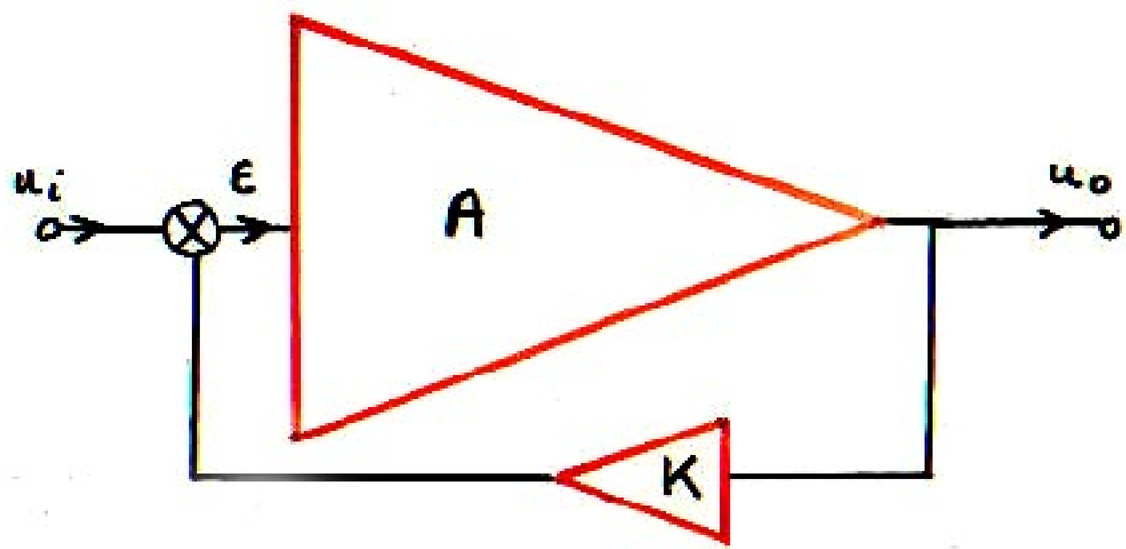
8

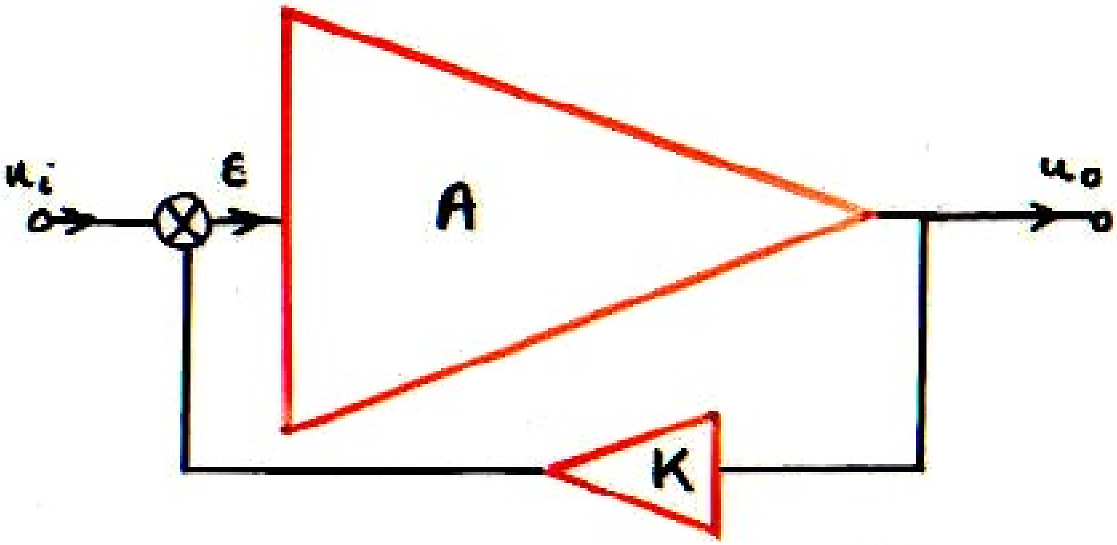
FEEDBACK:

AN IMPROVED FORMULA

Basic Properties



Basic Properties



Solution for closed-loop gain $G \equiv \frac{u_o}{u_i}$:

$$\varepsilon = u_i - K u_o \qquad u_o = A\varepsilon$$

$$u_o = A(u_i - K u_o)$$

$$\frac{u_o}{u_i} \equiv G = \frac{A}{1+AK}$$

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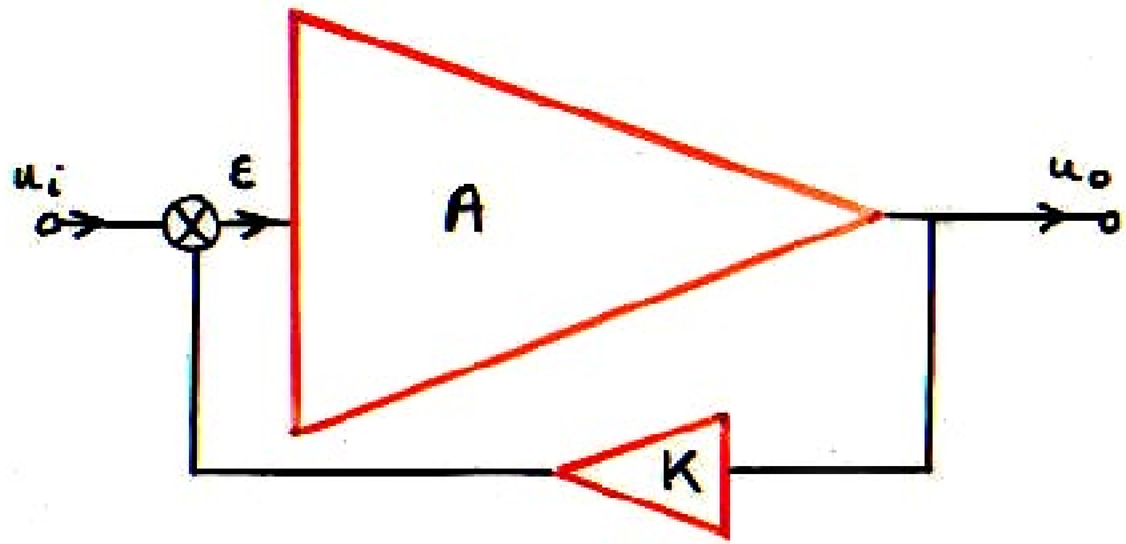
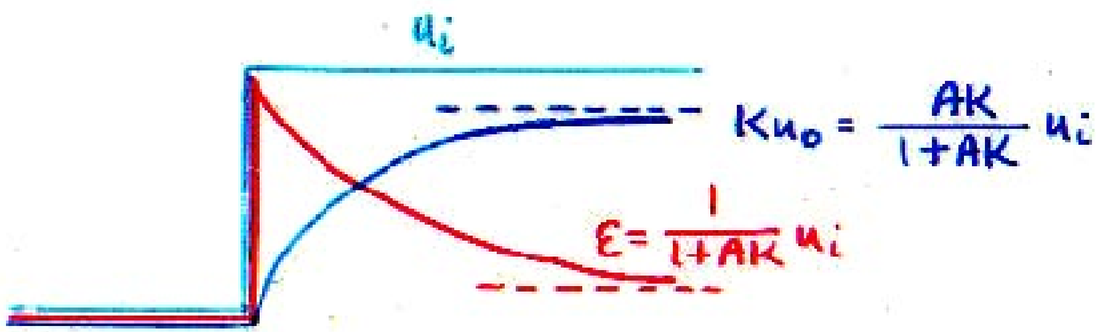


Response to a step in u_i:



