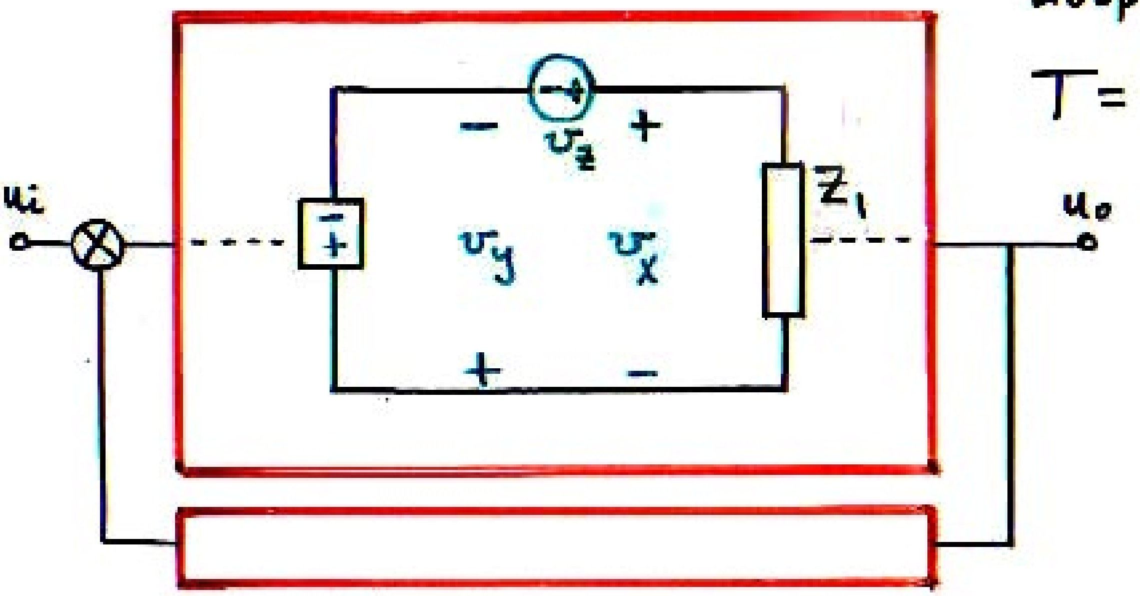
Two points satisfy these conditions

1. Inject a voltage in series with a controlled voltage generator:

$v_z = v_x + v_y$

Loop gain:

$$T = \frac{v_y}{v_x}\Big|_{u_i = 0}$$



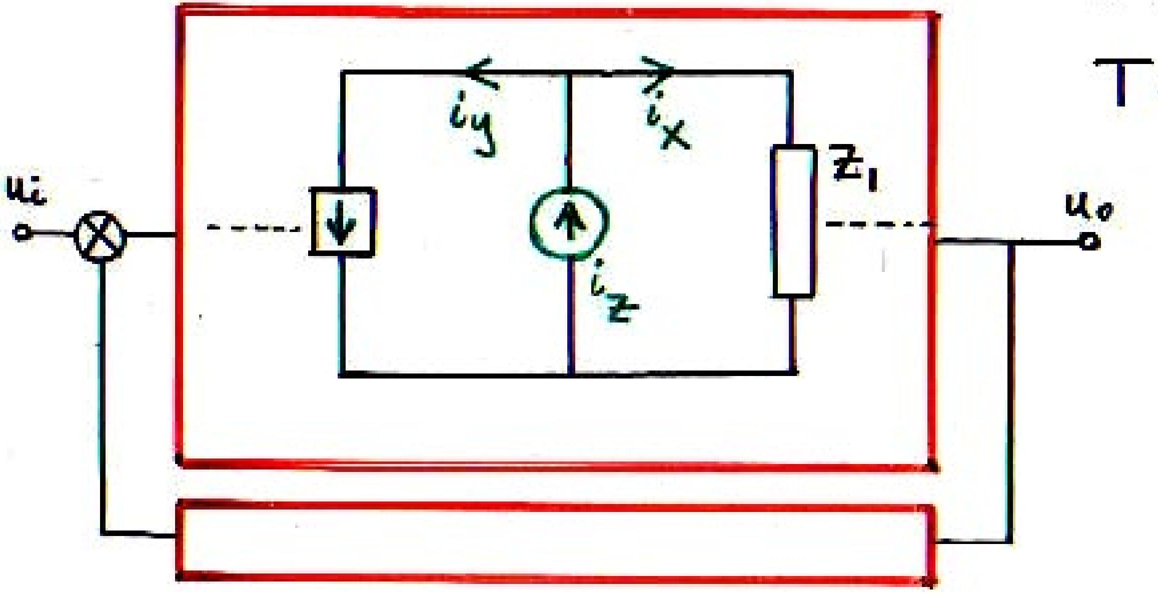
Two points satisfy these conditions

2. Inject a current in shunt with a controlled current generator:

$i_z = i_x + i_y$

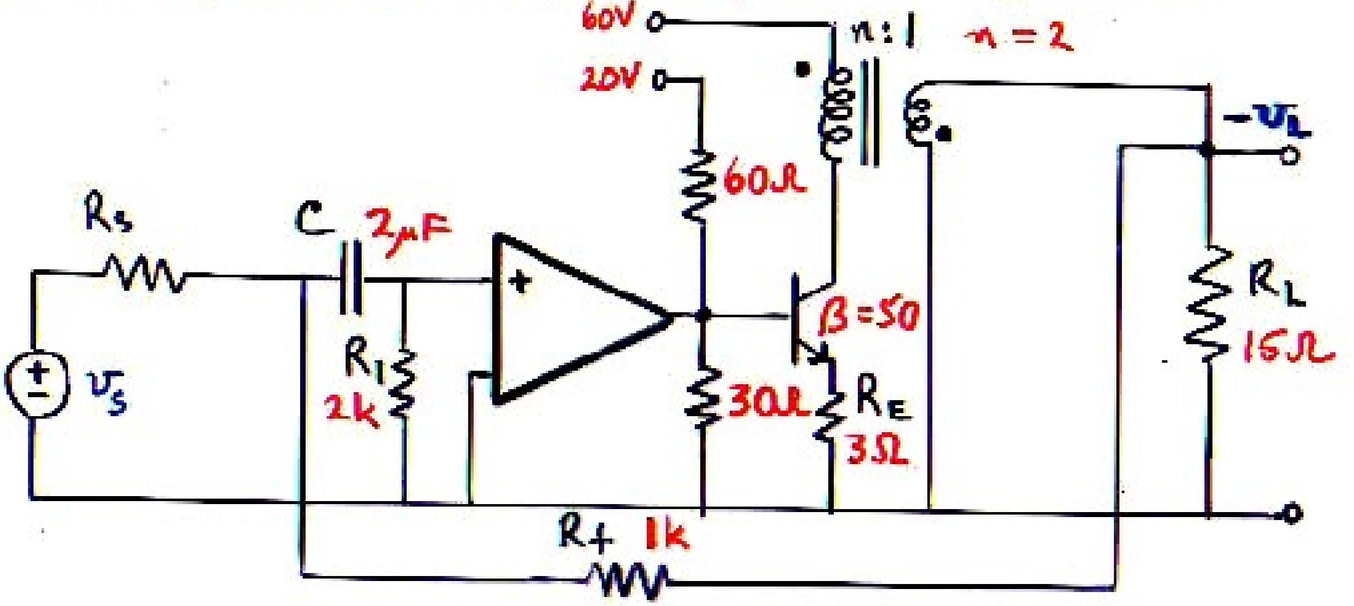
Loop gain:

