7

THE INPUT/OUTPUT IMPEDANCE THEOREM

Techniques of Design-Oriented Analysis

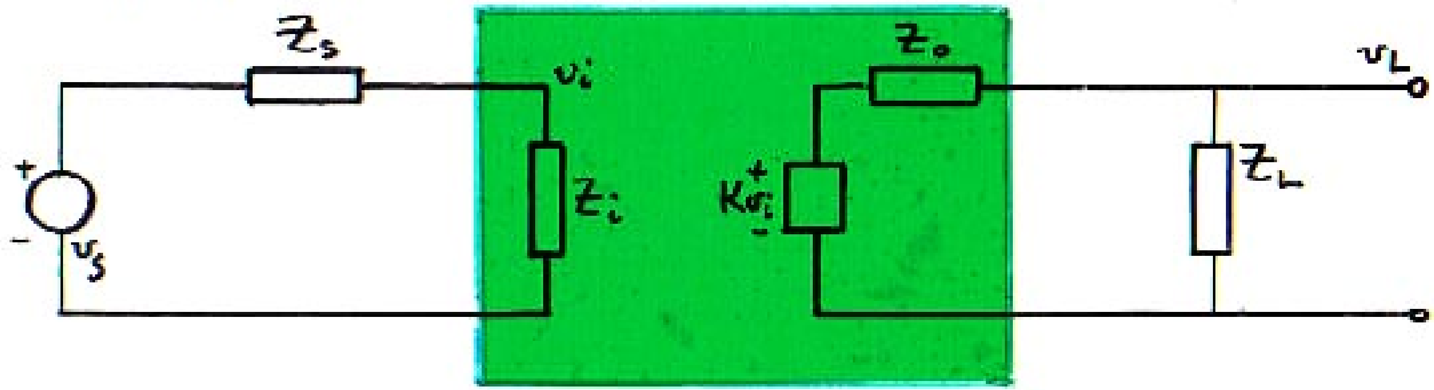
Lowering the Entropy of an expression
Doing the algebra on the circuit diagram.
Doing the algebra on the graph.
Using inverted poles and zeros.
Using numerical values to justify analytic approximations.
Improved formulas for quadratic roots
The Input/Output Impedance Theorem
The Feedback Theorem
Loop gain by injection of a test signal into the closed loop
Measurement of an unstable loop gain
The Extra Element Theorem (EET)

Motivation:

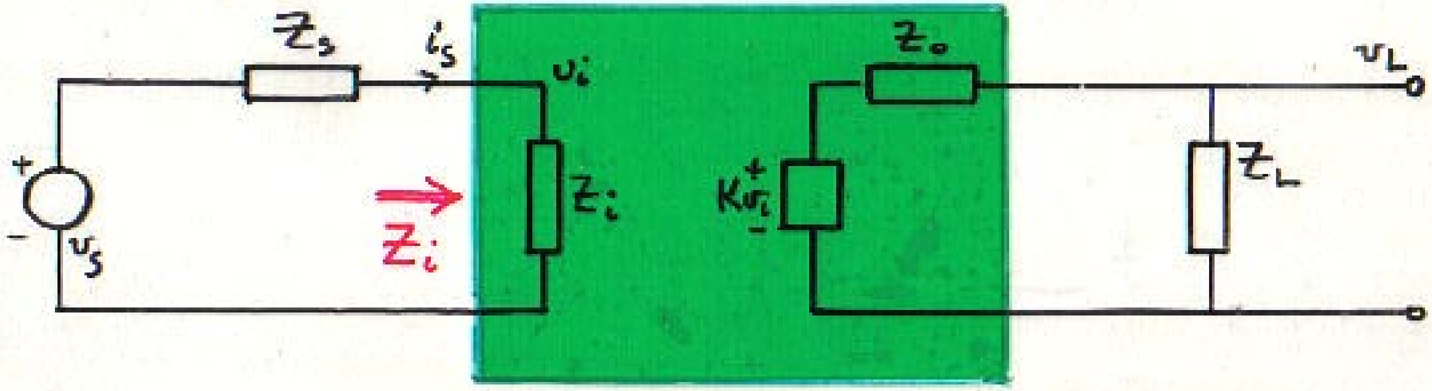
The most important result from analysis of a circuit model is usually the Gain. If the Input and Output impedances are also required, the usual approach is to perform two more separate and independent calculations each of which, especially if feedback is present, can be as long as that for the gain.

The Input/Output impedance theorem eliminates almost two-thirds of the total work by permitting the input and output impedances to be determined by taking simple limits upon the expression already obtained for the gain.

Input/Output Impedance Theorem



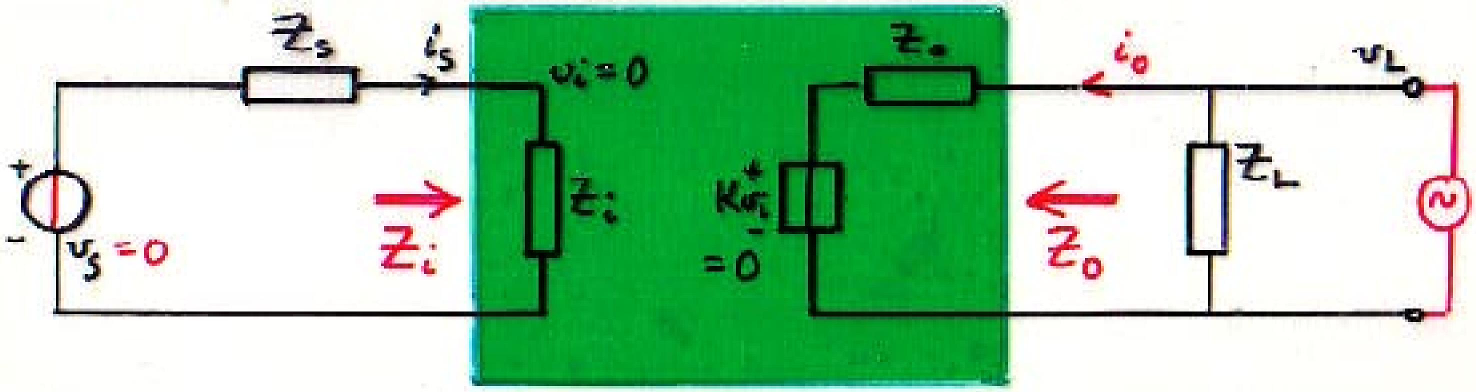
Input/Output Impedance Theorem



voltage gain $A = \frac{v_L}{v_s}$

input impedance $Z_i = \frac{v_i}{i_s}$

Input/Output Impedance Theorem



voltage gain $A = \frac{v_L}{v_s}$

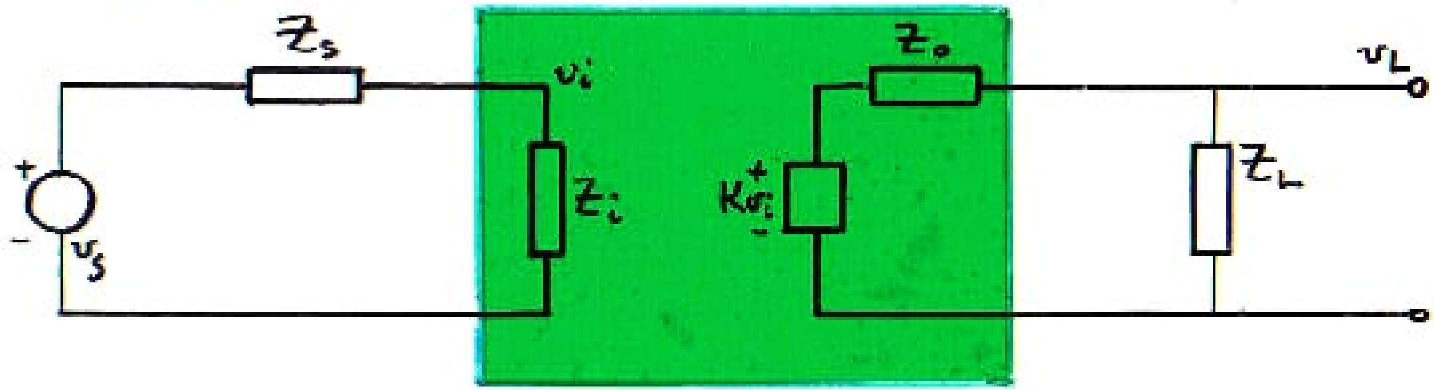
input impedance $Z_i = \frac{v_i}{i_s}$

output impedance $Z_o = \left.\frac{v_L}{i_o}\right|_{v_s=0}$

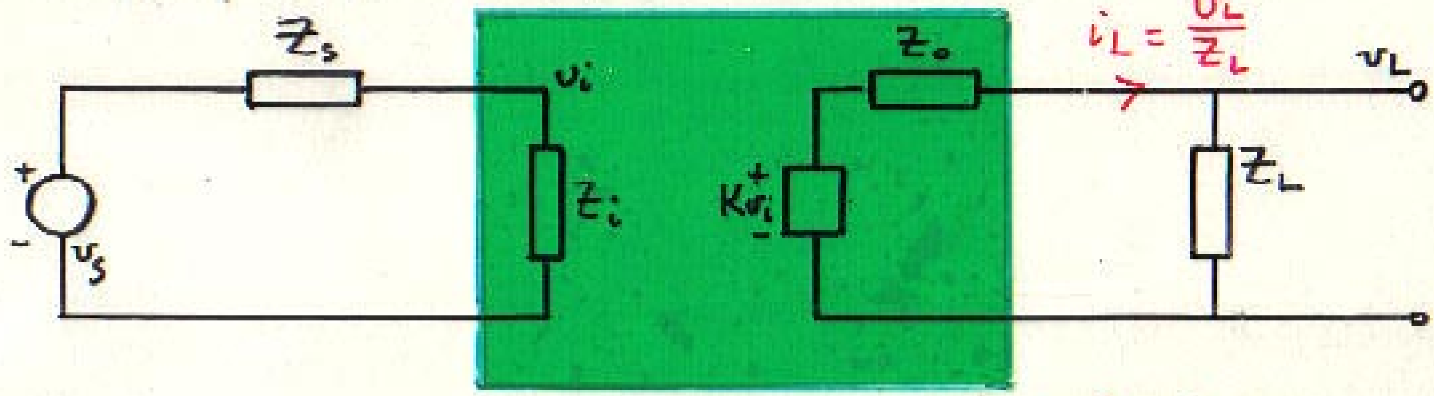
Three separate lengthy calculations are required.

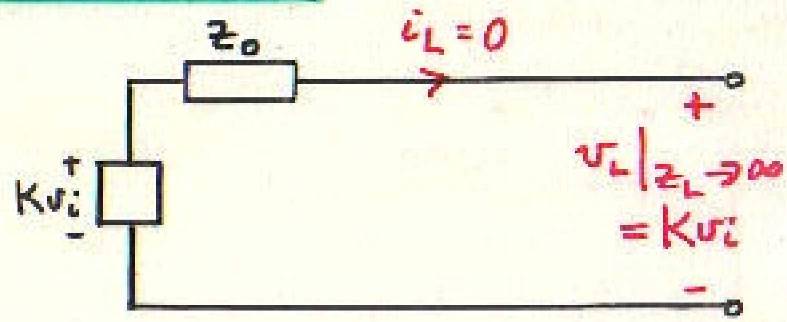
The Theorem permits Z_o and Z_i to be determined directly from the gain A, so only one lengthy calculation is required.