Other forms of the result:

internal gain
forward gain
open-loop gain

$$G = \frac{A}{1+AK} = \frac{A}{1+T} = \frac{A}{F}$$

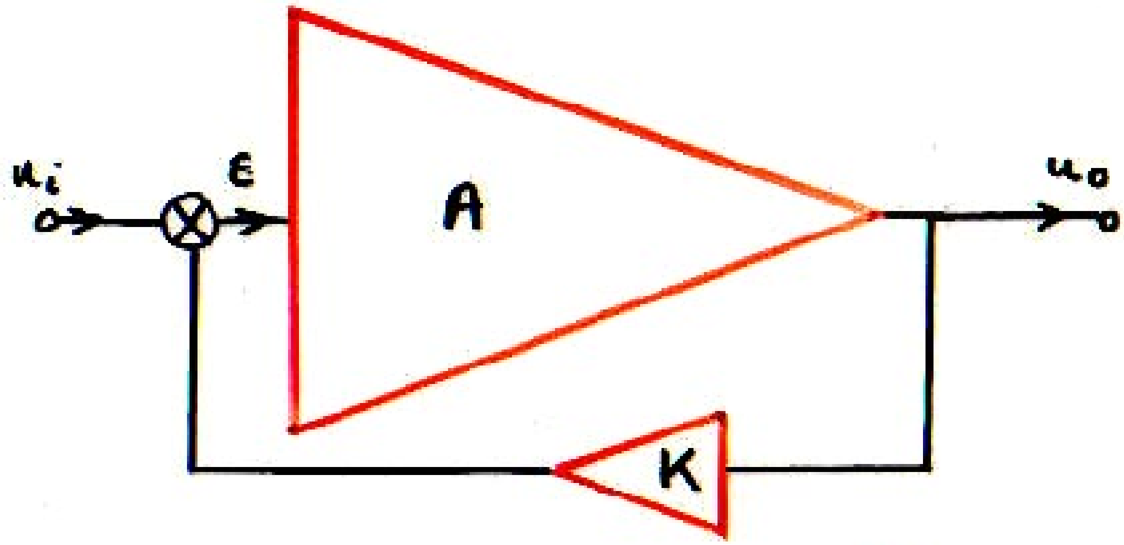
feedback ratio

closed-loop gain

return ratio
loop gain

return difference
feedback factor

Basic Properties



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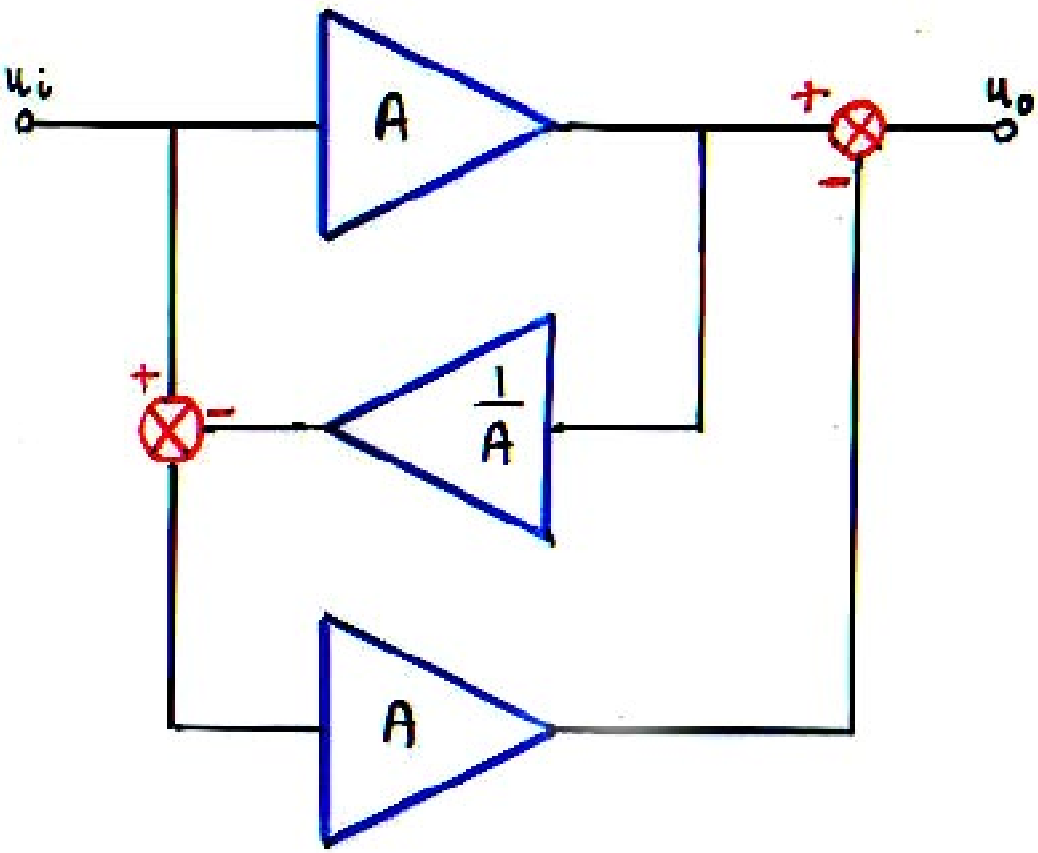
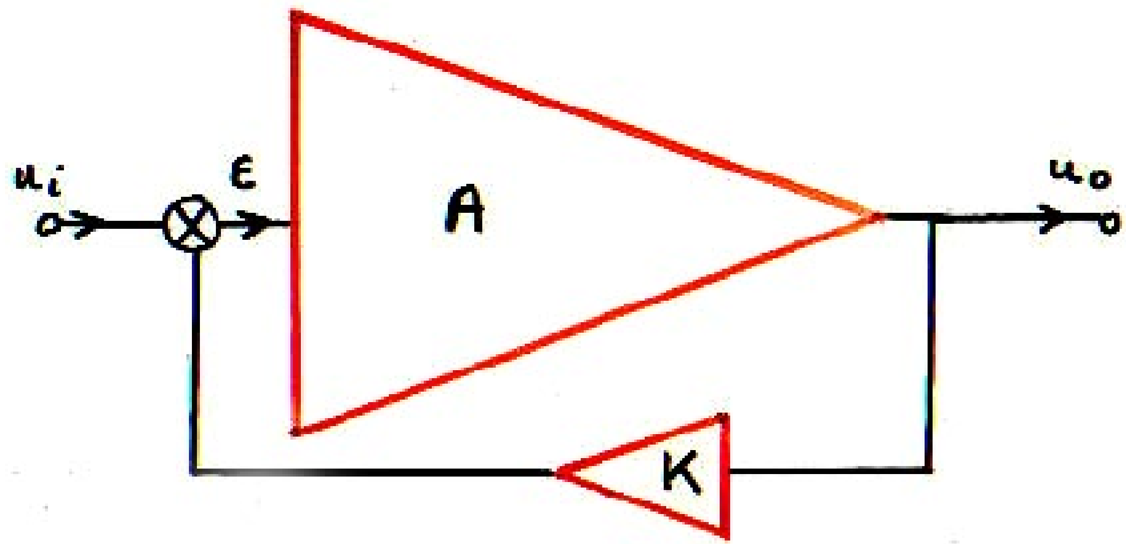


ui
A
+
uo
-
+
-
1/A
A

Basic Properties



exists because the residual transimmittance W_0 remains in the tube. For most circuits, however, the idea of bridge balance between input and output in the reference condition allows the problem to be much simplified. Since the balance cannot depend upon the input and output impedances, we can study the input to grid transmission for an arbitrary value of the impedance connected to the output terminals, or the plate to output transmission for an arbitrary value of the input impedance. By choosing the proper values in each case it is generally* possible to interrupt the residual feedback path.