$$T = \left.\frac{i_y}{i_x}\right|_{u_i=0}$$

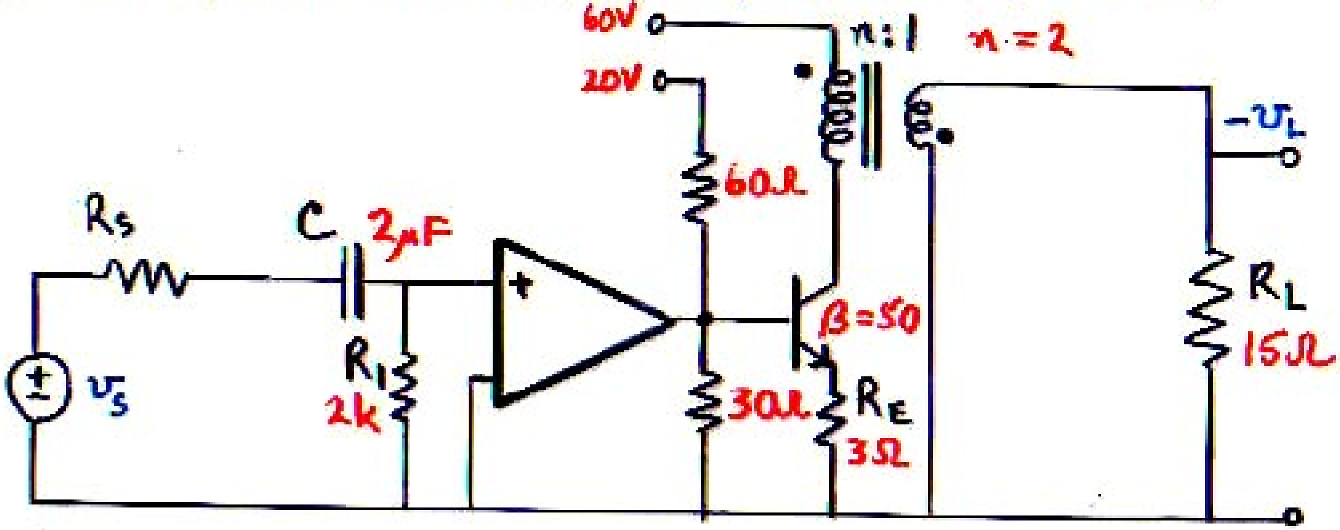


Determination of Feedback System Parameters – Example

Single-ended Class A audio feedback amplifier. The driver opamp has a gain $A_1 = A_{10}/(1+s/\omega_A)$, where $A_{10} = 8\,dB$ and $f_A = 2\,kHz$, and an output impedance of 4.5Ω

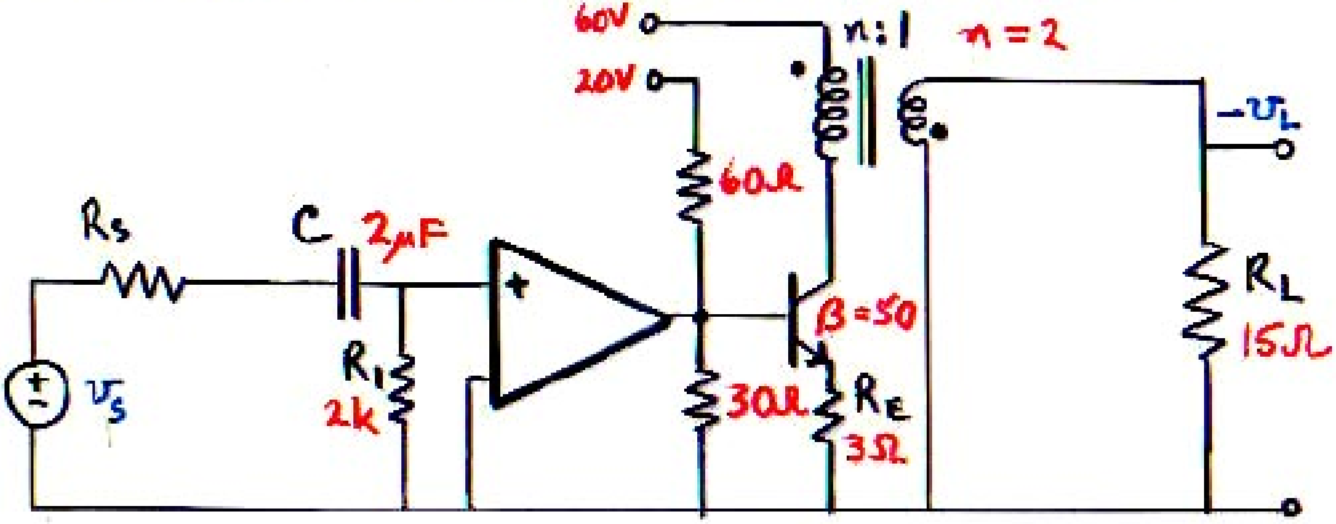


Single-ended Class A audio feedback amplifier. The driver opamp has a gain $A_1 = A_{10}/((1+s/\omega_A)$, where $A_{10} = 8dB$ and $f_A = 2kHz$, and an output impedance of 4.5Ω.



The conventional analysis method is to break the loop by opening the feedback path, then calculate A and K separately to get $T = AK$ and $G = A/(1+T)$.

