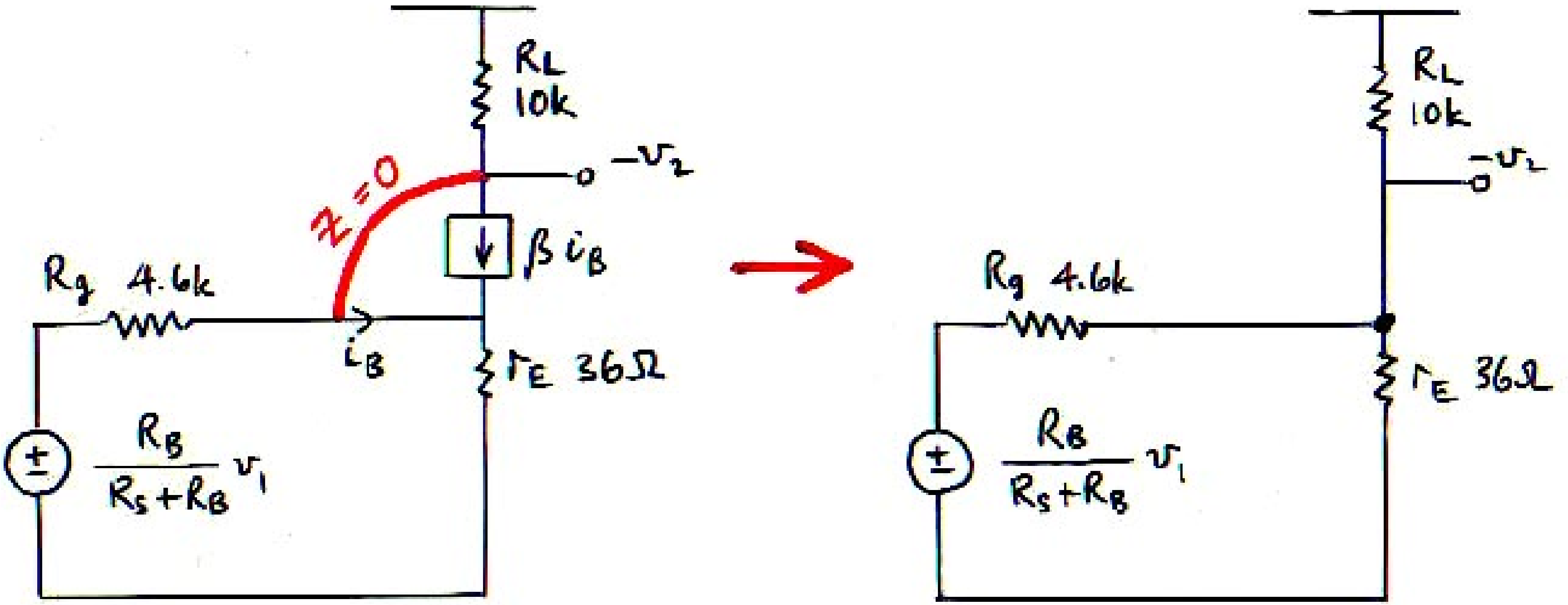
Midband model after Thevenin reduction of R_s, R_1, R_2:



Model for calculation of $A|_{z=0}$



$$A|_{z=0} = -\frac{R_B}{R_s+R_B}\,\frac{r_E \| R_L}{R_g + r_E \| R_L} = -\frac{R_B}{R_s+R_B}\,\frac{R_g \| r_E \| R_L}{R_g}$$

Hence: $$Z_d = R_d = R_n \frac{A|_{z=\infty}}{A|_{z=0}} = \frac{r_E}{\alpha}\,\frac{\beta R_L}{R_g + (1+\beta) r_E}\,\frac{R_g}{R_g \| r_E \| R_L} = \frac{R_g \| (1+\beta) r_E}{R_g \| r_E \| R_L} R_L$$

This is much easier than was the direct calculation of Z_d!

Generalization: Extra Element theorem — #5

The two reference gains and the two driving point impedances are related by:

$$\frac{A\big|_{Z=0}}{A\big|_{Z=\infty}} = \frac{Z_n}{Z_d}$$

One reference gain is always known or is easily found. Z_n is always easier to find than Z_d.

Therefore:

It is often easier to find the other reference gain and to use the above ratio relation for Z_d, than to find Z_d directly.