These possibilities are reasonably obvious physically, but it will simplify later analysis if we also verify them mathematically. To represent the effect of a change in the output line upon the input to grid transmission in the reference condition, then, we can rewrite (6–1) as

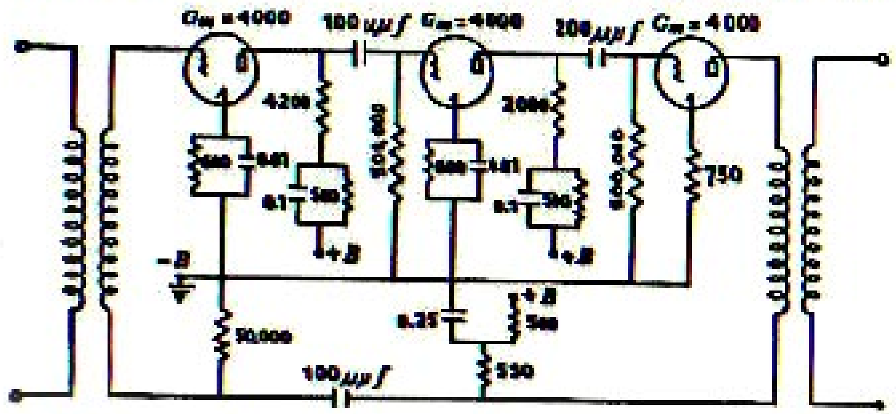
$$\frac{F}{S'} = \frac{\Delta_{12}}{\Delta^0} \frac{\Delta' + W_2 \Delta'_{22}}{\Delta'_{11} + W_2 \Delta'_{1122}}, \tag{6-2}$$

where W_2 is an arbitrary immittance added at the output terminals when the tube is in the reference condition. But we can also write

$$\Delta' \Delta'_{1122} = \Delta'_{11} \Delta'_{22}, \tag{6-3}$$

from the general identity (4–13), Chapter IV, if we recall that $\Delta'_{12} = 0$, since there is zero transmission from input to output in the reference state. It follows from (6–3) that (6–2) is independent of W_2, so that we can choose any value we like for this quantity without vitiating the original relationship between S' and F given by (6–1). In particular, then, we may give W_2 a value which will interrupt the return path from plate to grid, or in other words will make $\Delta_{22} = 0$. With this choice the second factor of (6–2) becomes independent of W_0, so that we are at liberty to suppose that the tube is dead rather than that it is in its reference condition.

550 ohm resistances are the low frequency values for the impedances of the two interstage networks and the β circuit. The other elements will be recognized as self-biasing units in the cathodes,† blocking condensers, grid



Other forms of the result:

$$\underset{\substack{\text{closed-loop}\\ \text{gain}}}{G} = \frac{\overset{\left\{\begin{array}{l}\text{internal gain}\\ \text{forward gain}\\ \text{open-loop gain}\end{array}\right.}{A}}{1+A\underset{\text{feedback ratio}}{K}} = \frac{A}{1+\underset{\left\{\begin{array}{l}\text{return ratio}\\ \text{loop gain}\end{array}\right.}{T}} = \frac{A}{\underset{\left\{\begin{array}{l}\text{return difference}\\ \text{feedback factor}\end{array}\right.}{F}}$$

Other forms of the result:

internal gain
forward gain
open-loop gain

$$G = \frac{A}{1+AK} = \frac{A}{1+T} = \frac{A}{F}$$

feedback ratio

closed-loop gain

return ratio
loop gain

return difference
feedback factor

ideal closed-loop gain

$$= \left(\frac{1}{K}\right)\left(\frac{T}{1+T}\right) \equiv G_\infty D$$

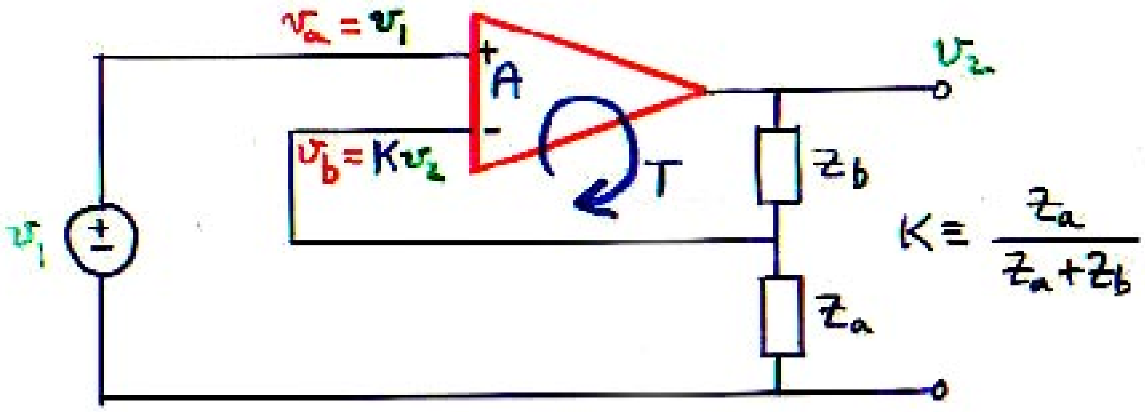
discrepancy factor

G_∞ is <u>given</u> (the <u>Specification</u>)

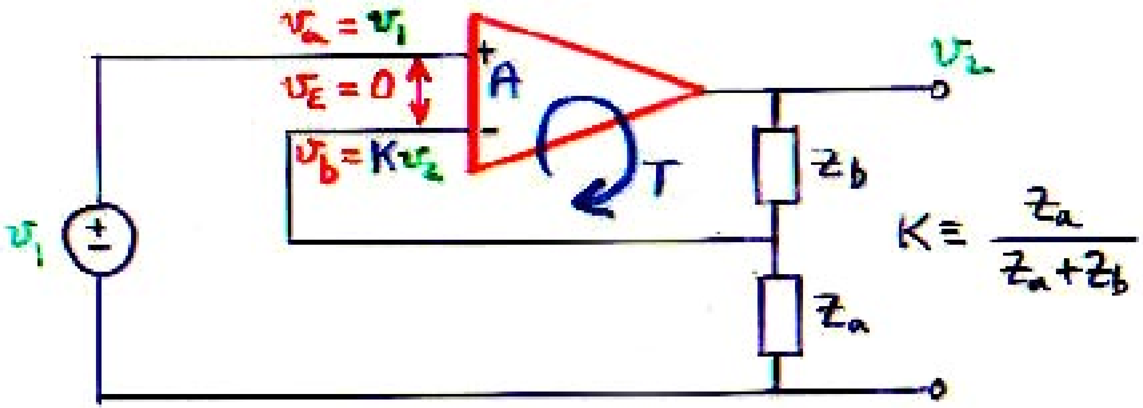
The <u>problem</u> of feedback design is how to make <u>D close enough to 1</u>, over the <u>specified frequency range</u>.