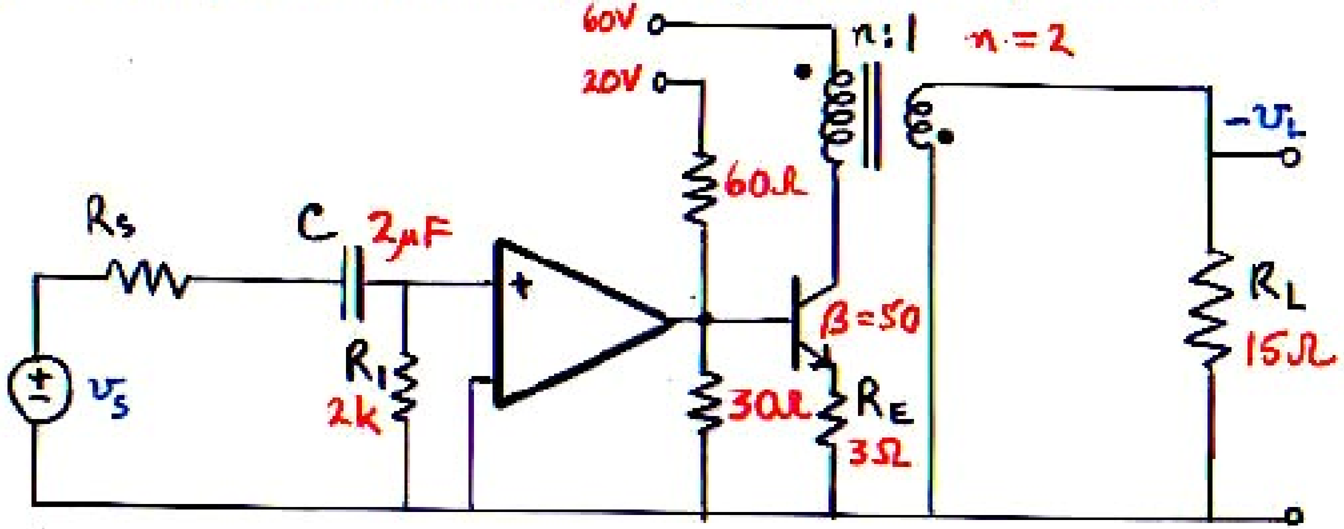
Single-ended Class A audio feedback amplifier. The driver opamp has a gain $A_1 = A_{10}/((1+s/\omega_A))$, where A_{10} = 8dB and f_A = 2kHz, and an output impedance of 4.5Ω.



This is wrong, because loading by the feedback network at output and input is ignored.

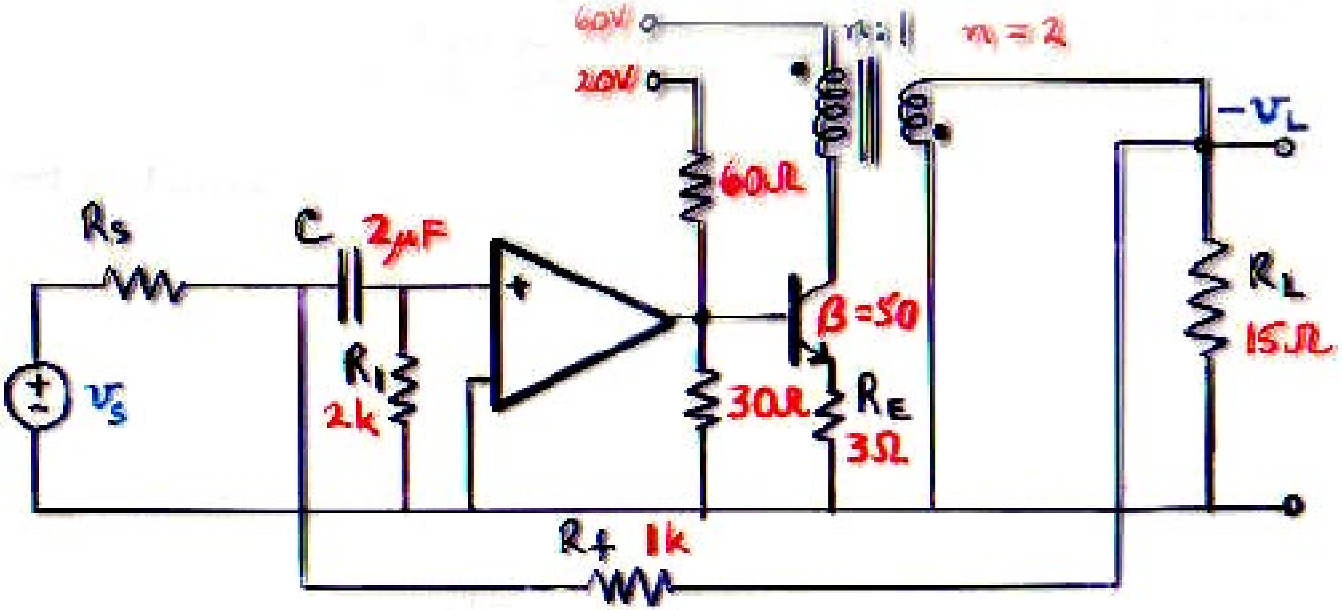
The loading could be simulated, but involves guesswork.

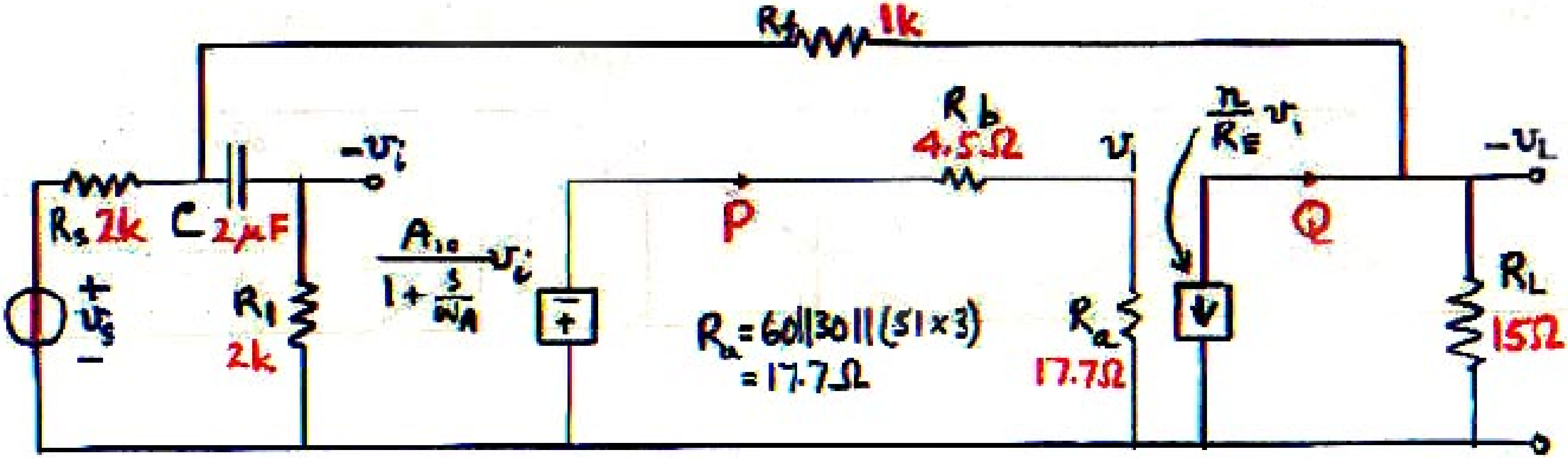
Single-ended Class A audio feedback amplifier. The driver opamp has a gain $A_1 = A_{10}/((1+s/\omega_A)$, where $A_{10} = 8$ dB and $f_A = 2$ kHz, and an output impedance of 4.5Ω