Input/Output Impedance Theorem

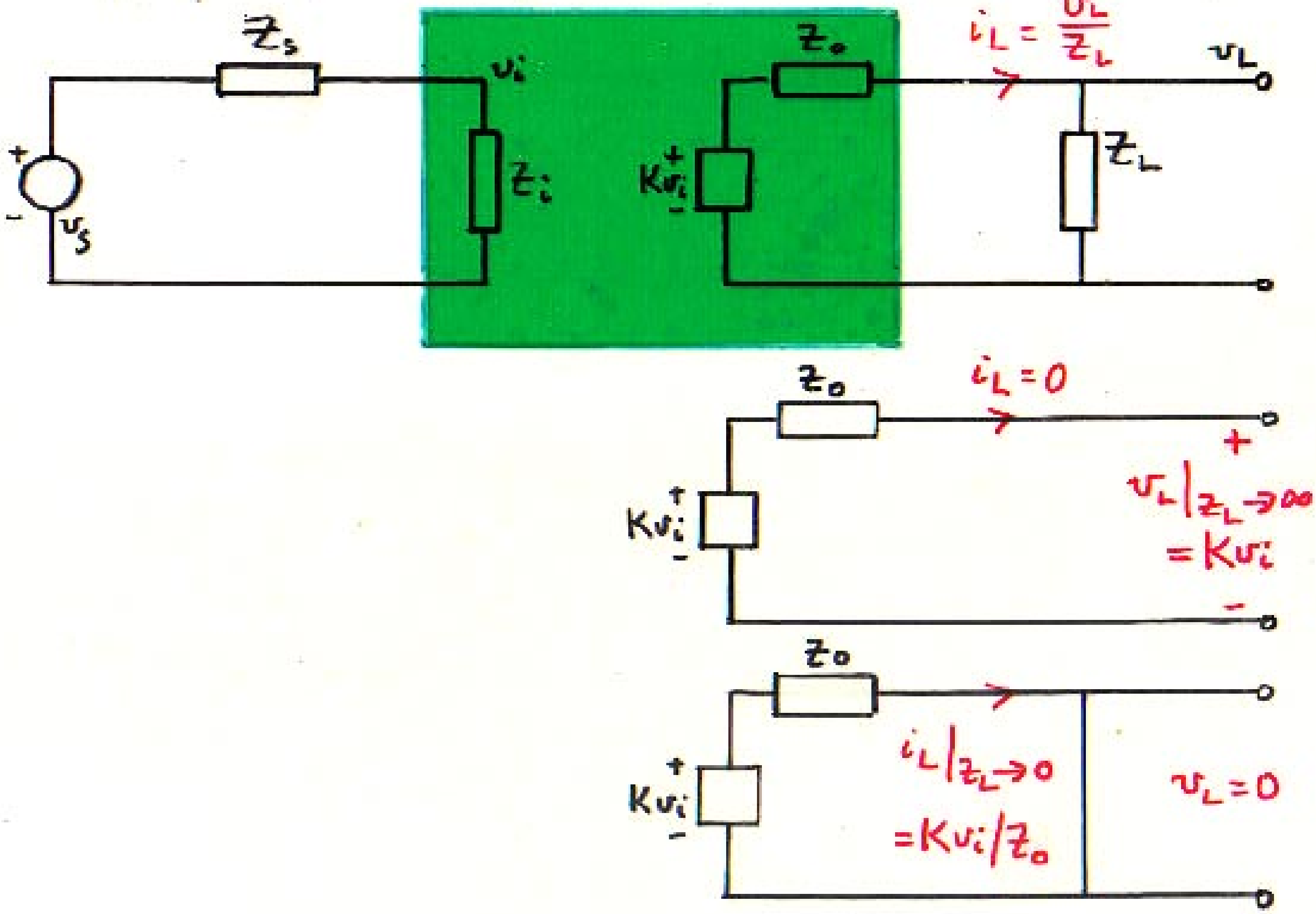


Input/Output Impedance Theorem

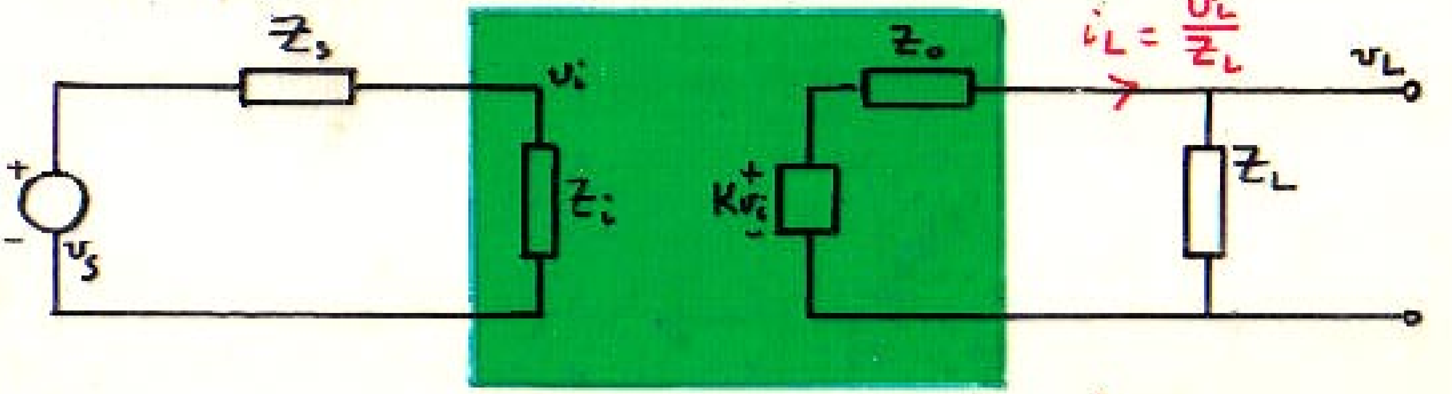




Input/Output Impedance Theorem

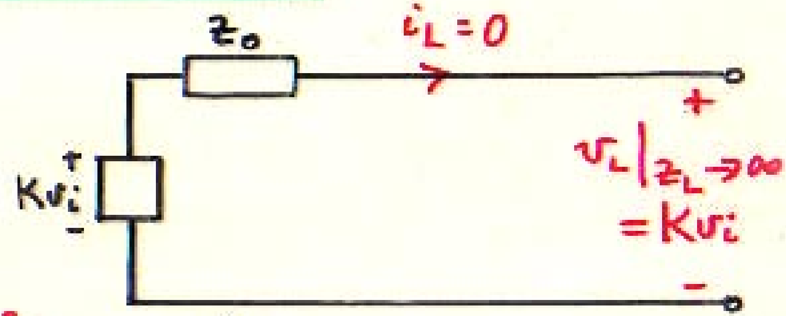


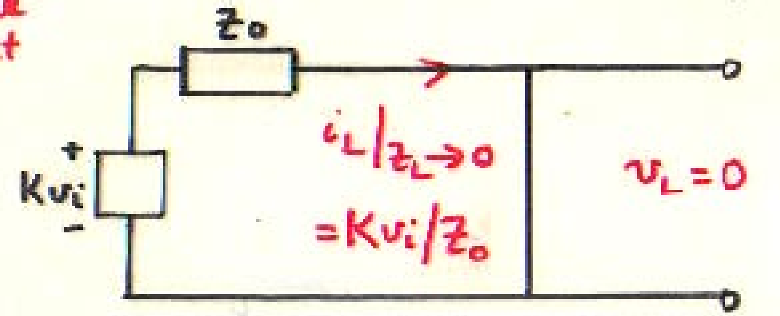
Input/Output Impedance Theorem



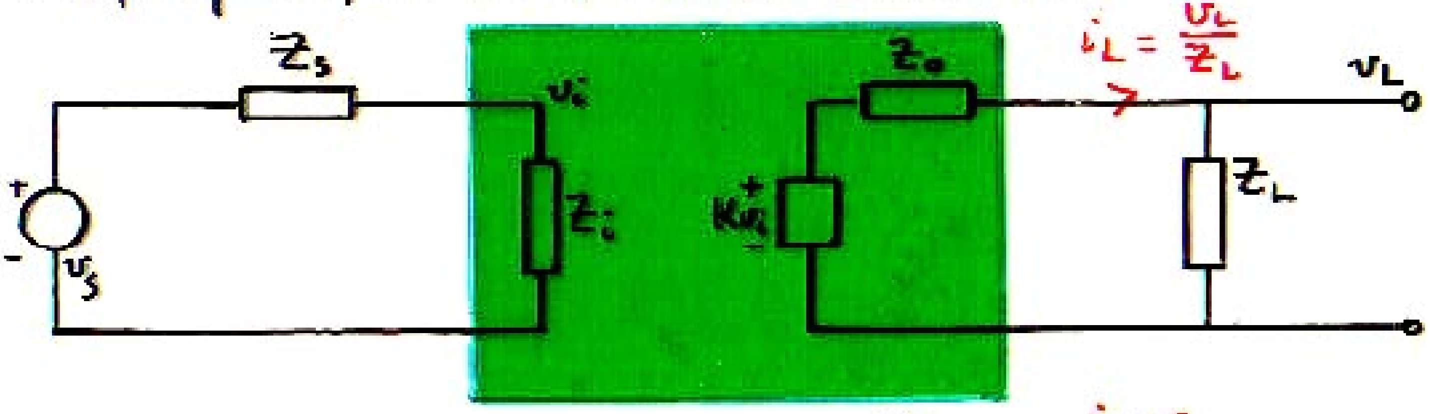
$$\frac{Kv_i}{Kv_i/Z_o} = Z_o = \frac{v_L|_{Z_L\to\infty}}{i_L|_{Z_L\to 0}}$$

$$= \frac{\text{o.c. output voltage}}{\text{s.c. output current}} \Bigg\} \text{ same input } v_S$$

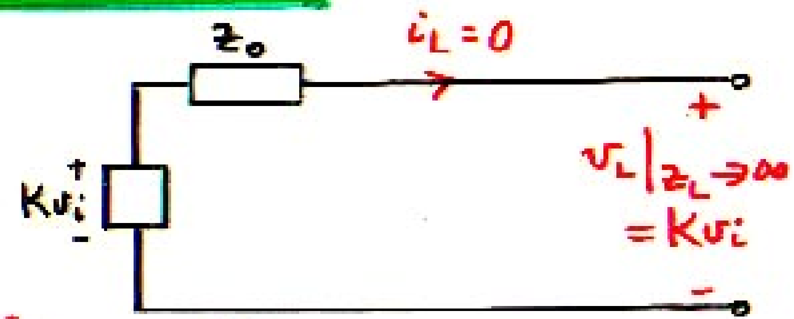




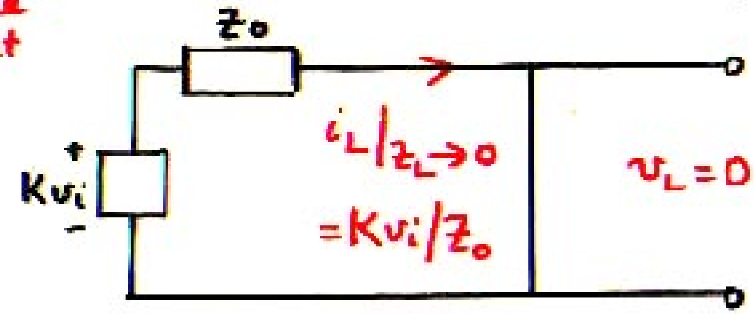
Input/Output Impedance Theorem



$$\frac{Kv_i}{Kv_i/Z_o} = Z_o = \frac{v_L|_{Z_L \to \infty}}{i_L|_{Z_L \to 0}}$$

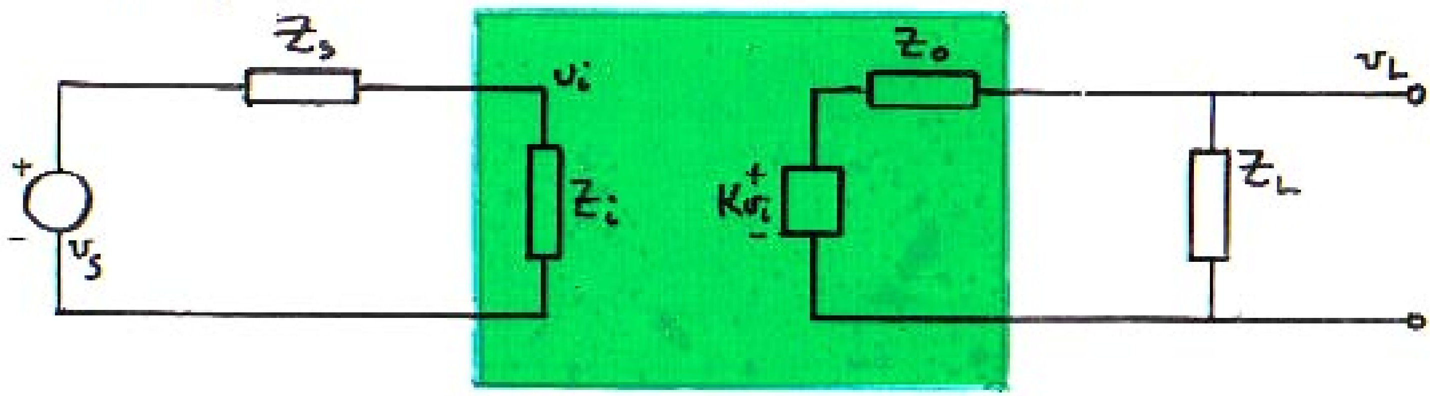


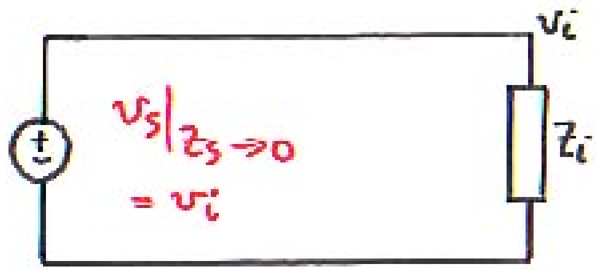
$$= \frac{\text{o.c. output voltage}}{\text{s.c. output current}} \Big\} \begin{array}{l}\text{same} \\ \text{input} \\ v_s\end{array}$$