G_∞ is the gain conventionally calculated

on the assumption of $\left\{ \begin{array}{l} \text{infinite loop gain } T \\ \text{infinite forward gain } A \\ \text{zero error signal } v_E \end{array} \right.$



$v_a = v_1$
+
A
-
$v_b = K v_2$
T
v_2
Z_b
Z_a
v_1
$K = \frac{Z_a}{Z_a + Z_b}$

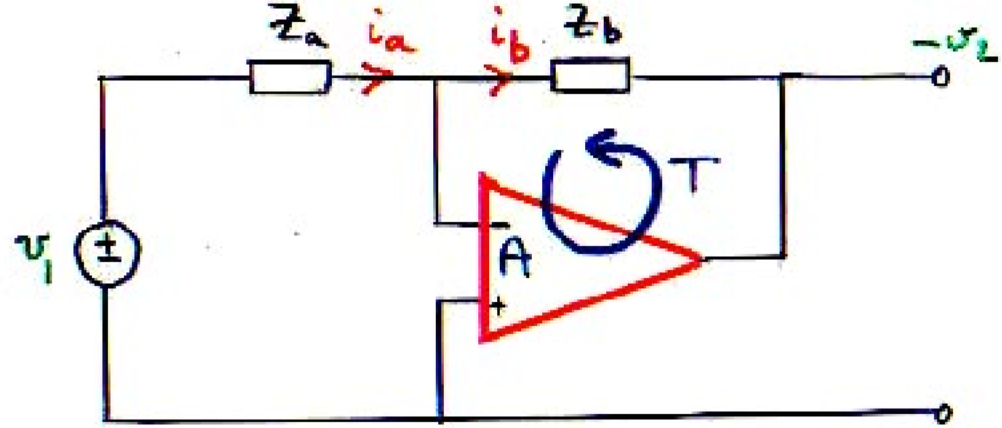


For zero error signal $v_\epsilon = 0$:

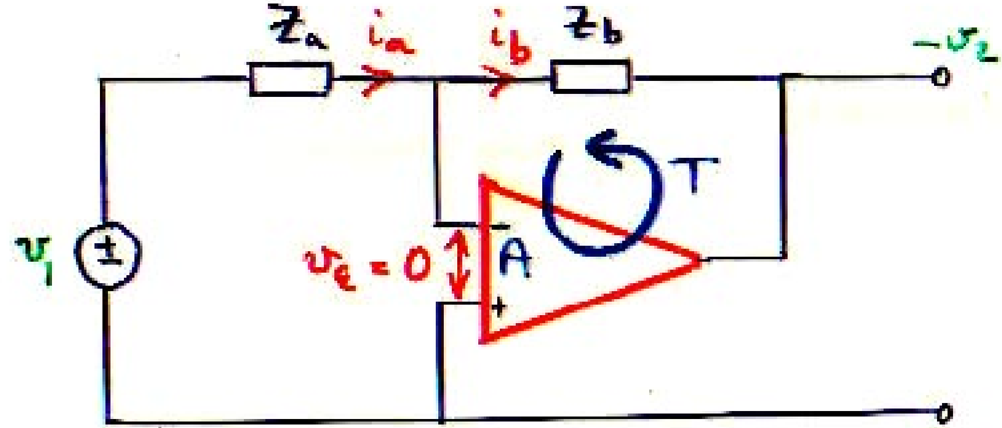
$$v_b = v_a$$

$$K v_2 = v_1$$

$$\frac{v_2}{v_1} = G_\infty = \frac{1}{K} = \frac{Z_a + Z_b}{Z_a}$$



Za
ia
ib
Zb
-v2
T
A
v1



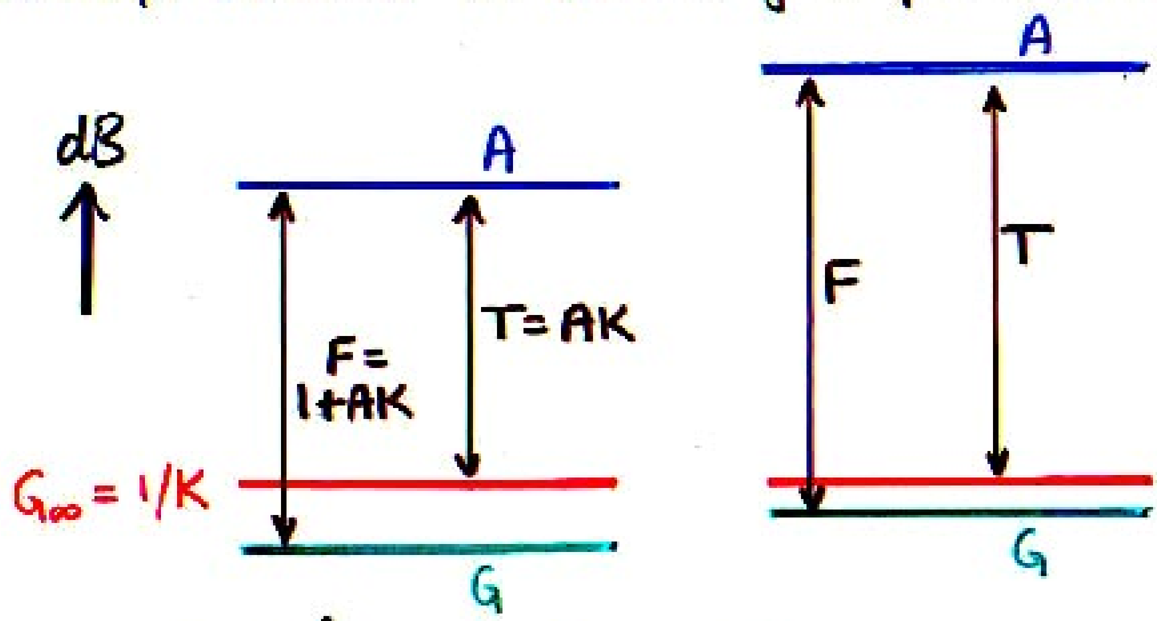
For zero error signal $v_\varepsilon = 0$:

$$i_b = i_a$$

$$\frac{v_2}{Z_b} = \frac{v_1}{Z_a}$$

$$\frac{v_2}{v_1} = G_\infty = \frac{Z_b}{Z_a}$$

Relationships between the various gain quantities:



$$G = \frac{A}{1+AK} = \frac{A}{1+T} = \frac{A}{F}$$

Note: to get the same output u_o closed-loop as open-loop, must increase input u_i from u_o/A to $F u_o/A$.

Principal effect of feedback:

$$G \xrightarrow{T \to 0} A \qquad\qquad G \xrightarrow{T \to \infty} \frac{1}{K}$$

loop gain:
$T \equiv AK$

Feedback transfers sensitivity from A to K:

$$G = \frac{A}{1+AK}$$

$$\ln G = \ln A - \ln(1+AK)$$

$$\frac{\Delta G}{G} = \frac{\Delta A}{A} - \frac{K\Delta A + A\Delta K}{1+AK}$$

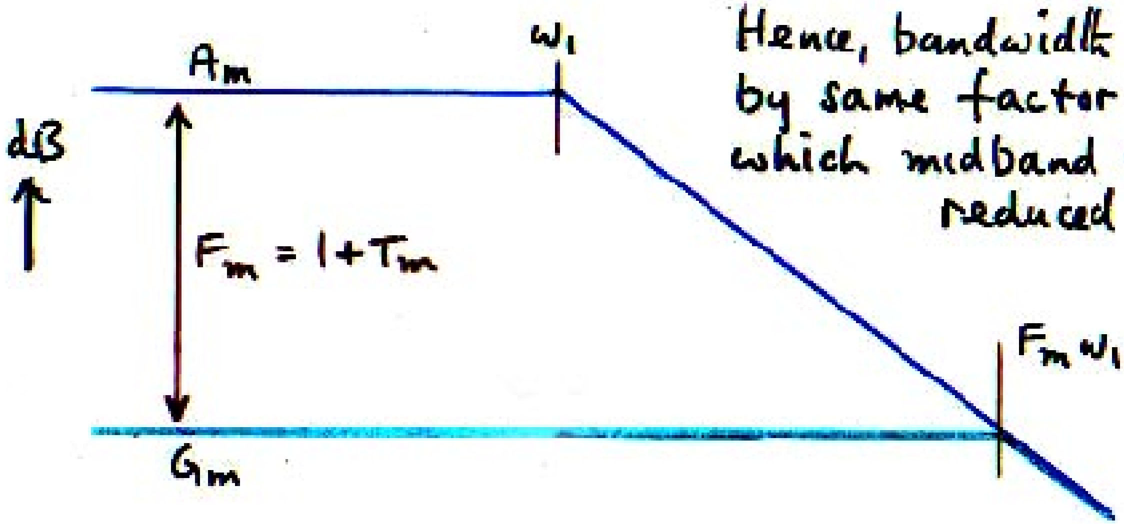
$$= \underbrace{\frac{1}{1+T}}_{\text{decreases}} \frac{\Delta A}{A} - \underbrace{\frac{T}{1+T}}_{\text{increases}} \frac{\Delta K}{K}$$

decreases / increases
for increasing T

Extension of bandwidth

Consider the simplest high-frequency rolloff, a single pole:
$$A = A_m \frac{1}{1+\frac{s}{\omega_1}}$$