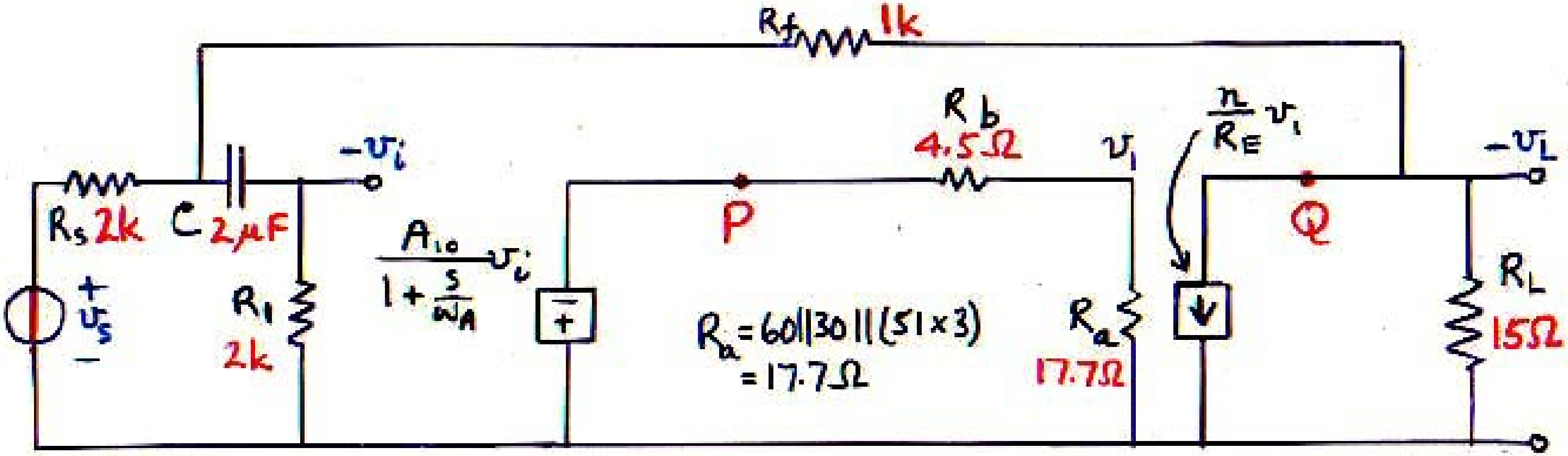


Instead, break the loop in the forward path and find T directly.

First, make an equivalent circuit model of the closed loop.

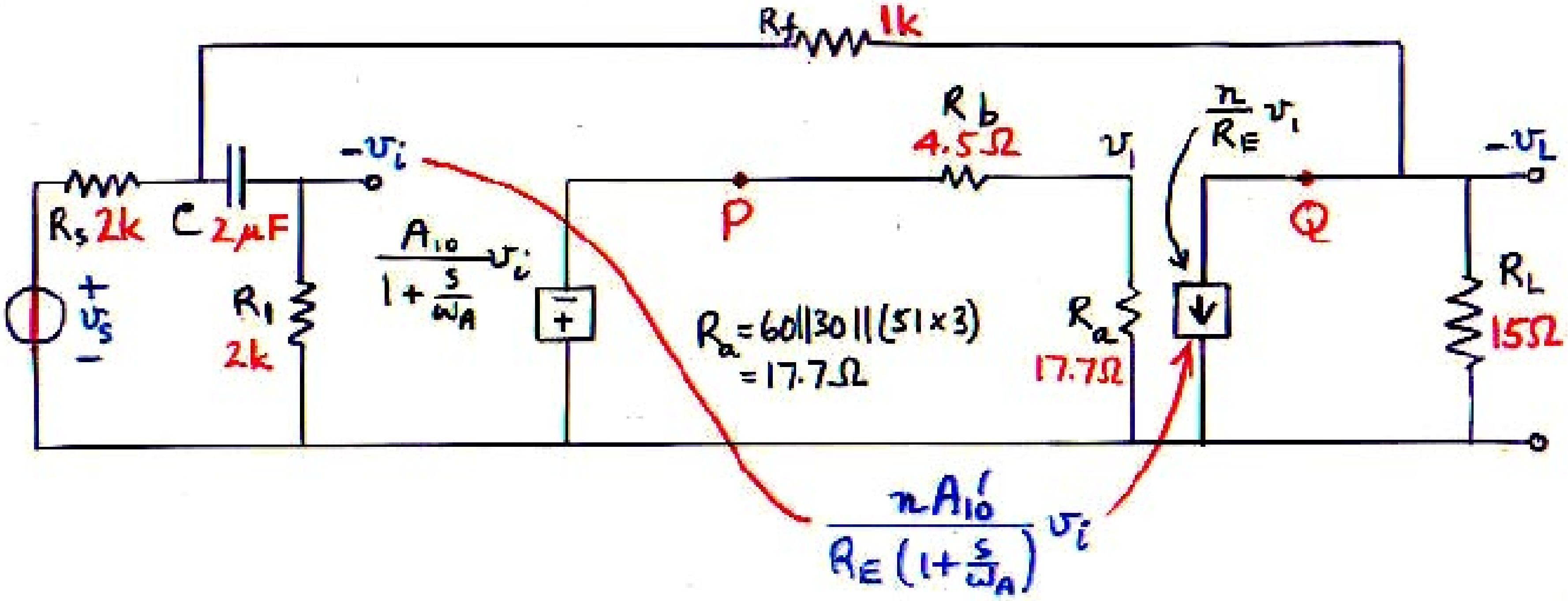

60V
20V
n:1
n = 2
60Ω
R_s
C 2μF
R_1
2k
v_s
β=50
30Ω
R_E
3Ω
R_L
15Ω
−v_L
R_f 1k


R_f 1k
R_b
4.5Ω
v_1
n/R_E v_1
−v_L
−v_i
R_s 2k
C 2μF
R_1
2k
+ v_s −
A_10 / (1 + s/ω_A) v_i
P
Q
R_a = 60||30||(51×3)
= 17.7Ω
R_a
17.7Ω
R_L
15Ω



For all calculations concerning loop gain, the input voltage v_s is zero.