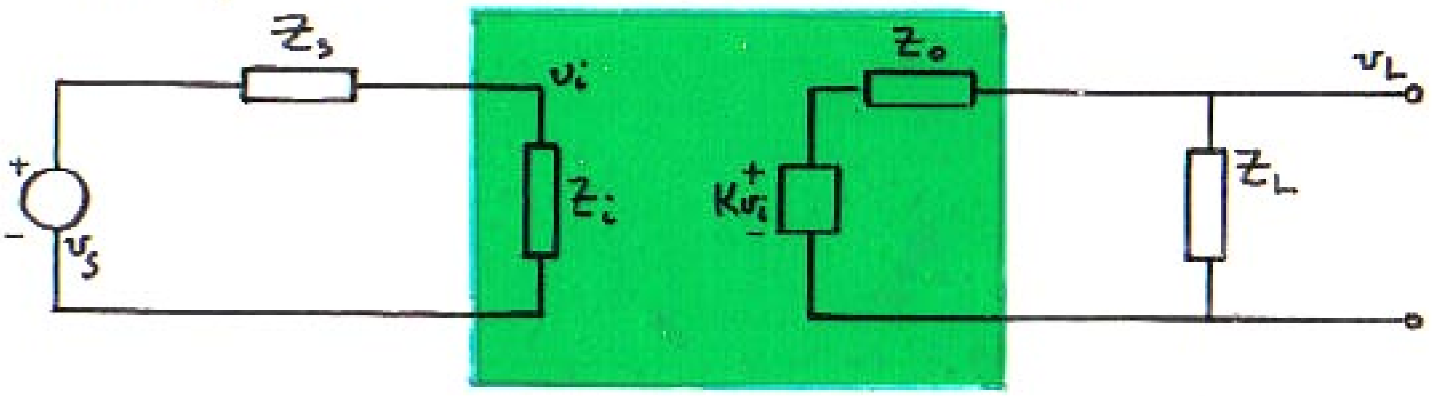
$$Z_o = \frac{v_L|_{Z_L \to \infty}}{i_L|_{Z_L \to 0}} = \frac{\frac{v_L}{v_s}\Big|_{Z_L \to \infty}}{\frac{v_L}{v_s Z_L}\Big|_{Z_L \to 0}} = \frac{A|_{Z_L \to \infty}}{\frac{A}{Z_L}\Big|_{Z_L \to 0}}$$

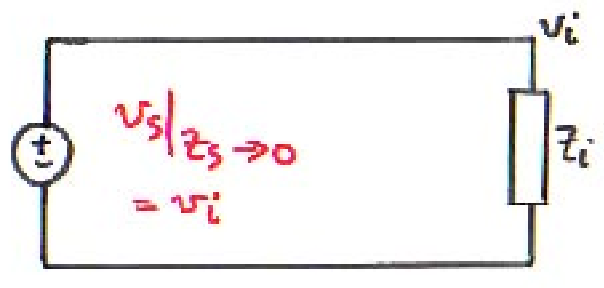
Input/Output Impedance Theorem

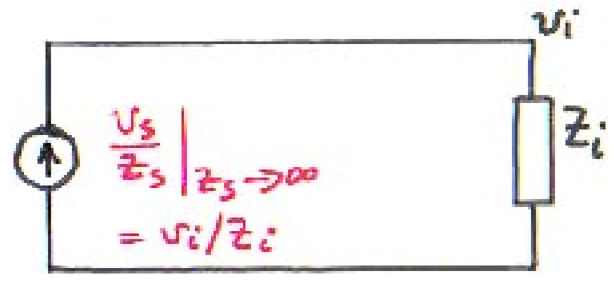




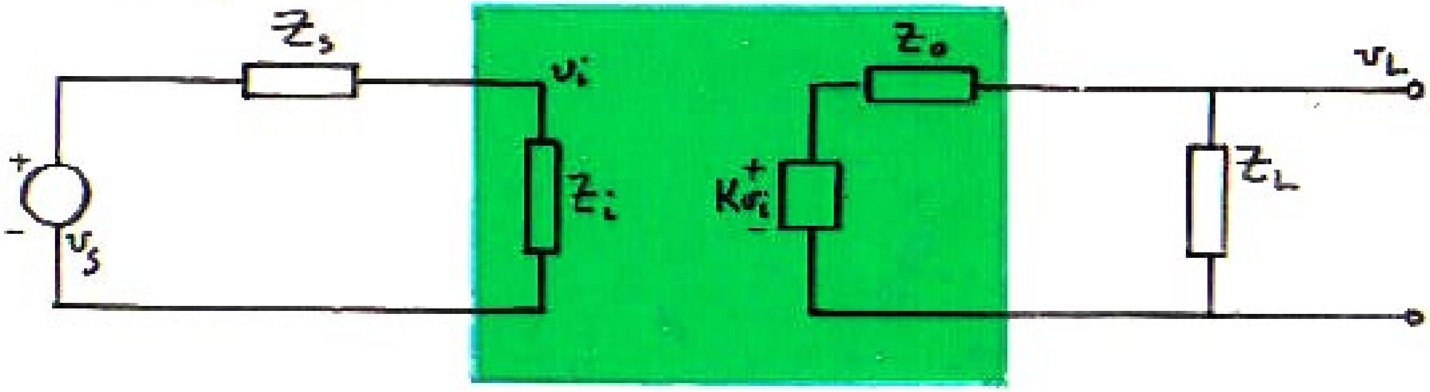
Input/Output Impedance Theorem

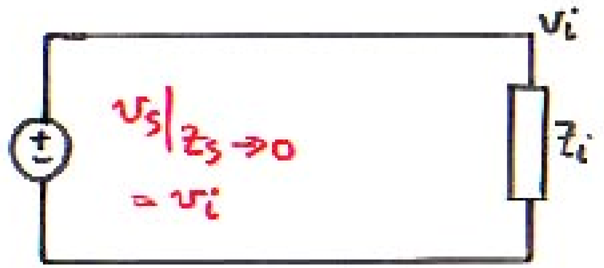


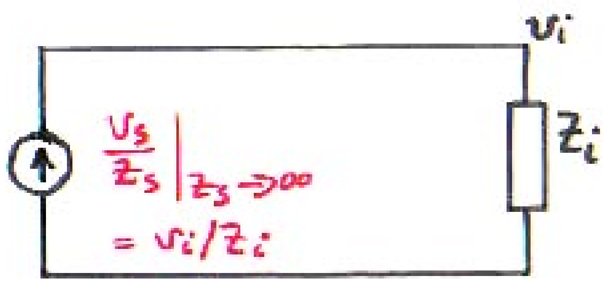




Input/Output Impedance Theorem



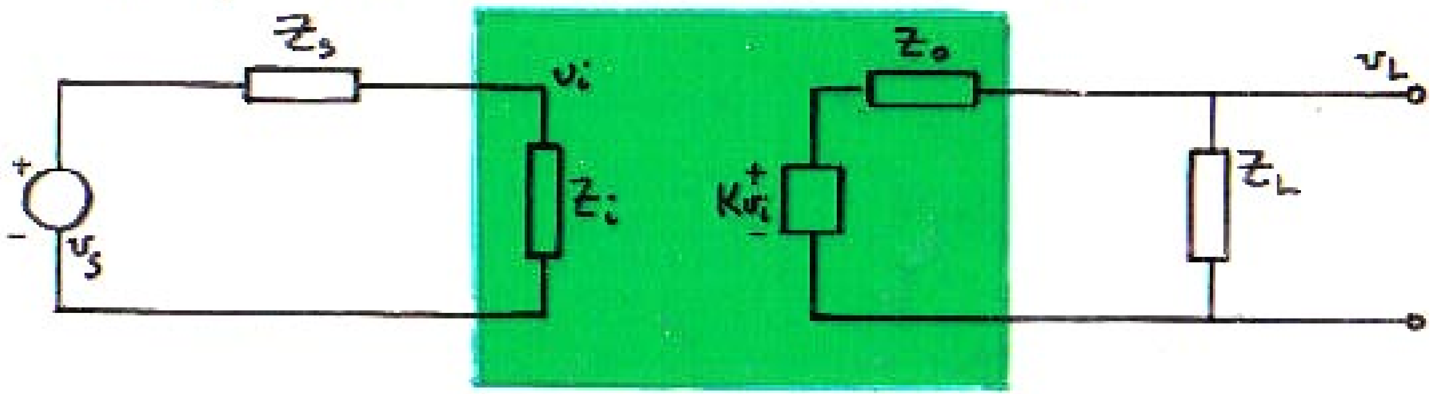


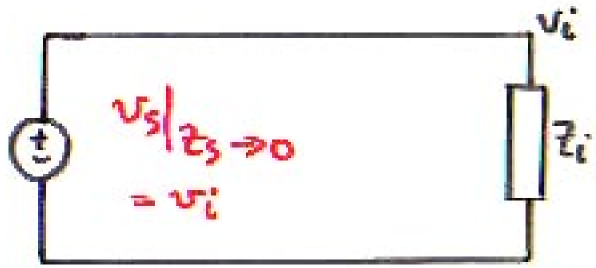


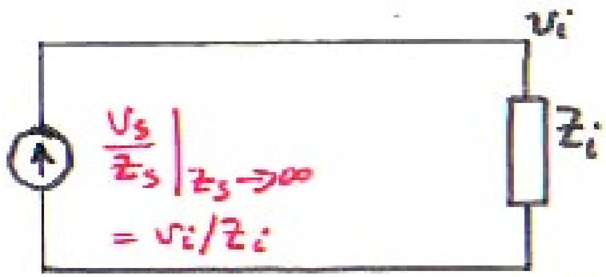
$$\frac{v_i}{v_i/Z_i} = Z_i = \frac{v_s|_{Z_s \to 0}}{\left.\frac{v_s}{Z_s}\right|_{Z_s \to \infty}}$$

$$= \frac{\text{input voltage}}{\text{input current}} \Big\} \text{ same output } v_L \text{ (same } v_i)$$

Input/Output Impedance Theorem







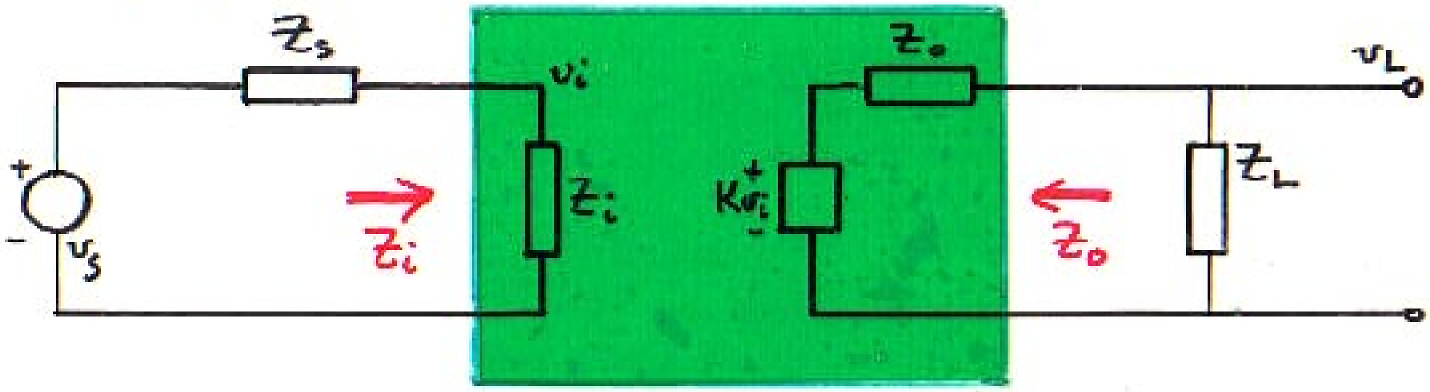
$$\frac{v_i}{v_i/Z_i} = Z_i = \frac{v_s|_{z_s\to 0}}{\frac{v_s}{z_s}\Big|_{z_s\to\infty}}$$

$$= \frac{\text{input voltage}}{\text{input current}}$$

same output v_L (same v_i)

$$Z_i = \frac{v_s|_{z_s\to 0}}{\frac{v_s}{z_s}\Big|_{z_s\to\infty}} = \frac{\frac{v_s}{v_L}\Big|_{z_s\to 0}}{\frac{v_s}{v_L z_s}\Big|_{z_s\to\infty}} = \frac{\frac{1}{A}\Big|_{z_s\to 0}}{\frac{1}{z_s A}\Big|_{z_s\to\infty}} = \frac{z_s A|_{z_s\to\infty}}{A|_{z_s\to 0}}$$