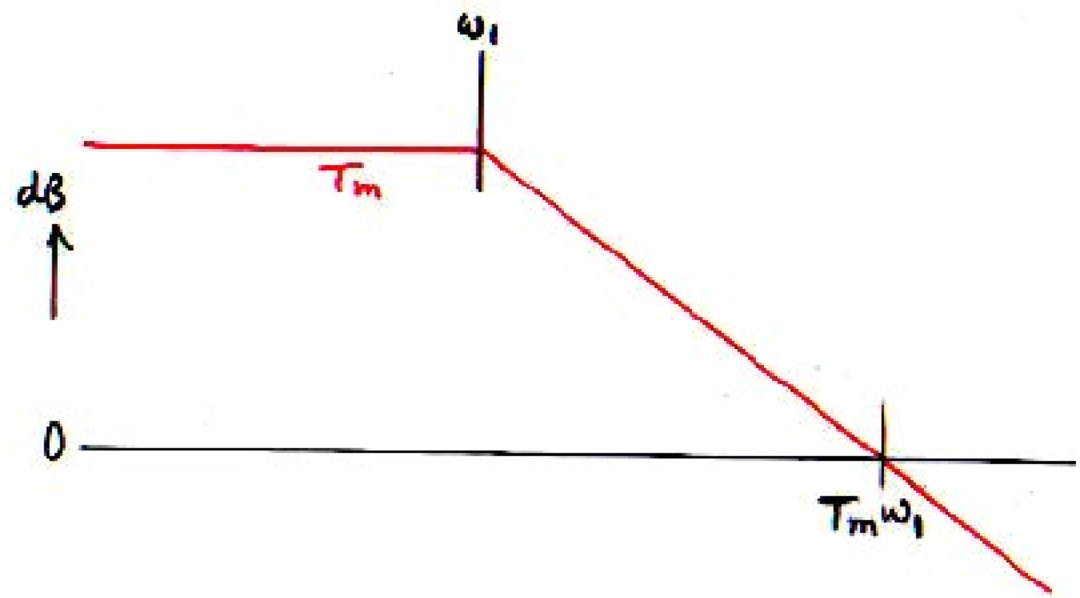
Then
$$G = \frac{A_m \frac{1}{1+\frac{s}{\omega_1}}}{1 + A_m K \frac{1}{1+\frac{s}{\omega_1}}} = \frac{A_m}{1 + A_m K + \frac{s}{\omega_1}} = \frac{A_m}{1+T_m} \frac{1}{1+\frac{s}{(1+T_m)\omega_1}}$$
$$= G_m \frac{1}{1+\frac{s}{F_m \omega_1}} \quad \text{where} \quad G_m = \frac{A_m}{F_m}$$



Hence, bandwidth is increased by same factor F_m by which midband gain is reduced

An alternative approach emphasizes T:

$$T = A_m K \frac{1}{1+\frac{s}{\omega_1}} = T_m \frac{1}{1+\frac{s}{\omega_1}}$$



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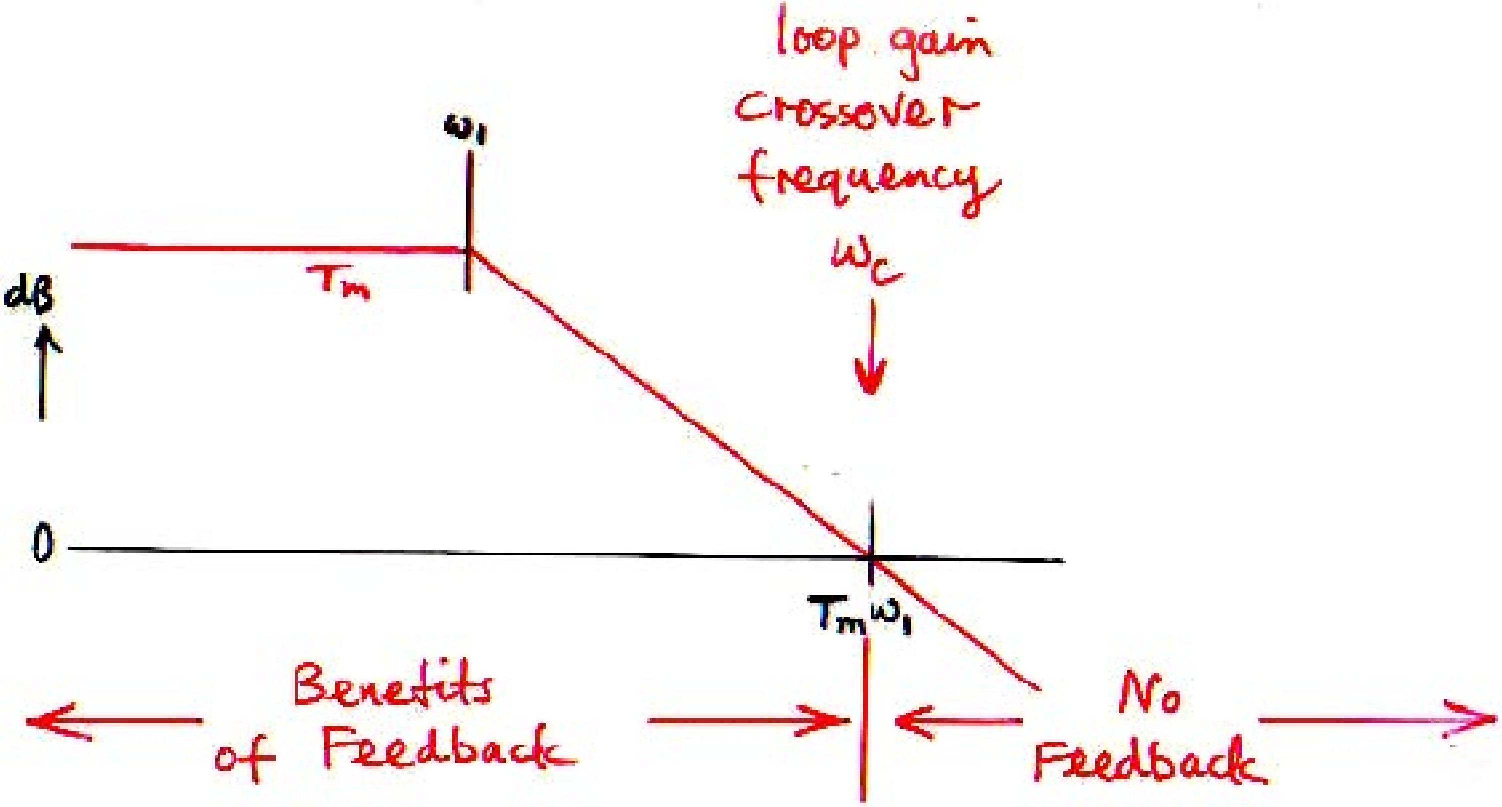
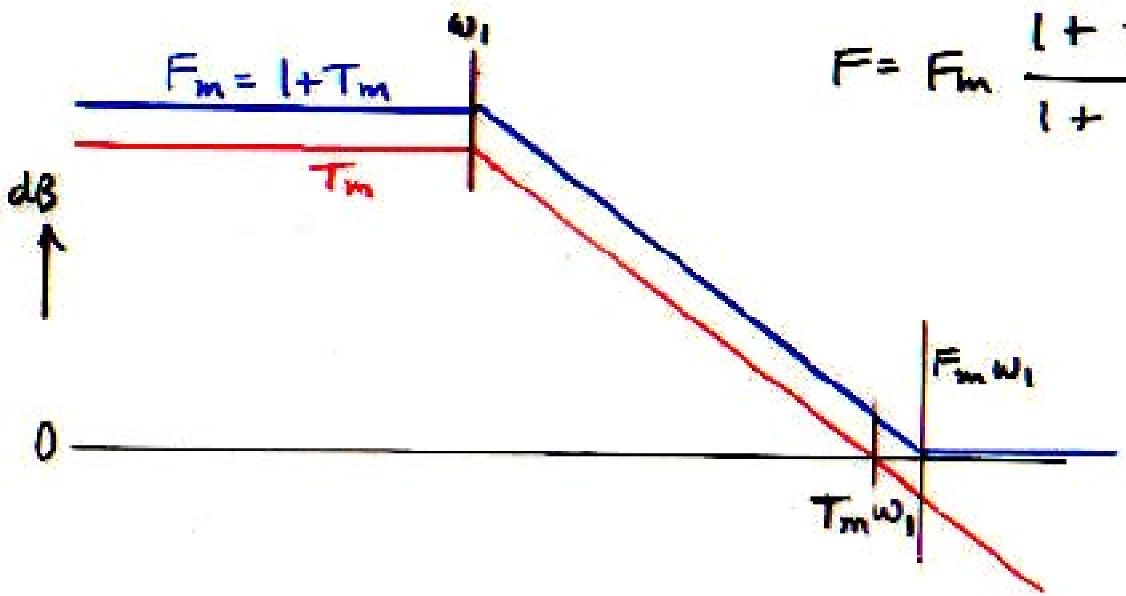