Suitable injection points:
- Series voltage at point P
- Shunt current at point Q



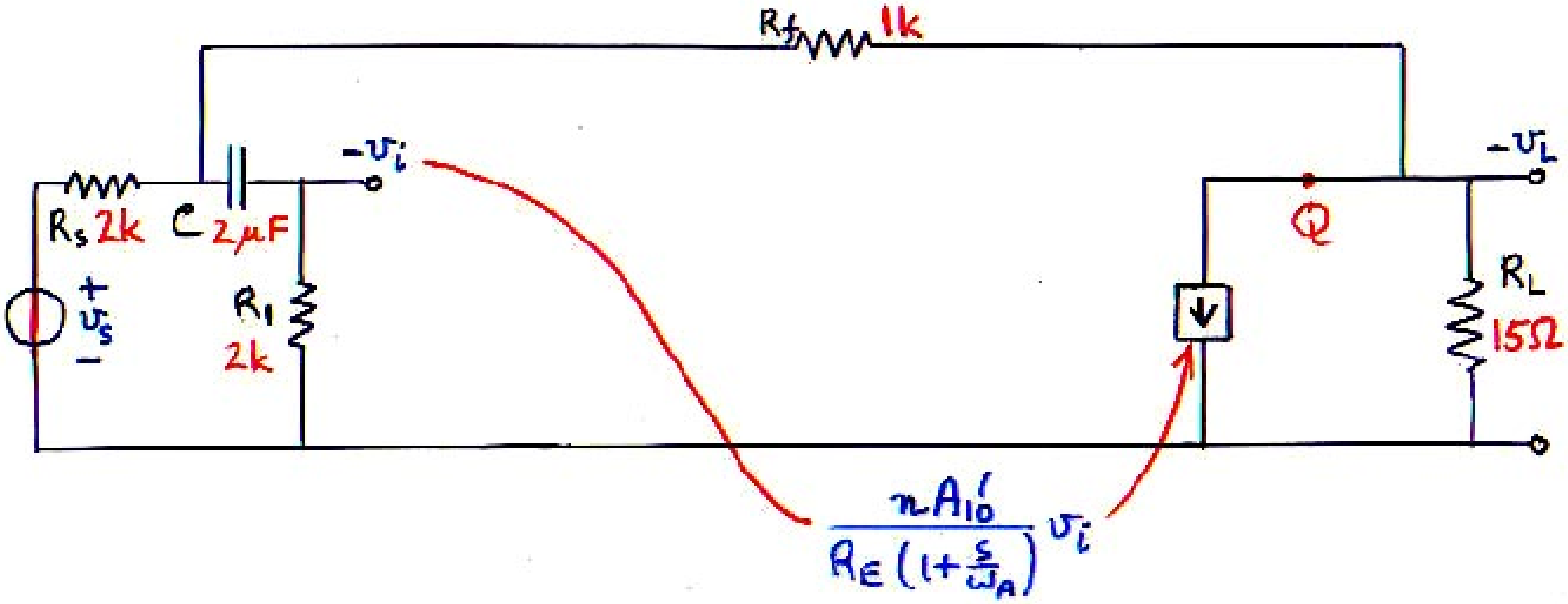
If current injection is chosen, the gain from v_i to the power stage output generator can be condensed into a single factor:

$$\frac{A_{10}}{1+\frac{s}{\omega_A}}\,\frac{R_a}{R_a+R_b}\,\frac{n}{R_E}\,v_i = \frac{nA_{10}'}{R_E\left(1+\frac{s}{\omega_A}\right)}v_i$$

where

$$A_{10}' = A_{10}\frac{R_a}{R_a+R_b} = 8\,\mathrm{dB}\times\frac{17.7}{17.7+4.5} = 2.51\times0.80$$
$$= 2.0 \Rightarrow 6\,\mathrm{dB}$$

is the loaded gain of the opamp.



If current injection is chosen, the gain from v_i to the power stage output generator can be condensed into a single factor:

$$\frac{A_{10}}{1+\frac{s}{\omega_A}} \frac{R_a}{R_a+R_b} \frac{n}{R_E} v_i = \frac{n A_{10}'}{R_E\left(1+\frac{s}{\omega_A}\right)} v_i$$

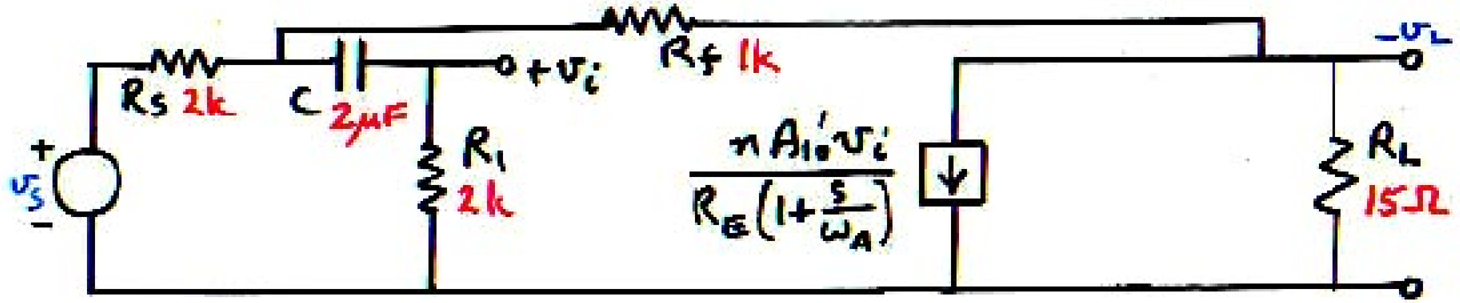
where

$$A_{10}' = A_{10}\frac{R_a}{R_a+R_b} = 8\,\text{dB} \times \frac{17.7}{17.7+4.5} = 2.51 \times 0.80$$