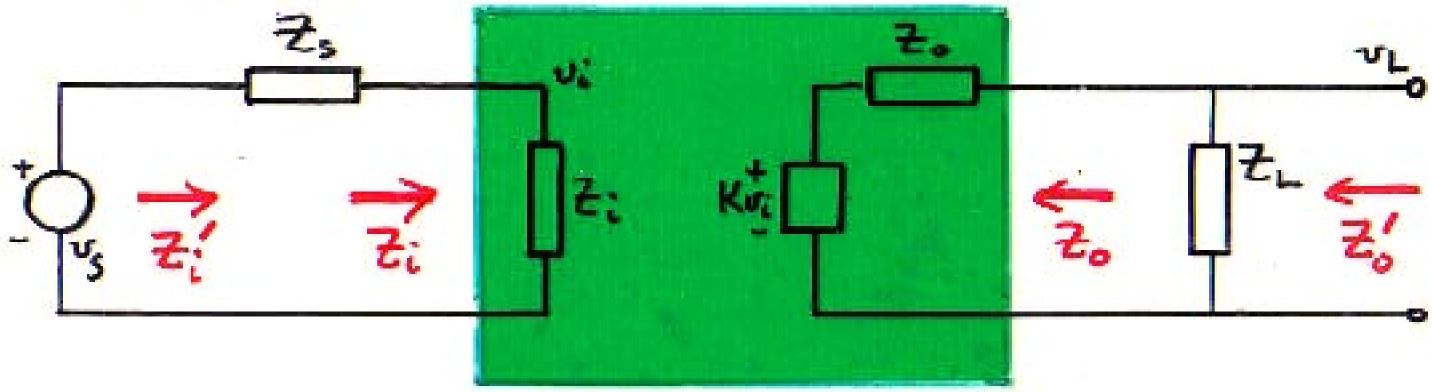
Input/Output Impedance Theorem



Result for "inside" input and output impedances:

$$Z_i = \frac{Z_s A|_{Z_s \to \infty}}{A|_{Z_s \to 0}} \qquad Z_o = \frac{A|_{Z_L \to \infty}}{\left.\frac{A}{Z_L}\right|_{Z_L \to 0}}$$

Input/Output Impedance Theorem



Result for "inside" input and output impedances:

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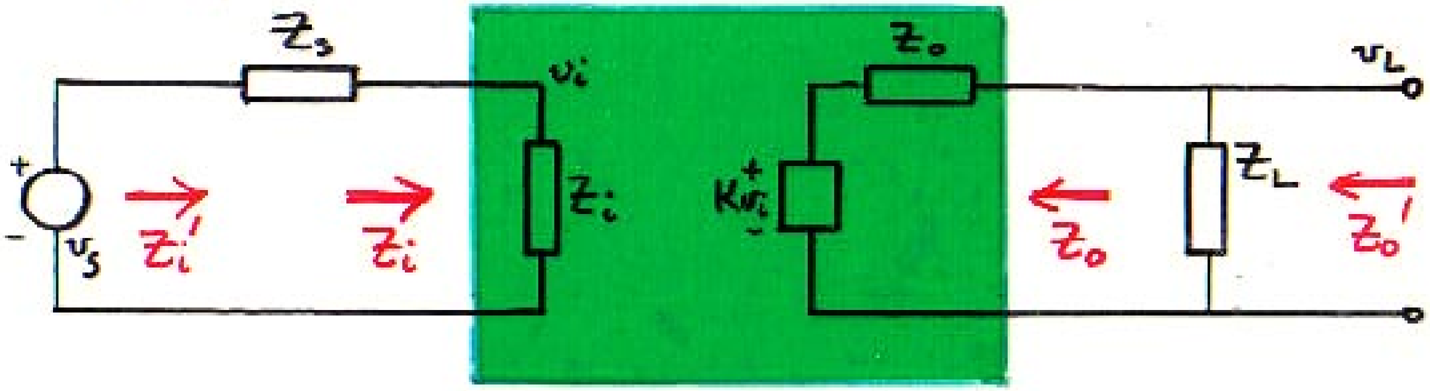
The "outside" input and output impedances may be found as

$$Z_i' = Z_s + Z_i \qquad\qquad Z_o' = Z_L \| Z_o$$

but this is inconvenient because of required refactoring for pole-zero form.

Instead:

Input/Output Impedance Theorem



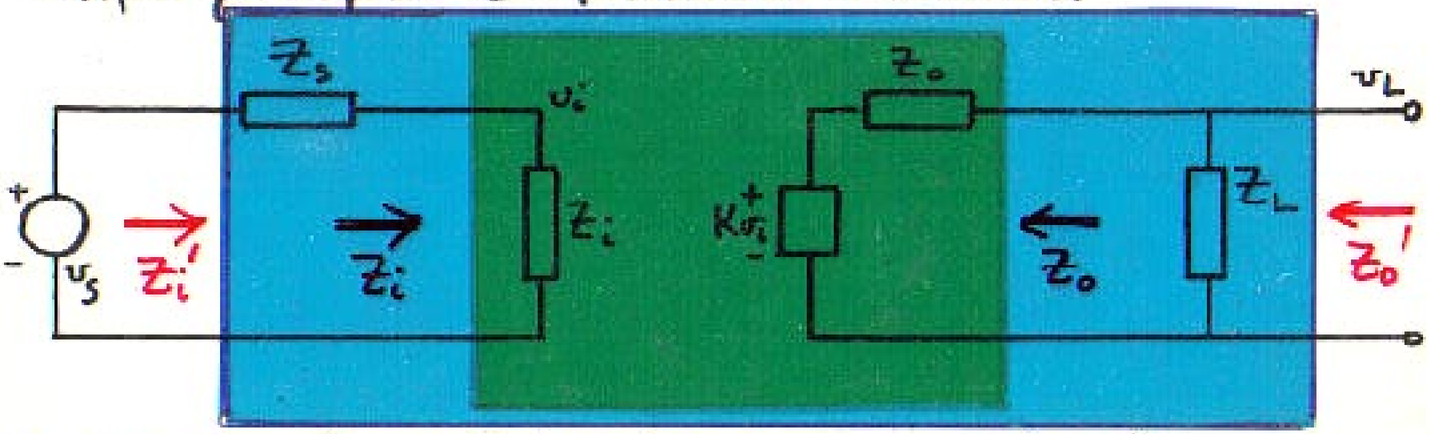
Result for "inside" input and output impedances:

$$Z_i = \frac{Z_s A|_{Z_s \to \infty}}{A|_{Z_s \to 0}} \qquad Z_o = \frac{A|_{Z_L \to \infty}}{\frac{A}{Z_L}\Big|_{Z_L \to 0}}$$

The "outside" input and output impedances can be found directly from:

$$Z_i' = \frac{Z_s A|_{Z_s \to \infty}}{A} \qquad Z_o' = \frac{A}{\frac{A}{Z_L}\Big|_{Z_L \to 0}}$$

Input/Output Impedance Theorem



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The "outside" input and output impedances can be found directly from:

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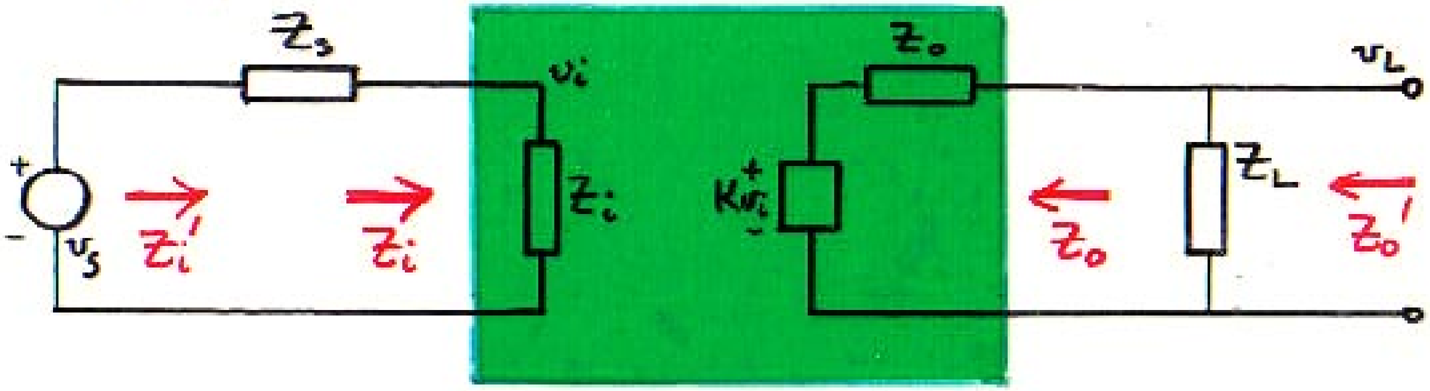
Input/Output Impedance Theorem



Result for "inside" input and output impedances:

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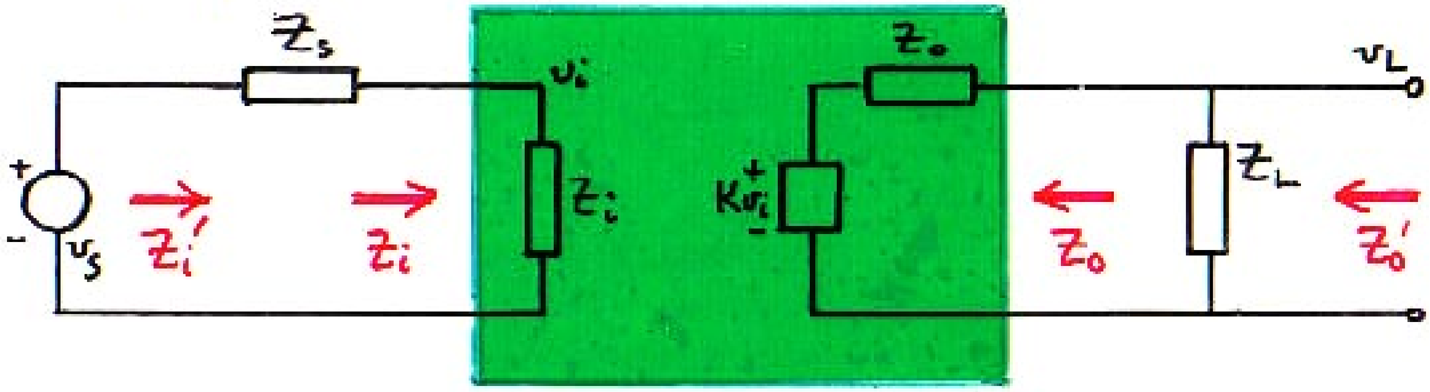
The "outside" input and output impedances can be found directly from:

$$Z_i' = \frac{Z_s A|_{Z_s \to \infty}}{A} \qquad Z_o' = \frac{A}{\frac{A}{Z_L}\Big|_{Z_L \to 0}}$$

Input/Output Impedance Theorem



In practice, it is easier to calculate the "outside" impedances first:

$$Z_i' = \frac{Z_s A|_{Z_s \to \infty}}{A} \qquad Z_o' = \frac{A}{\left.\frac{A}{Z_L}\right|_{Z_L \to 0}}$$

then to find the "inside" impedances from

$$Z_i = Z_i'|_{Z_s \to 0} \qquad Z_o = Z_o'|_{Z_L \to \infty}$$

$$= \frac{Z_s A|_{Z_s \to \infty}}{A|_{Z_s \to 0}} \qquad = \frac{A|_{Z_L \to \infty}}{\left.\frac{A}{Z_L}\right|_{Z_L \to 0}}$$