An alternative approach emphasizes T:

$$T = A_m K \frac{1}{1+\frac{s}{\omega_1}} = T_m \frac{1}{1+\frac{s}{\omega_1}}$$

Construct $F = 1 + T$



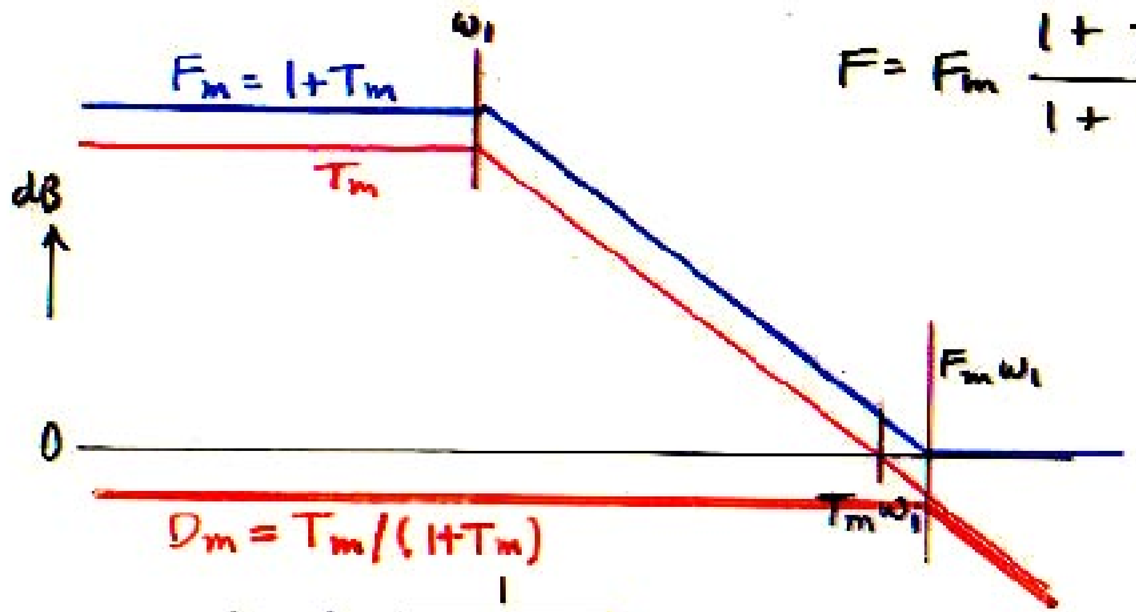
$$F = F_m \frac{1+\frac{s}{F_m\omega_1}}{1+\frac{s}{\omega_1}}$$

An alternative approach emphasizes T:

$$T = A_m K \frac{1}{1+\frac{s}{\omega_1}} = T_m \frac{1}{1+\frac{s}{\omega_1}}$$