$$= 2.0 \Rightarrow 6\,\text{dB}$$

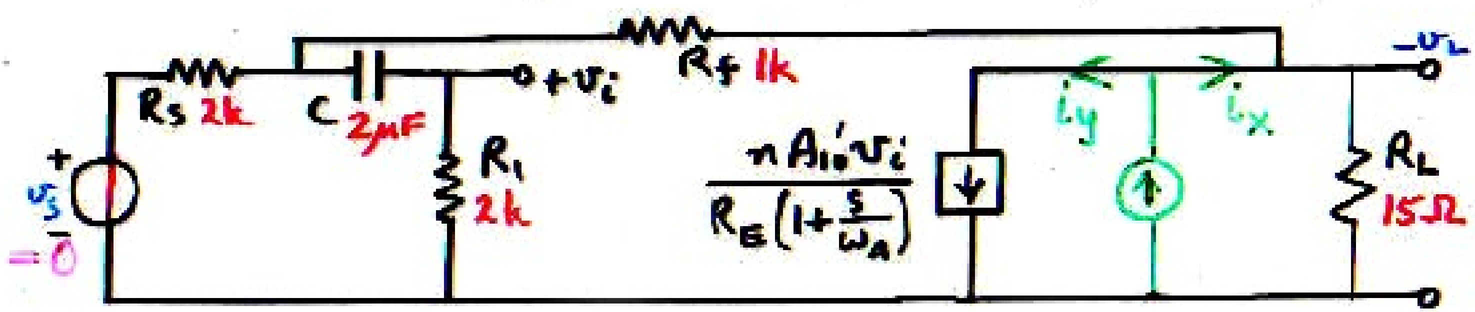
is the loaded gain of the opamp.

Ac model



Rs 2k
C 2μF
R1 2k
+ vi
Rf 1k
+
vs
-
$\frac{n A_{10}' v_i}{R_E(1+\frac{s}{\omega_A})}$
-vL
RL 15Ω

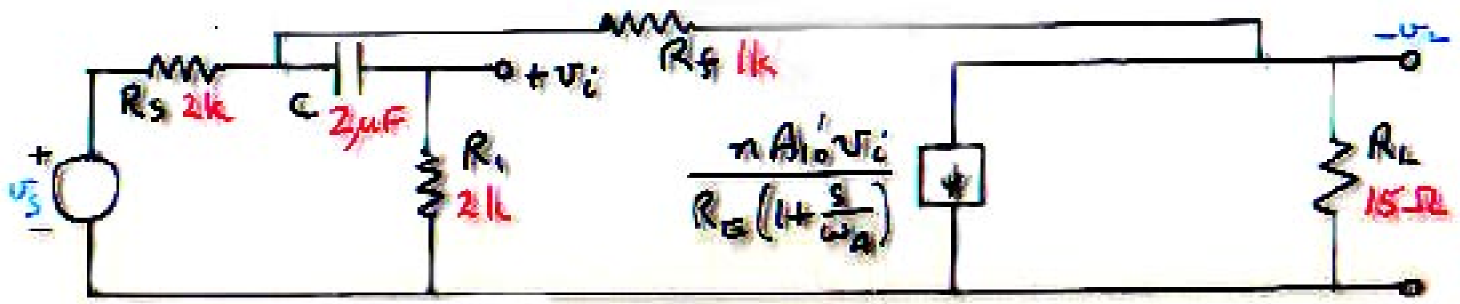
AC model



$$T = \left.\frac{i_y}{i_x}\right|_{v_s=0} = \frac{R_L}{\cancel{R_L}+R_f+R_s \| R_1(1+\frac{\omega_1}{s})}\;\frac{R_s}{R_s+R_1(1+\frac{\omega_1}{s})}\;\frac{R_1 n A_{10}'}{R_E(1+\frac{s}{\omega_A})}$$

where

$$\omega_1 \equiv \frac{1}{CR_1} \qquad f_1 = \frac{159}{2\times2} = 40\text{Hz}$$



This method requires additional algebraic force to find the corner frequencies. Therefore, choose

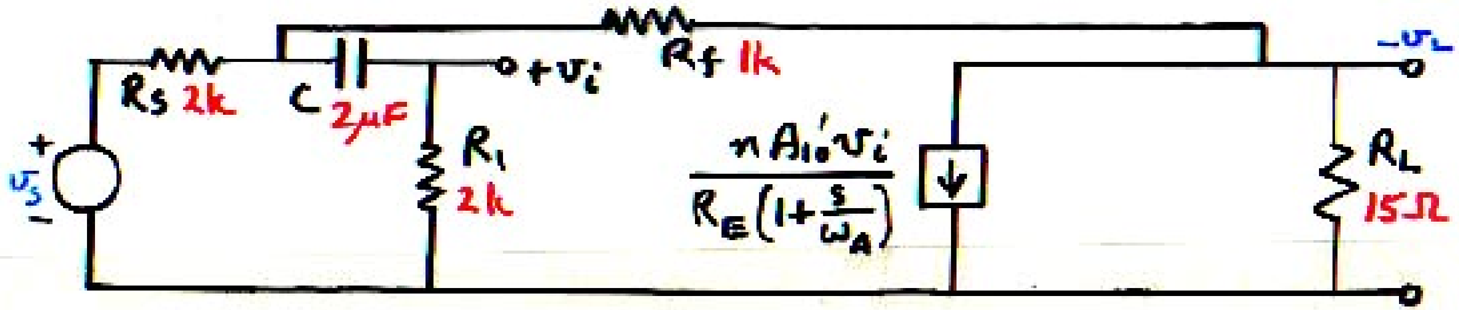
Better Method:

First, find T_m:

$$T_m = \left.\frac{i_y}{i_x}\right|_{v_s=0} = \frac{R_L}{\cancel{R_L} + R_f + R_S \| R_1} (R_S \| R_1) \frac{n A_{10}'}{R_E}$$

$$= \frac{(R_f \| R_S \| R_1) n A_{10}' R_L}{R_f R_E} = \frac{0.5 \times 2 \times 2 \times 15}{1 \times 3}$$

$$= 10 \Rightarrow 20\text{dB}$$



Second, find corner frequencies due to C by use of Extra Element Theorem.

Apply test signal u_z in place of C.