Bottom Line

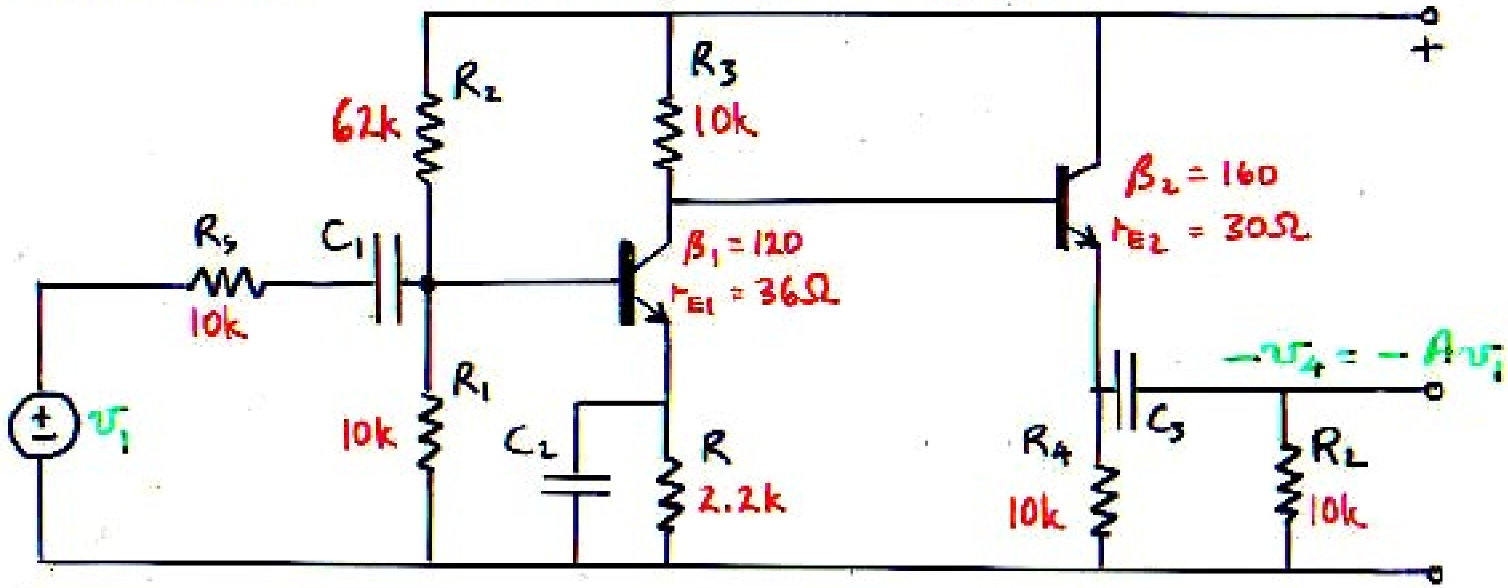
The Input/Output Impedance Theorem eliminates almost two-thirds of the work by permitting the input and output impedances Z_i', Z_i and Z_o', Z_o to be determined by taking simple limits upon the expression already obtained for the gain A.

Since the limits can be taken factor by factor in A, the theorem affords two free bonuses:

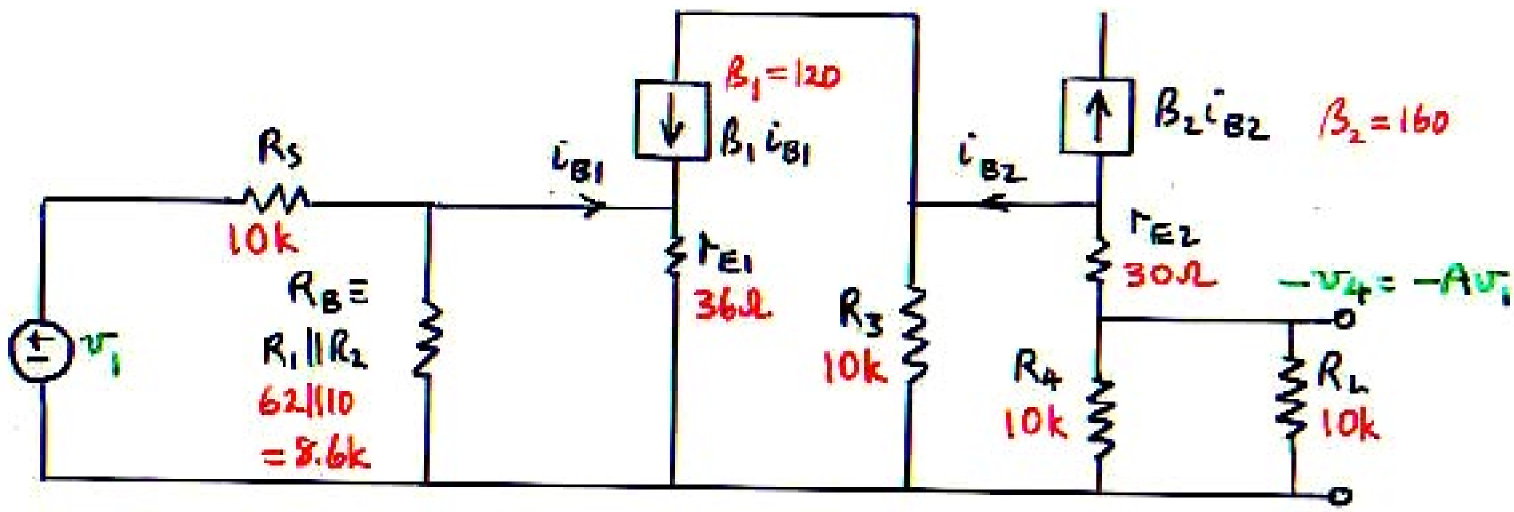
1. Any factor in A not containing Z_S or Z_L cancels out of the result.
2. Since A is already in factored pole-zero form, Z_i', Z_i and Z_o', Z_o are automatically obtained in factored pole-zero form, with poles and zeros different from those in A, but obtained from them by simple limits.

Example

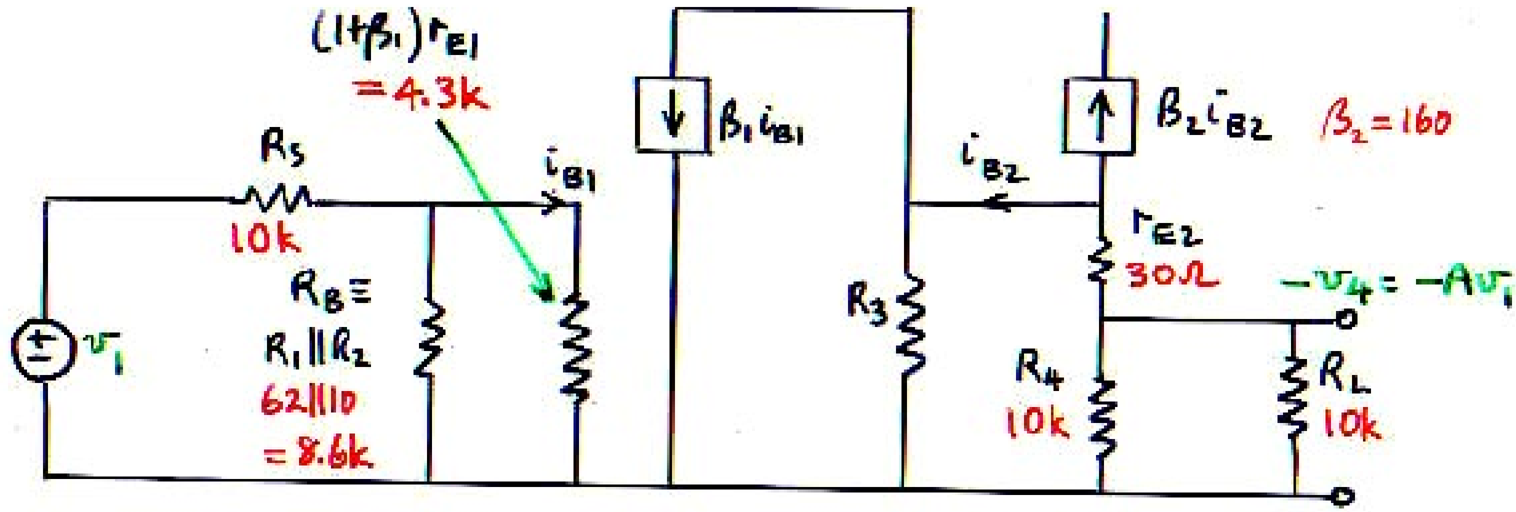
Draw the small-signal equivalent circuit model of the common-emitter — emitter-follower amplifier:



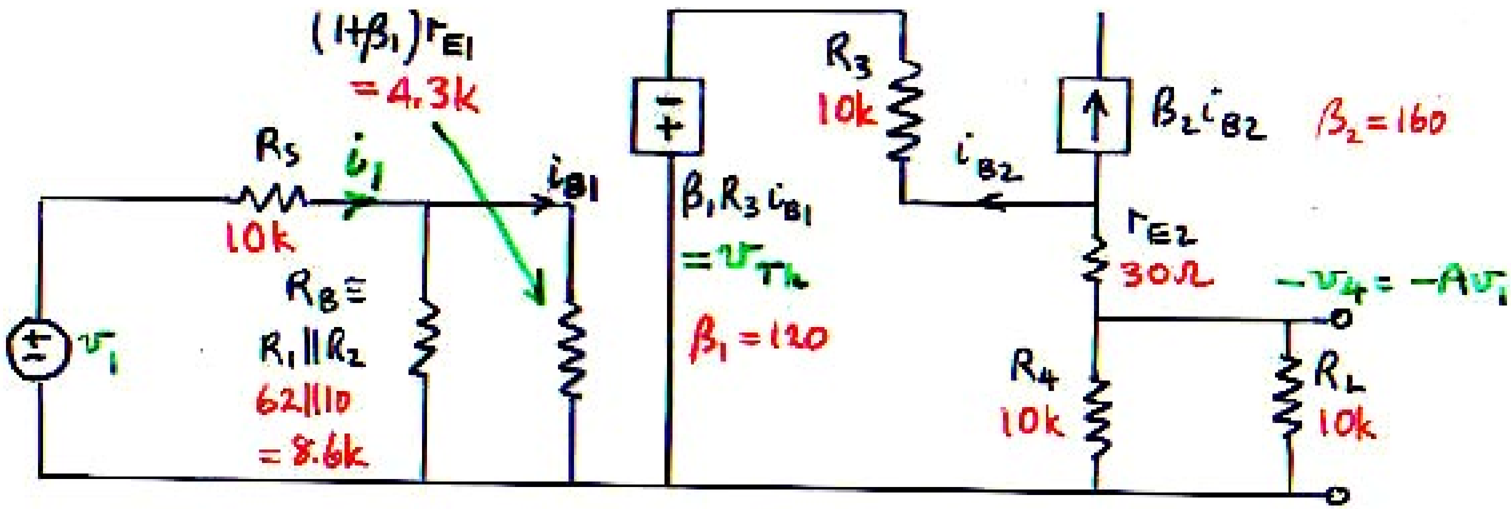
Find the midband gain $A \equiv v_4/v_i$ analytically and numerically.