Construct $F = 1+T$

Construct $D = \frac{T}{1+T} = \frac{T}{F}$

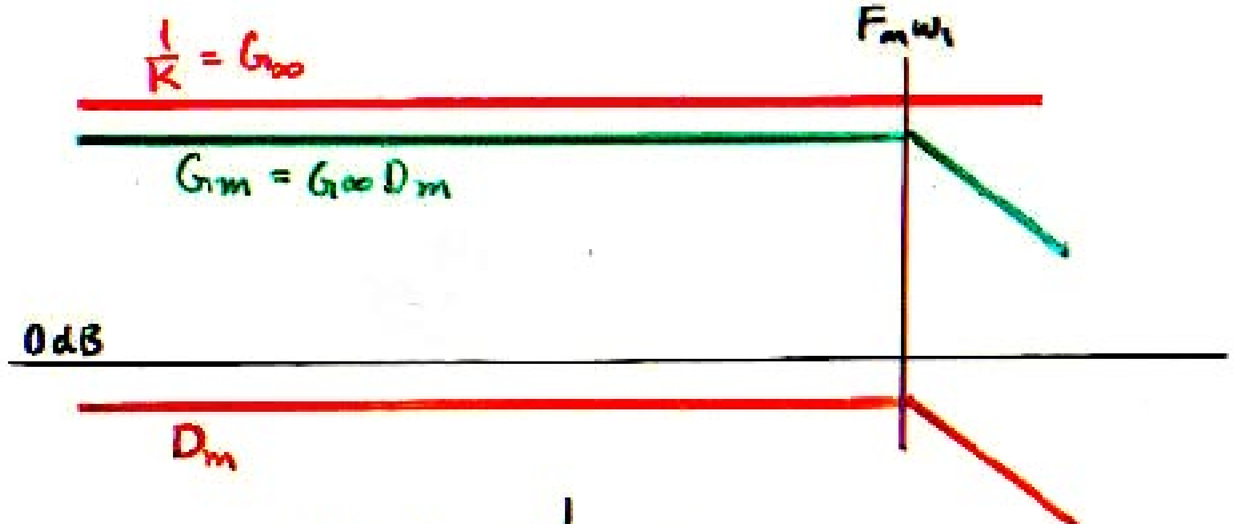


$$F = F_m \frac{1+\frac{s}{F_m\omega_1}}{1+\frac{s}{\omega_1}}$$

$$D = D_m \frac{1}{1+\frac{s}{F_m\omega_1}}$$

Hence the closed-loop gain G can be obtained from

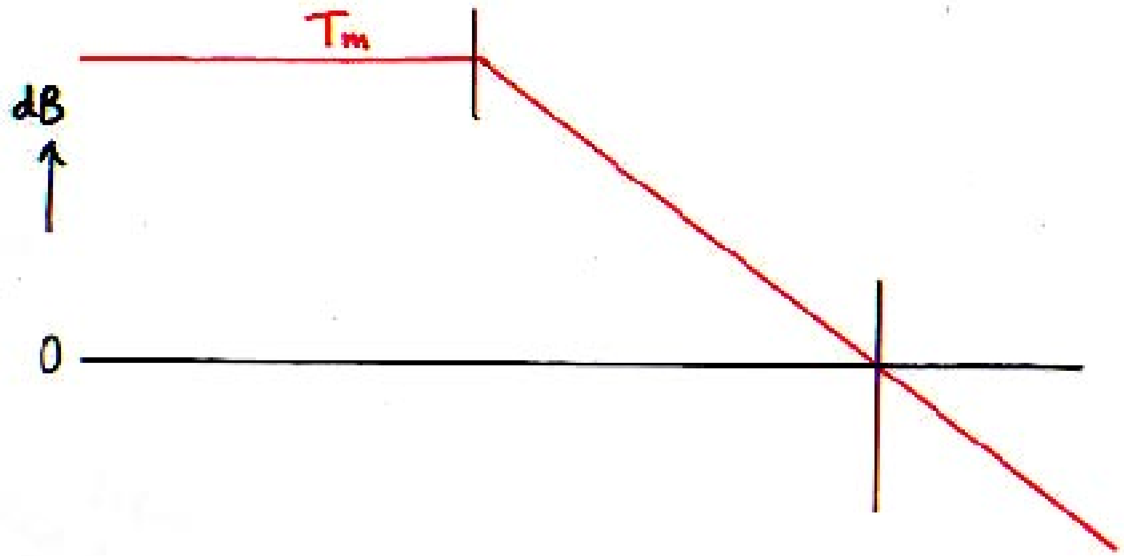
$$G = \frac{1}{K} D = G_{\infty} D$$



$$G = G_m \frac{1}{1 + \frac{s}{F_m \omega_1}}$$

Another way to obtain D is directly from T as

$$D = \frac{1}{1 + \frac{1}{T}}$$



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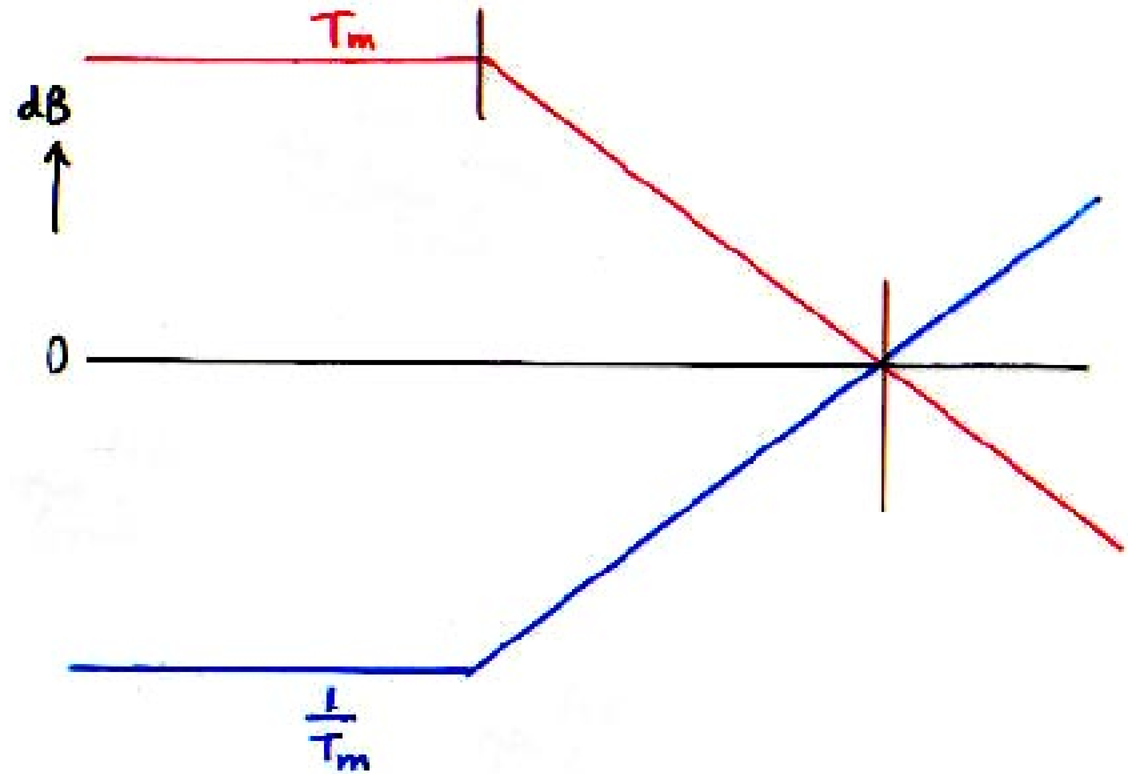

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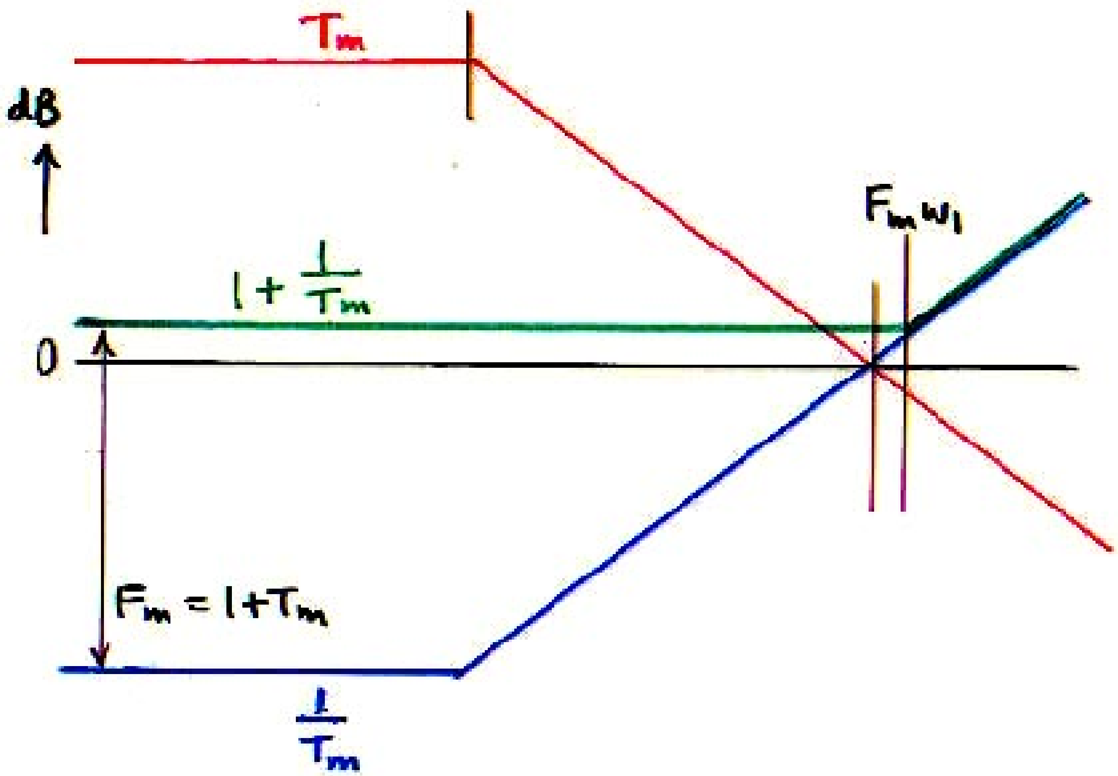