To find $Z_n = R_n$:

"input" signal for T ($v_s = 0$)

Adjust u_z in presence of i_x to null i_y

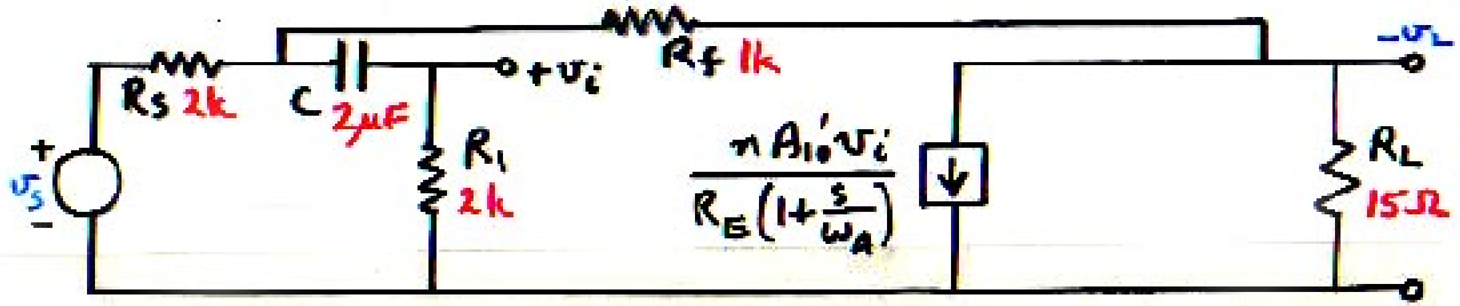
"output" signal for T

Hence: $v_i = 0$, and $R_n = \infty$

To find $Z_d = R_d$:

Apply u_z with $i_x = 0$

Hence: $R_d = (\cancel{R_L} + R_f) \| R_s + R_1$



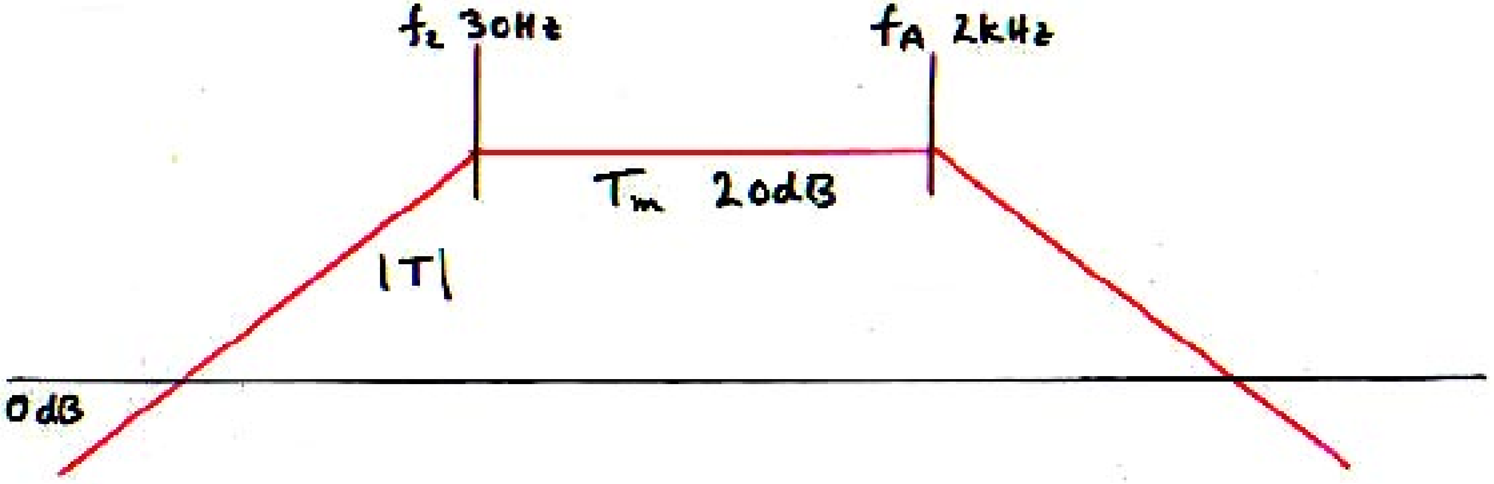
Then:

$$T = T_m \frac{1+\frac{Z}{\overset{\infty}{Z_n}}}{1+\frac{\underset{\frac{1}{sC}}{Z}}{\underset{R_d}{Z_d}}} \; \frac{1}{1+\frac{s}{\omega_A}} \longleftarrow \text{already explicit}$$

$$= T_m \frac{1}{\left(1+\frac{\omega_2}{s}\right)\left(1+\frac{s}{\omega_A}\right)}$$

where

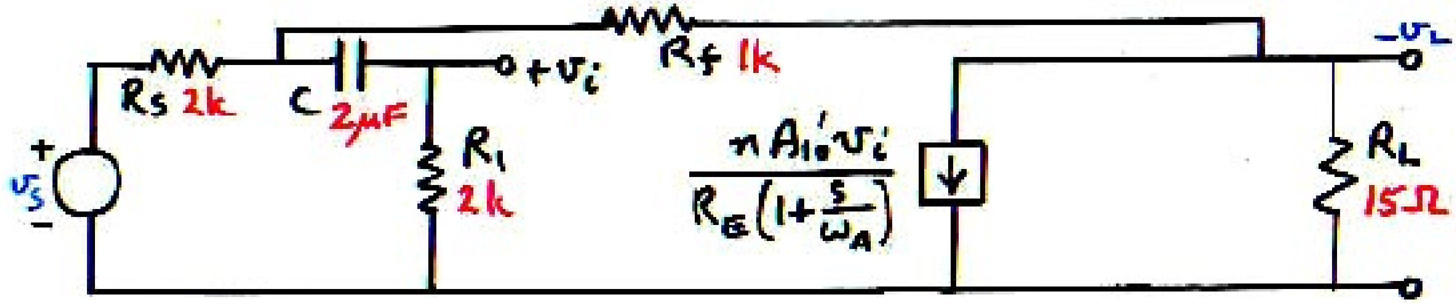
$$\omega_2 \equiv \frac{1}{CR_d} = \frac{1}{C[R_f \| R_s + R_1]} \qquad f_2 = \frac{159}{2[1\|2+2]} = 30\text{Hz}$$