$\beta_1 = 120$
$\beta_1 i_{B1}$
$\beta_2 i_{B2}$
$\beta_2 = 160$
R_S
10k
i_{B1}
i_{B2}
r_{E1}
36Ω
r_{E2}
30Ω
$-v_4 = -Av_i$
$R_B \equiv$
$R_1 \| R_2$
$62 \| 110$
$= 8.6k$
v_i
R_3
10k
R_4
10k
R_L
10k



$(1+\beta_1)r_{E1}$
$=4.3k$
$\beta_1 i_{B1}$
$\beta_2 i_{B2}$
$\beta_2=160$
R_S
10k
i_{B1}
i_{B2}
$R_B \equiv$
$R_1 \| R_2$
$62\|10$
$=8.6k$
v_i
R_3
r_{E2}
30Ω
$-v_4 = -Av_i$
R_4
10k
R_L
10k



$$A = \frac{v_4}{v_1} = \frac{i_{B1}}{i_1} \times \frac{i_1}{v_1} \times \frac{v_{Th}}{i_{B1}} \times \frac{v_4}{v_{Th}}$$

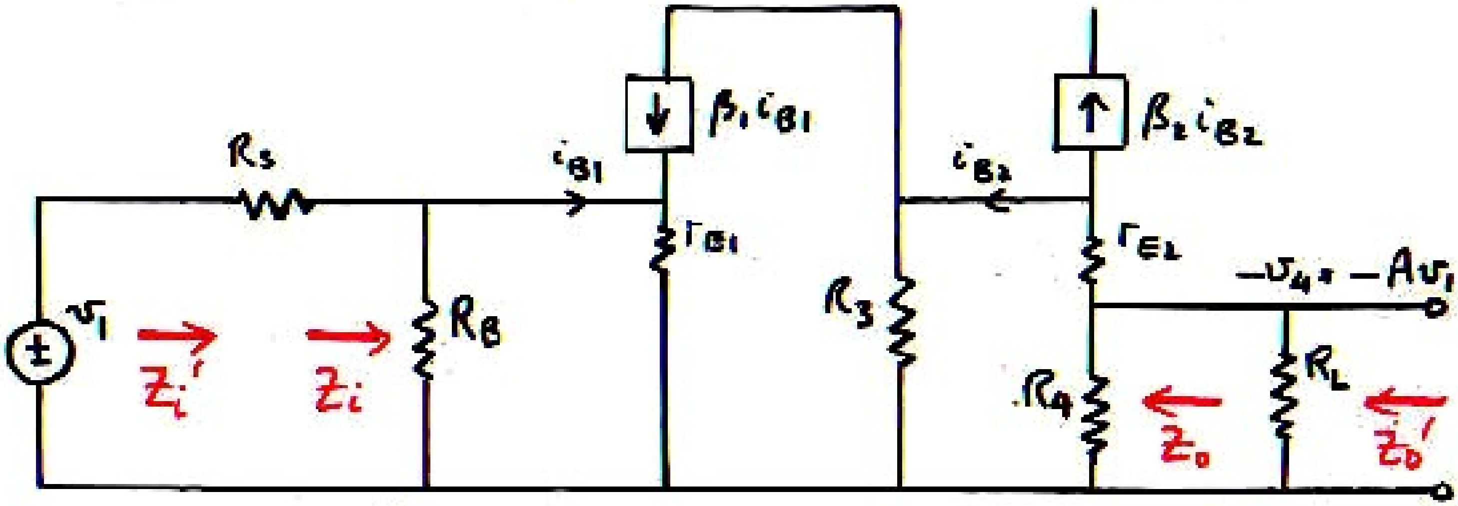
$$= \frac{R_B}{R_B + (1+\beta_1) r_{E1}} \; \frac{1}{R_S + R_B \| (1+\beta_1) r_{E1}} \; \beta_1 R_3 \; \frac{R_4 \| R_L}{R_4 \| R_L + r_{E2} + R_3 / (1+\beta_2)}$$

$$= \underbrace{\frac{8.6}{8.6 + 4.3} \; \frac{1}{10 + 8.6 \| 4.3} \; 1{,}200}_{62} \; \frac{10\|10}{5 + 0.03 + 0.06}$$

$$= 62 \times 0.98$$

$$= 61 \Rightarrow 36\,\text{dB}$$

Example: the CE plus emitter follower amplifier



$$A_m = \underbrace{\frac{R_B}{R_B + (1+\beta_1) r_{E1}}}_{K_1} \cdot \underbrace{\frac{\beta_1 R_3}{R_s + R_B \| (1+\beta_1) r_{E1}}}_{K_2} \cdot \underbrace{\frac{R_4 \| R_L}{R_4 \| R_L + r_{E2} + R_3/(1+\beta_2)}}_{K_3}$$

$$A_m = \frac{R_B}{R_B + (1+\beta_1) r_{E1}} \cdot \frac{\beta_1 R_3}{R_S + R_B \| (1+\beta_1) r_{E1}} \cdot \frac{R_4 \| R_L}{R_4 \| R_L + r_{E2} + R_3/(1+\beta_2)}$$

$K_1 \uparrow$ $K_2 \uparrow$ $K_3 \uparrow$

$$Z_o' \to R_{om}' = \frac{A_m}{\left.\frac{A_m}{R_L}\right|_{R_L \to 0}} = \frac{\cancel{(K_1 K_2)} K_3}{\left.\cancel{(K_1 K_2)} \frac{K_3}{R_L}\right|_{R_L \to 0}} = \frac{K_3}{\left.\frac{K_3}{R_L}\right|_{R_L \to 0}}$$

$$A_m = \frac{R_B}{R_B + (1+\beta_1) r_{E1}} \cdot \frac{\beta_1 R_B}{R_S + R_B \| (1+\beta_1) r_{E1}} \cdot \frac{R_4 \| R_L}{R_4 \| R_L + r_{E2} + R_3/(1+\beta_2)}$$

$$\quad K_1 \uparrow \qquad\qquad K_2 \uparrow \qquad\qquad K_3 \uparrow$$

$$Z_o' \to R_{om}' = \frac{A_m}{\left.\frac{A_m}{R_L}\right|_{R_L \to 0}} = \frac{\cancel{(K_1 K_2)} K_3}{\left.\cancel{(K_1 K_2)} \frac{K_3}{R_L}\right|_{R_L \to 0}} = \frac{K_3}{\left.\frac{K_3}{R_L}\right|_{R_L \to 0}}$$

NOTE: All factors in A that do not contain R_L

CANCEL OUT

$$\left.\frac{K_3}{R_L}\right|_{R_L \to 0} = \left.\frac{\frac{R_4 \cancel{R_L}}{R_4 + R_L}}{\cancel{R_L}\left[R_4 \| R_L + r_{E2} + R_3/(1+\beta_2)\right]}\right|_{R_L \to 0} = \frac{1}{r_{E2} + R_3/(1+\beta_2)}$$

$$A_m = \frac{R_B}{R_B + (1+\beta_1) r_{E1}} \cdot \frac{\beta_1 R_B}{R_S + R_B \| (1+\beta_1) r_{E1}} \cdot \frac{R_4 \| R_L}{R_4 \| R_L + r_{E2} + R_3/(1+\beta_2)}$$

$$\quad K_1 \uparrow \qquad\qquad K_2 \uparrow \qquad\qquad K_3 \uparrow$$

$$Z_o' \rightarrow R_{om}' = \frac{A_m}{\left.\frac{A_m}{R_L}\right|_{R_L \to 0}} = \frac{\cancel{(K_1 K_2)} K_3}{\left.\cancel{(K_1 K_2)} \frac{K_3}{R_L}\right|_{R_L \to 0}} = \frac{K_3}{\left.\frac{K_3}{R_L}\right|_{R_L \to 0}}$$

NOTE: All factors in A that do not contain R_L

CANCEL OUT

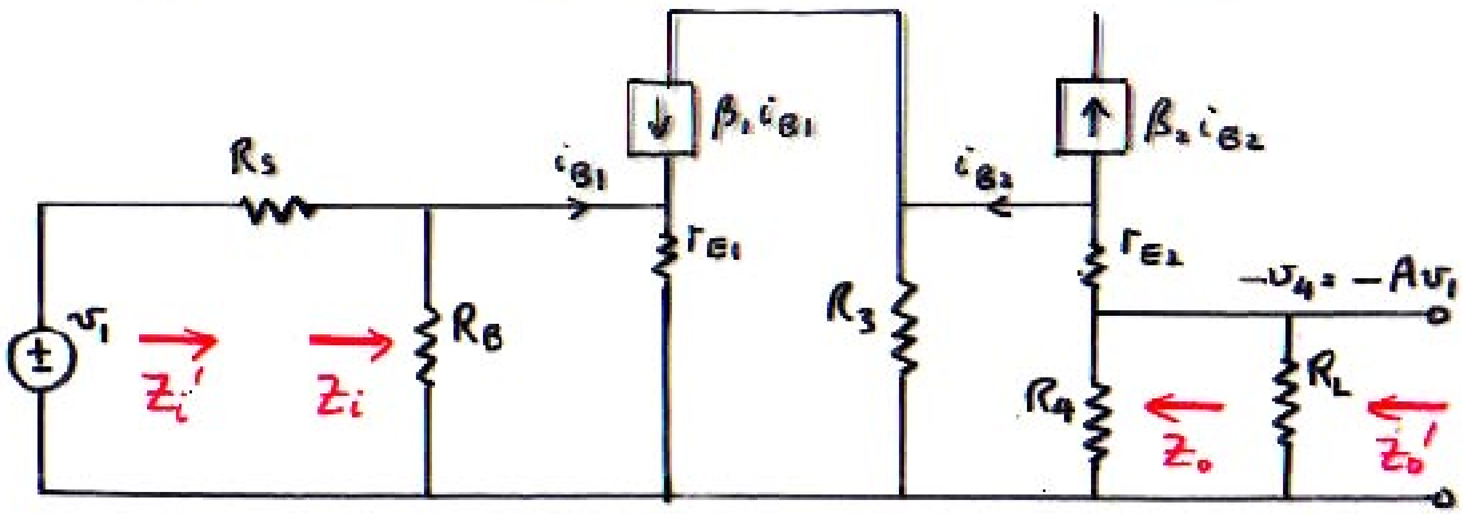
$$\left.\frac{K_3}{R_L}\right|_{R_L \to 0} = \left.\frac{\frac{R_4 \cancel{R_L}}{R_4 + R_L}}{\cancel{R_L} \left[R_4 \| R_L + r_{E2} + R_3/(1+\beta_2)\right]}\right|_{R_L \to 0} = \frac{1}{r_{E2} + R_3/(1+\beta_2)}$$

$$R_{om}' = \frac{R_4 \| R_L}{R_4 \| R_L + r_{E2} + R_3/(1+\beta_2)} \left[r_{E2} + R_3/(1+\beta_2)\right]$$

$$= R_4 \| R_L \| \left[r_{E2} + R_3/(1+\beta_2)\right] = 10 \| 10 \| \left[0.03 + \frac{10}{160}\right] = 90\Omega$$

$$R_{om} = \left. R_{om}' \right|_{R_L \to \infty} = R_4 \| \left[r_{E2} + R_3/(1+\beta_2)\right] = 90\Omega$$

Example: the CE plus emitter follower amplifier



$$A_m = \frac{R_B}{R_B + (1+\beta_1) r_{E1}} \cdot \frac{\beta_1 R_3}{R_S + R_B \| (1+\beta_1) r_{E1}} \cdot \frac{R_4 \| R_L}{R_4 \| R_L + r_{E2} + R_3/(1+\beta_2)}$$

$K_1 \uparrow$ $K_2 \uparrow$ $K_3 \uparrow$

$$R_{om}' = \frac{R_4 \| R_L}{R_4 \| R_L + r_{E2} + R_3/(1+\beta_2)} \left[r_{E2} + R_3/(1+\beta_2) \right]$$

$$= R_4 \| R_L \| \left[r_{E2} + R_3/(1+\beta_2) \right] = 10 \| 10 \| \left[0.03 + \frac{10}{160} \right] = 90\Omega.$$

$$R_{om} = R_{om}' \Big|_{R_L \to \infty} = R_4 \| \left[r_{E2} + R_3/(1+\beta_2) \right] = 90\Omega$$

$$A_m = \underset{K_1 \uparrow}{\frac{R_B}{R_B + (1+\beta_1) r_{E1}}} \cdot \underset{K_2 \uparrow}{\frac{\beta_1 R_B}{R_S + R_B \| (1+\beta_1) r_{E1}}} \cdot \underset{K_3 \uparrow}{\frac{R_4 \| R_L}{R_4 \| R_L + r_{E2} + R_3/(1+\beta_2)}}$$

$$Z_i' \rightarrow R'_{im} = \frac{R_S A_m \big|_{R_S \to \infty}}{A_m} = \frac{\cancel{K_1 K_3} (R_S K_2) \big|_{R_S \to \infty}}{\cancel{K_1 K_3} (K_2)} = \frac{R_S K_2 \big|_{R_S \to \infty}}{K_2}$$

$$A_m = \underbrace{\frac{R_B}{R_B + (1+\beta_1) r_{E1}}}_{K_1\uparrow} \cdot \underbrace{\frac{\beta_1 R_3}{R_S + R_B \| (1+\beta_1) r_{E1}}}_{K_2\uparrow} \cdot \underbrace{\frac{R_4 \| R_L}{R_4 \| R_L + r_{E2} + R_3/(1+\beta_2)}}_{K_3\uparrow}$$

$$Z_i' \to R_{im}' = \frac{R_S A_m \big|_{R_S \to \infty}}{A_m} = \frac{\cancel{K_1 K_3} (R_S K_2) \big|_{R_S \to \infty}}{\cancel{K_1 K_3} (K_2)} = \frac{R_S K_2 \big|_{R_S \to \infty}}{K_2}$$