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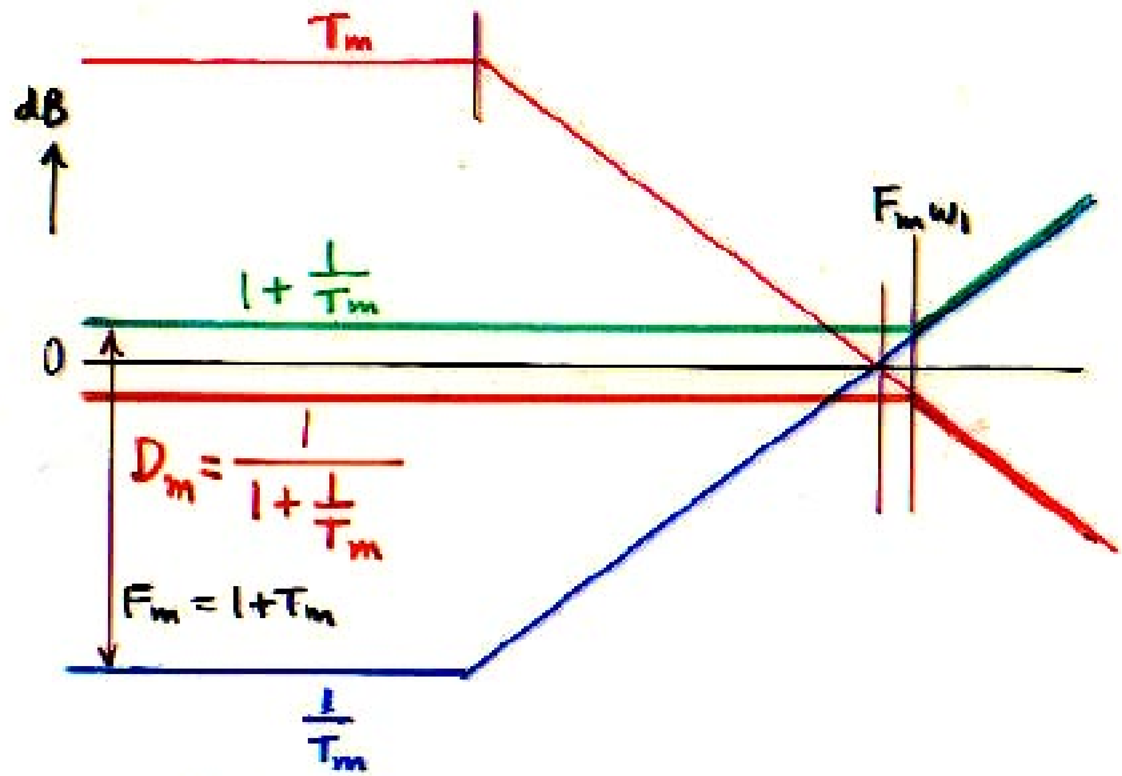


which gives the same result for D.

Although the factored pole-zero forms for F and D could easily have been obtained analytically in this case, the above graphical procedure saves much algebra in more complicated cases because suitable approximations can be seen immediately.

Notice that in finding $F = 1 + T$ and $1/D = 1 + 1/T$ a sum (or in general a difference) is determined from the asymptotes on log scales. This is an example of the powerful technique of doing the algebra on the graph.

Generalization: Doing the Algebra on the Graph

The log-log scales of dB vs. log frequency graphs permit determination of

Not only:

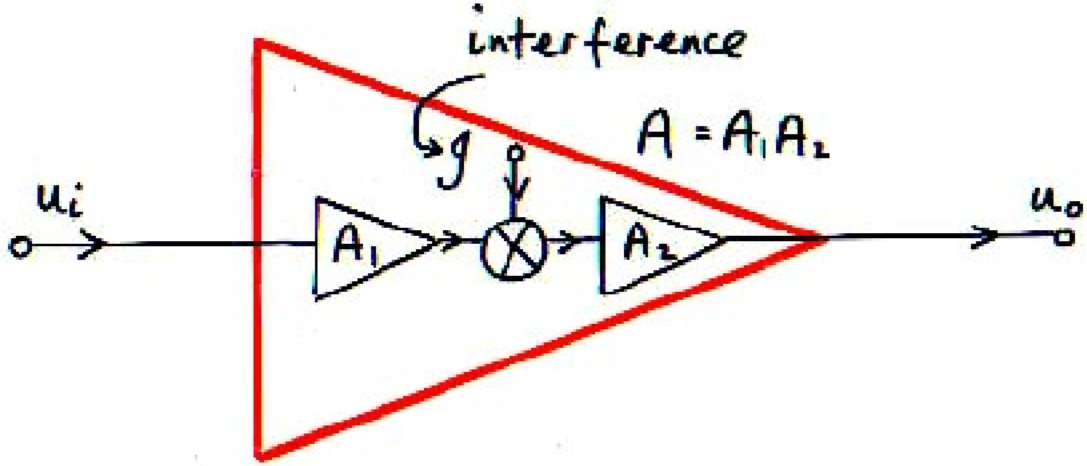
Exact combinations of products and quotients of constituent factors

But also:

Approximate combinations of sums and differences of constituent factors: whichever is the larger dominates. This technique permits approximate analytic results to be obtained, in which algebraic approximations are replaced by graphical approximations.

Examples: Analytic determination from T of $F = 1+T$ and $D = T/F$.

Reduction of interference signal



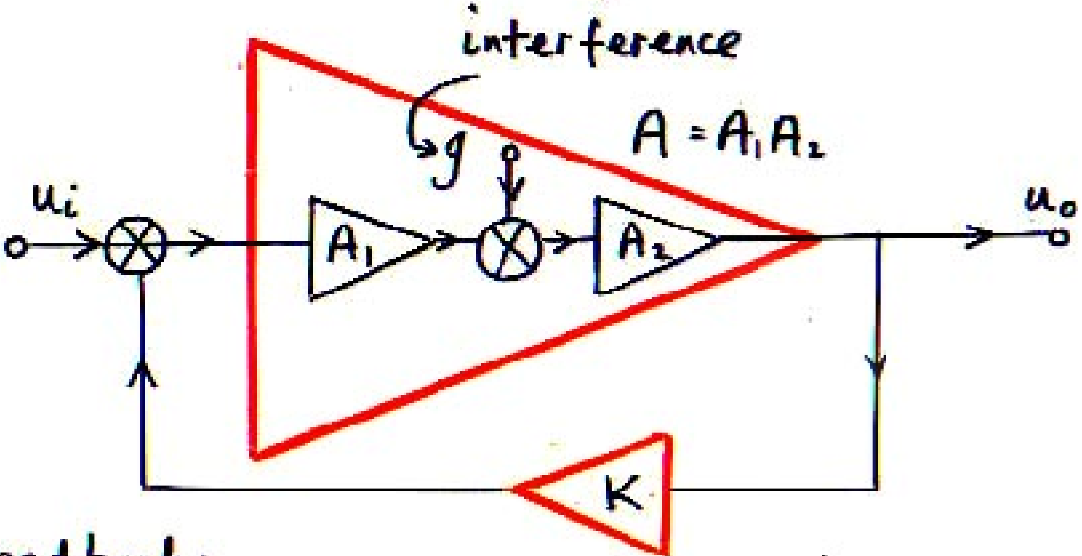
No feedback:

$$u_o = A_1 A_2 u_i + A_2 g$$

Interference-to-output transfer function:

$$\frac{u_o}{g} = A_2$$

Reduction of interference signal



No feedback:

$$u_o = A_1 A_2 u_i + A_2 g$$

Interference - to - output transfer function:

$$\frac{u_o}{g} = A_2$$

With feedback:

$$u_o = \frac{A_1 A_2 u_i + A_2 g}{1+AK}$$

Interference - to - output transfer function:

$$\frac{u_o}{g} = \frac{A_2}{1+AK} = \frac{A_2}{1+T} = \frac{A_2}{F}$$

Hence

$$\left.\frac{u_o}{g}\right|_{\substack{\text{with}\\ \text{fbk}}} = \frac{1}{F} \cdot \left.\frac{u_o}{g}\right|_{\substack{\text{no}\\ \text{fbk}}}$$

Examples of interference-to-output transfer functions:

Amplifier power supply variations (tolerance, ripple) that show up in the output.

Power supply line variations (tolerance, ripple) that show up in the regulated output (audio susceptibility).