NOTE: All factors in A that do not contain R_S

CANCEL OUT

$$R_S K_2 \big|_{R_S \to \infty} = \frac{R_S \beta_1 R_3}{R_S + \cancel{R_B \| (1+\beta_1) r_{E1}}} \Bigg|_{R_S \to \infty} = \beta_1 R_3$$

$$A_m = \underbrace{\frac{R_B}{R_B + (1+\beta_1) r_{E1}}}_{K_1 \uparrow} \cdot \underbrace{\frac{\beta_1 R_3}{R_S + R_B \| (1+\beta_1) r_{E1}}}_{K_2 \uparrow} \cdot \underbrace{\frac{R_4 \| R_L}{R_4 \| R_L + r_{E2} + R_3/(1+\beta_2)}}_{K_3 \uparrow}$$

$$Z_i' \to R_{im}' = \frac{R_S A_m |_{R_S \to \infty}}{A_m} = \frac{\cancel{K_1 K_3} (R_S K_2) |_{R_S \to \infty}}{\cancel{K_1 K_3} (K_2)} = \frac{R_S K_2 |_{R_S \to \infty}}{K_2}$$

NOTE: All factors in A that do not contain R_S

CANCEL OUT

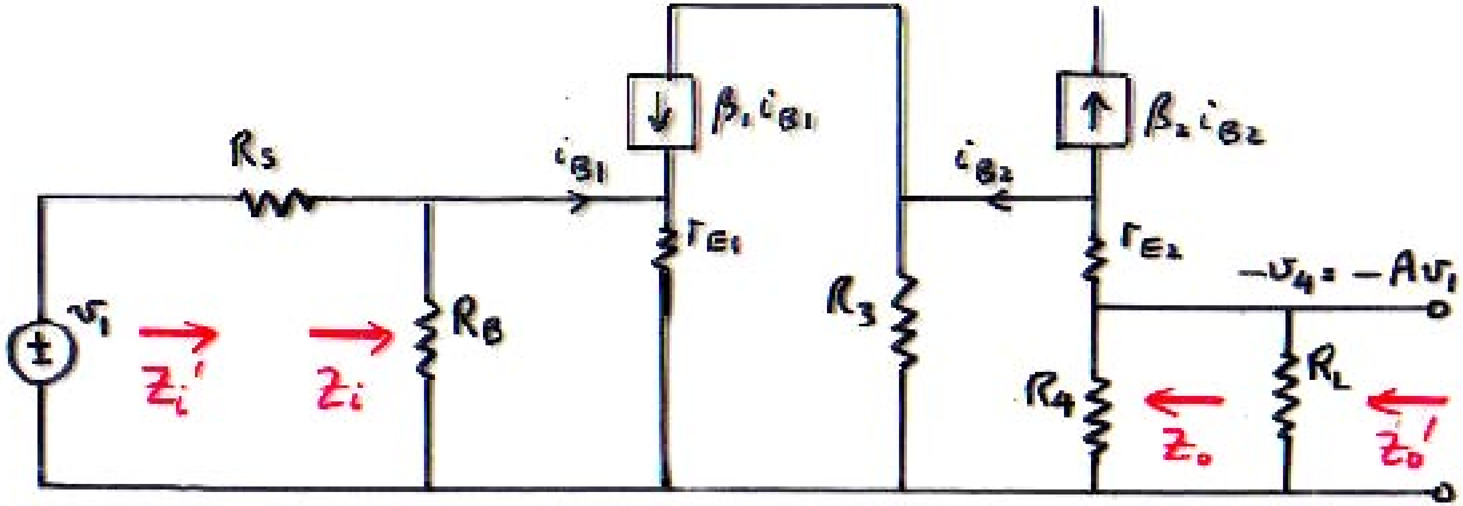
$$R_S K_2 |_{R_S \to \infty} = \left. \frac{R_S \beta_1 R_3}{R_S + \cancel{R_B \| (1+\beta_1) r_{E1}}} \right|_{R_S \to \infty} = \beta_1 R_3$$

$$R_{im}' = \frac{\beta_1 R_3}{\dfrac{\beta_1 R_3}{R_S + R_B \| (1+\beta_1) r_{E1}}} = R_S + R_B \| (1+\beta_1) r_{E1}$$

$$= 10 + 8.6 \| 4.3 = 13k$$

$$R_{im} = R_{im}' |_{R_S \to 0} = R_B \| (1+\beta_1) r_{E1} = 3k$$

Example: the CE plus emitter follower amplifier



$$A_m = \underbrace{\frac{R_B}{R_B + (1+\beta_1) r_{E1}}}_{K_1 \uparrow} \cdot \underbrace{\frac{\beta_1 R_3}{R_S + R_B \| (1+\beta_1) r_{E1}}}_{K_2 \uparrow} \cdot \underbrace{\frac{R_4 \| R_L}{R_4 \| R_L + r_{E2} + R_3/(1+\beta_2)}}_{K_3 \uparrow}$$

$$R_{im}' = \frac{\beta_1 R_3}{\dfrac{\beta_1 R_3}{R_S + R_B \| (1+\beta_1) r_{E1}}} = R_S + R_B \| (1+\beta_1) r_{E1}$$

$$= 10 + 8.6 \| 4.3 = 13k$$

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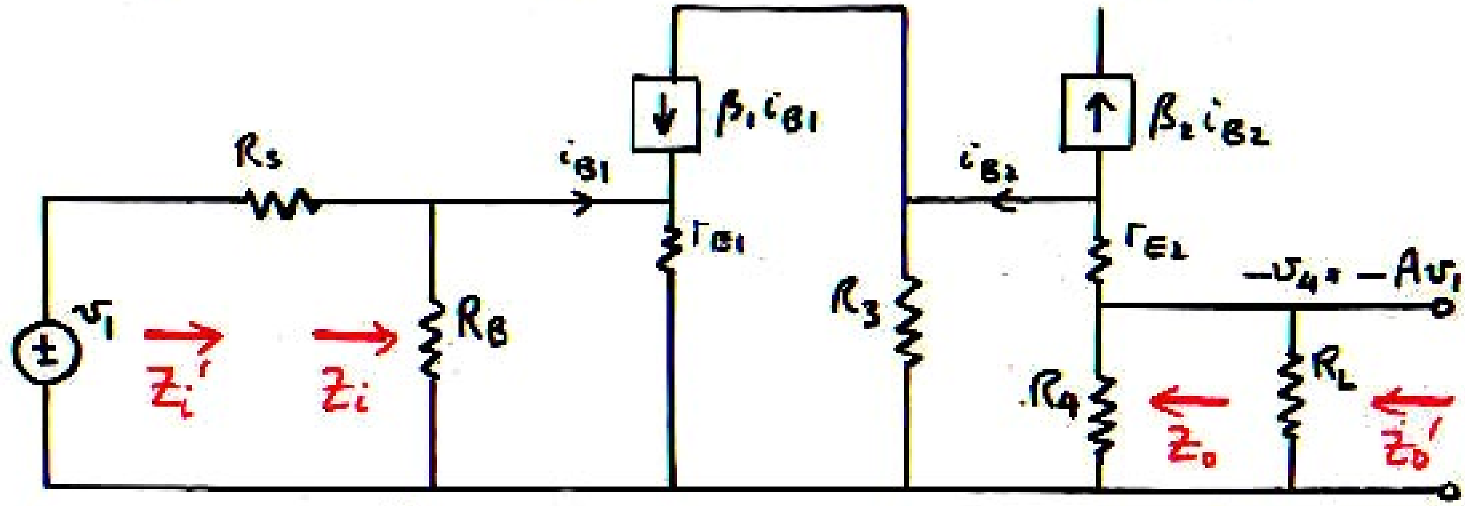
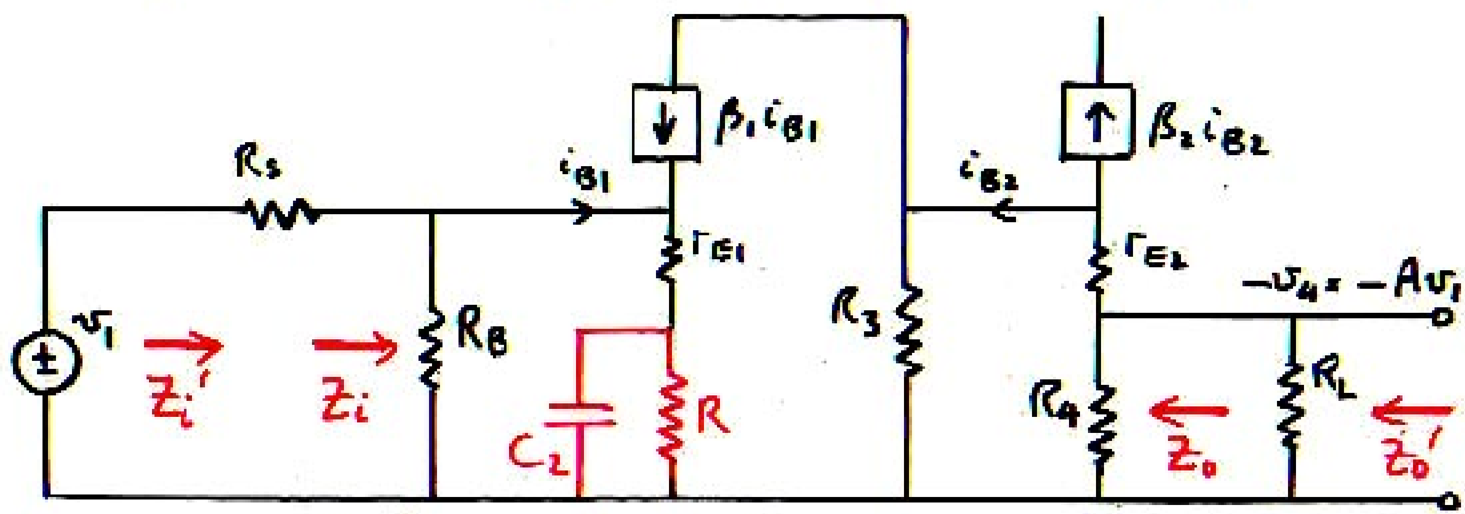
Example: the CE plus emitter follower amplifier



$$A_m = \underbrace{\frac{R_B}{R_B + (1+\beta_1) r_{E1}}}_{K_1 \uparrow} \cdot \underbrace{\frac{\beta_1 R_3}{R_s + R_B \| (1+\beta_1) r_{E1}}}_{K_2 \uparrow} \cdot \underbrace{\frac{R_4 \| R_L}{R_4 \| R_L + r_{E2} + R_3/(1+\beta_2)}}_{K_3 \uparrow}$$

Example: the CE plus emitter follower amplifier



$$A_m = \underbrace{\frac{R_B}{R_B + (1+\beta_1) r_{E1}}}_{K_1} \cdot \underbrace{\frac{\beta_1 R_3}{R_S + R_B \| (1+\beta_1) r_{E1}}}_{K_2} \cdot \underbrace{\frac{R_4 \| R_L}{R_4 \| R_L + r_{E2} + R_3/(1+\beta_2)}}_{K_3}$$

Include the emitter bypass capacitance C_2:

$$A = A_m \frac{1 + \frac{\omega_1}{s}}{1 + \frac{\omega_2}{s}} \qquad \omega_1 \equiv \frac{1}{C_2 R} \qquad \omega_2 \equiv \frac{1}{C_2 \{R \| [r_{E1} + (R_S \| R_B)/(1+\beta_1)]\}}$$

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$$Z_o' = \frac{A}{\left.\frac{A}{R_L}\right|_{R_L \to 0}} = \frac{\cancel{K_1 K_2 \frac{1+\frac{\omega_1}{s}}{1+\frac{\omega_2}{s}}}(K_3)}{\left.\cancel{K_1 K_2 \frac{1+\frac{\omega_1}{s}}{1+\frac{\omega_2}{s}}}\left(\frac{K_3}{R_L}\right)\right|_{R_L \to 0}} = R_{om}'$$

$$Z_o = Z_o'|_{R_L \to \infty} = R_{om}$$

Include the emitter bypass capacitance C_2:

$$A = A_m \frac{1+\frac{\omega_1}{s}}{1+\frac{\omega_2}{s}} \qquad \omega_1 \equiv \frac{1}{C_2 R} \qquad \omega_2 \equiv \frac{1}{C_2\{R\|[r_{E1} + (R_S\|R_B)/(1+\beta_1)]\}}$$

$$Z_i' = \frac{R_S A|_{R_S\to\infty}}{A} = \frac{\cancel{K_1 K_3}(R_S K_2)|_{R_S\to\infty} \frac{\cancel{1+\frac{\omega_1}{s}}}{1+\frac{\omega_2|_{R_S\to\infty}}{s}}}{\cancel{K_1K_3}(K_2)\frac{\cancel{1+\frac{\omega_1}{s}}}{1+\frac{\omega_2}{s}}}$$

$$= R_{im}' \frac{1+\frac{\omega_2}{s}}{1+\frac{\omega_2|_{R_S\to\infty}}{s}} \qquad \omega_2|_{R_S\to\infty} = \frac{1}{C_2\left[R\|\left(r_{E1} + \frac{R_B}{1+\beta_1}\right)\right]}$$

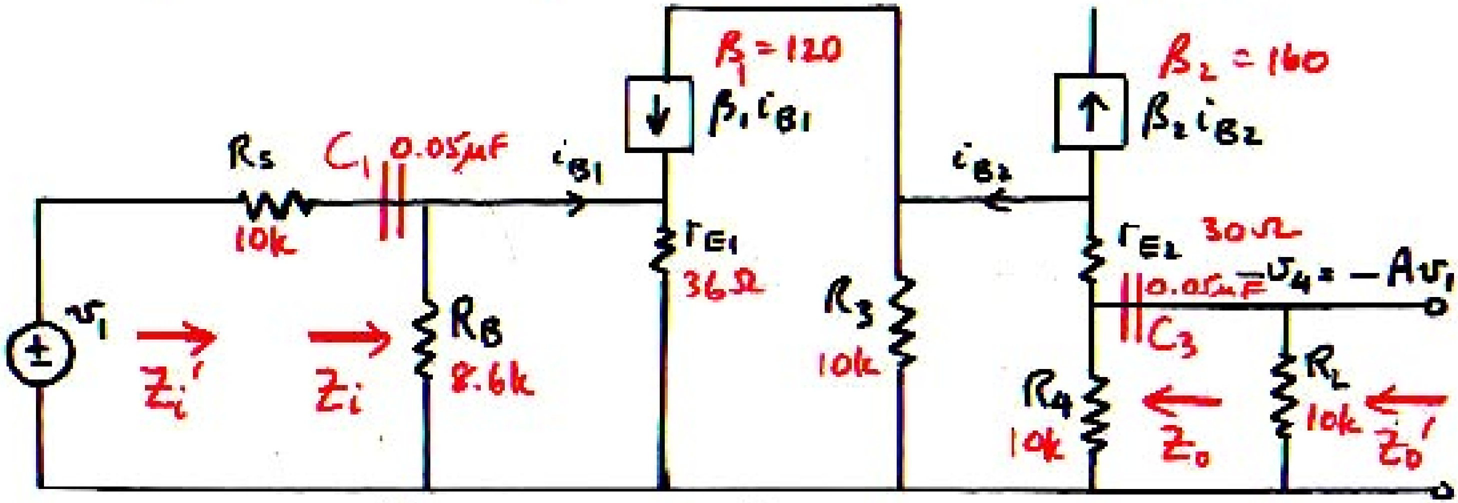
$$Z_i = Z_i'|_{R_S\to 0} = R_{im} \frac{1+\frac{\omega_2|_{R_S\to 0}}{s}}{1+\frac{\omega_2|_{R_S\to\infty}}{s}}$$

$$\omega_2|_{R_S\to 0} = \frac{1}{C_2(R\|r_{E1})}$$

NOTE:

If A is in factored pole-zero form, the input and output impedances are automatically also in factored pole-zero form.

Example: the CE plus emitter follower amplifier



$$A_m = \underbrace{\frac{R_B}{R_B + (1+\beta_1) r_{E1}}}_{K_1} \cdot \underbrace{\frac{\beta_1 R_3}{R_S + R_B \| (1+\beta_1) r_{E1}}}_{K_2} \cdot \underbrace{\frac{R_4 \| R_L}{R_4 \| R_L + r_{E2} + R_3/(1+\beta_2)}}_{K_3}$$

Exercise

Find the gain in factored pole-zero form when the coupling capacitances C_1 and C_3 are included. Evaluate the corner frequencies, and sketch the magnitude asymptotes.

Exercise Solution

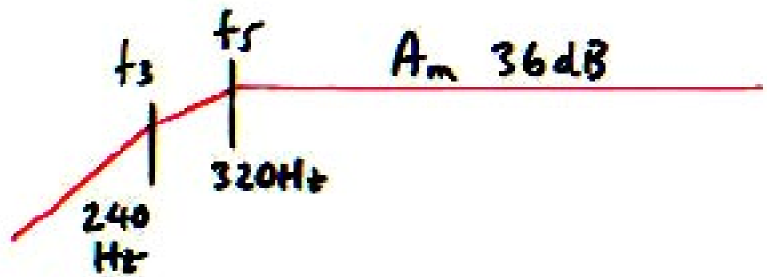
$$A = A_m \frac{1}{\left(1+\frac{\omega_3}{s}\right)\left(1+\frac{\omega_5}{s}\right)}$$

$$\omega_3 \cong \frac{1}{C_1\left[R_S + R_B \| (1+\beta_1) r_{E1}\right]} = \frac{159}{0.05\left[10 + \underbrace{8.6 \| 4.3}_{2.9}\right]}$$

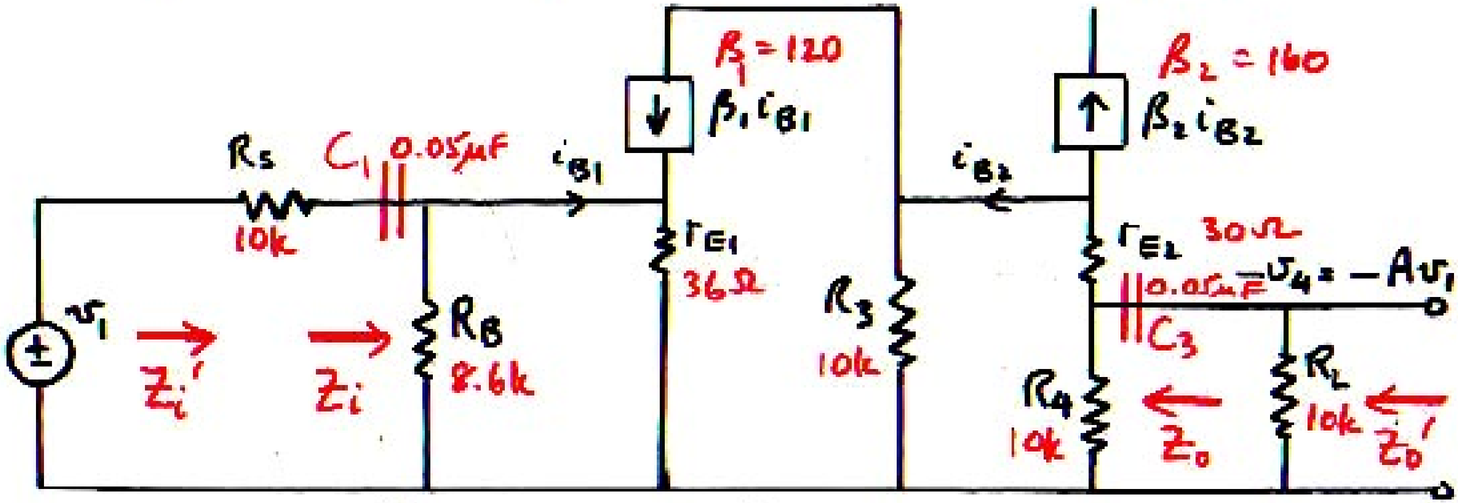
$$\approx 240 \text{ Hz}$$

$$\omega_5 \cong \frac{1}{C_3\left[\left(r_{E2} + \frac{R_3}{1+\beta_2}\right) \| R_4 + R_L\right]} = \frac{159}{0.05\left[\cancel{(0.03+0.06)} \| 10 + 10\right]}$$

$$= 320 \text{ Hz}$$



Example: the CE plus emitter follower amplifier

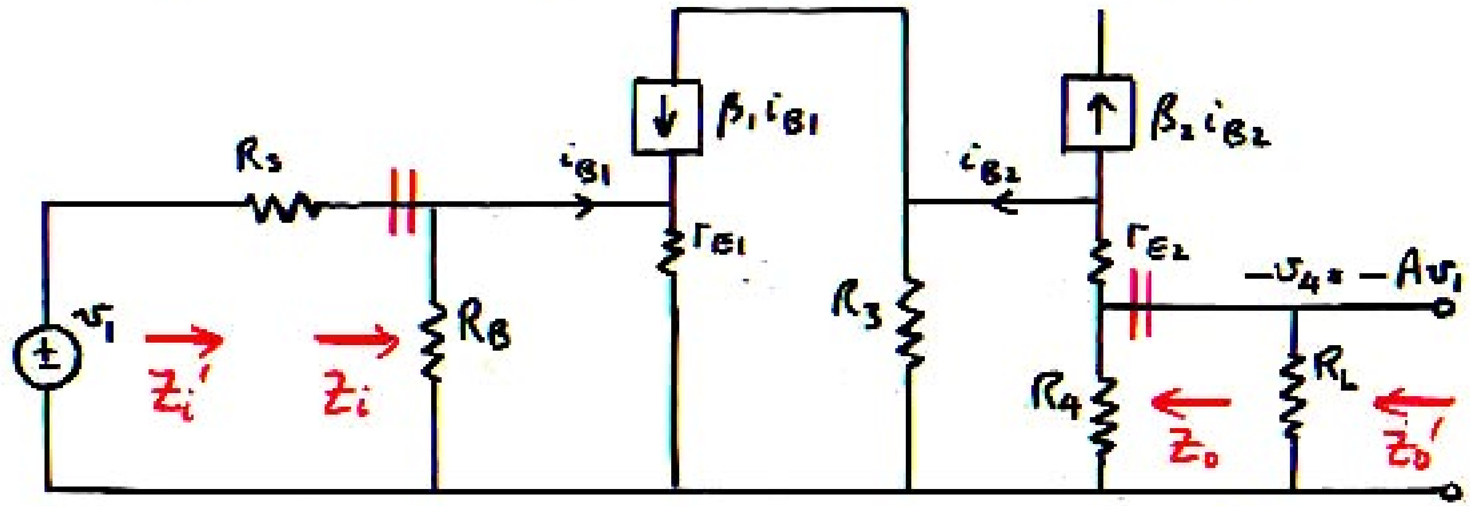


$$A_m = \underbrace{\frac{R_B}{R_B + (1+\beta_1) r_{E1}}}_{K_1 \uparrow} \cdot \underbrace{\frac{\beta_1 R_3}{R_S + R_B \| (1+\beta_1) r_{E1}}}_{K_2 \uparrow} \cdot \underbrace{\frac{R_4 \| R_L}{R_4 \| R_L + r_{E2} + R_3/(1+\beta_2)}}_{K_3 \uparrow}$$

Exercise

Find the gain in factored pole-zero form when the coupling capacitances C_1 and C_3 are included. Evaluate the corner frequencies, and sketch the magnitude asymptotes.

Example: the CE plus emitter follower amplifier



$$A_m = \frac{R_B}{R_B + (1+\beta_1) r_{E1}} \cdot \frac{\beta_1 R_3}{R_S + R_B \| (1+\beta_1) r_{E1}} \cdot \frac{R_4 \| R_L}{R_4 \| R_L + r_{E2} + R_3/(1+\beta_2)}$$

$K_1 \uparrow$ $K_2 \uparrow$ $K_3 \uparrow$

Exercise

Find the impedances Z_o', Z_i', Z_o, Z_i in factored pole-zero form in the presence of C_1 and C_3

Generalization: Input/Output Impedance Theorem

The Theorem permits the input and output impedances to be determined directly from the gain, by taking simple limits with respect to the source and load impedances, instead of from separate calculations on the model.

If the gain is in factored pole-zero form, so are the results for the input and output impedances.

Only gain factors containing the source or load impedance are needed for the calculations; all others cancel out.

The "outside" input and output impedance formulas require only one limit to be taken; the "inside" input and output impedance formulas require one additional limit.