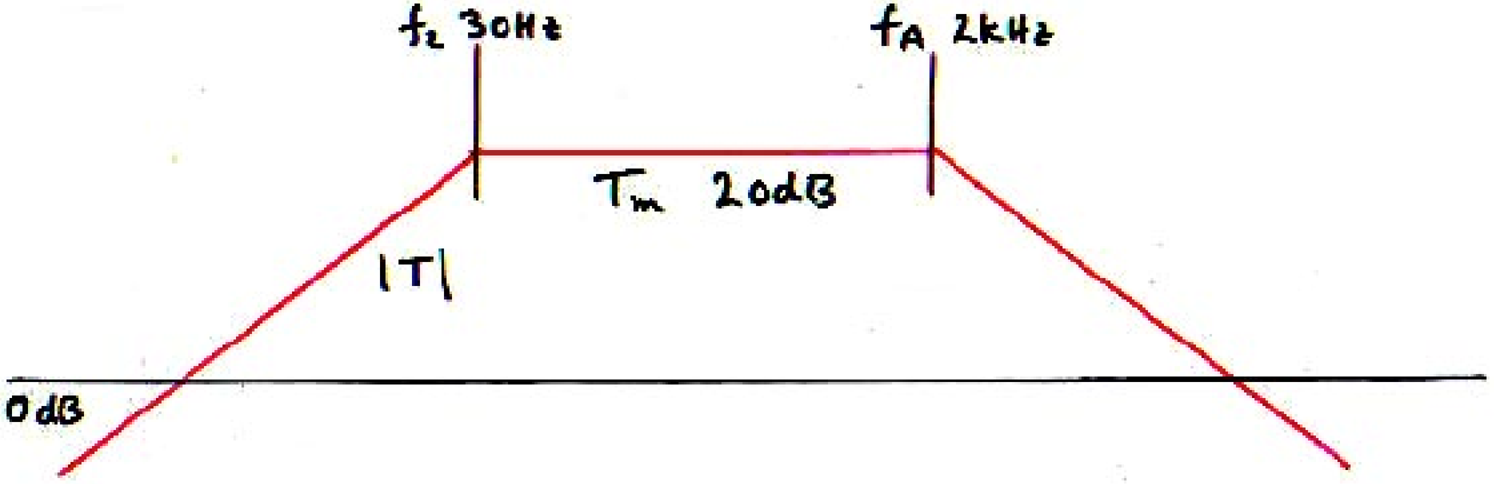
Exercise

By "doing the algebra on the picture," find the analytic pole-zero forms for $F = 1+T$ and $D = T/(1+T)$

Ac model



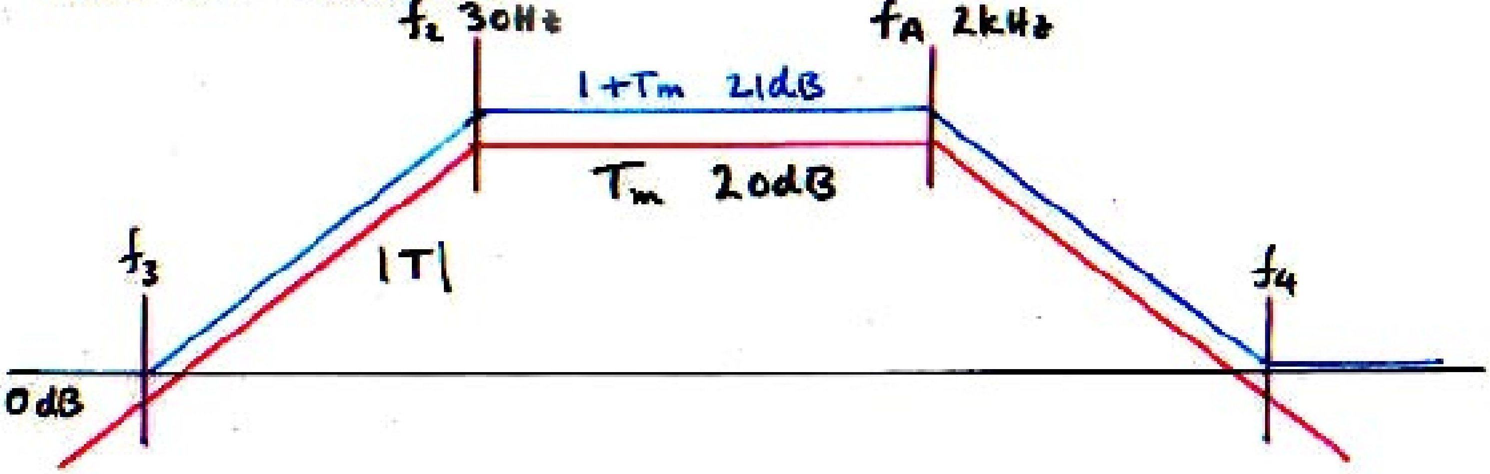
Rs 2k
C 2μF
+v_i
Rf 1k
R_1 2k
$\frac{n A_{10}' v_i}{R_E\left(1+\frac{s}{\omega_A}\right)}$
R_L 15Ω
−v_L
v_s
+
−



Exercise

By "doing the algebra on the picture," find the analytic pole-zero forms for $F = 1+T$ and $D = T/(1+T)$

Exercise Solution



Exercise

By "doing the algebra on the picture," find the analytic pole-zero forms for $F=1+T$ and $D=T/(1+T)$

$$F=(1+T_m)\,\frac{\left(1+\frac{\omega_3}{s}\right)\left(1+\frac{s}{\omega_4}\right)}{\left(1+\frac{\omega_2}{s}\right)\left(1+\frac{s}{\omega_A}\right)}$$

where $\omega_3 \equiv \dfrac{\omega_2}{1+T_m}$ $\qquad f_3 = \dfrac{30}{1+10} = 2.7\,\text{Hz}$

$\omega_4 \equiv (1+T_m)\,\omega_A$ $\qquad f_4 = 11 \times 2 = 22\,\text{kHz}$

Calculation of $F = 1+T$ the Hard Way (by algebra):

$$F = 1+T = 1 + \frac{T_m \frac{s}{\omega_2}}{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_A}\right)} = \frac{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_A}\right) + T_m \frac{s}{\omega_2}}{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_A}\right)}$$

$$= \frac{1 + \left(\frac{1+T_m}{\omega_2} + \frac{1}{\omega_A}\right)s + \left(\frac{1}{\omega_2 \omega_A}\right)s^2}{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_A}\right)}$$

$$= \frac{\left(1 + \left[\frac{1}{2}\left(\frac{1+T_m}{\omega_2} + \frac{1}{\omega_A}\right) + \frac{1}{2}\sqrt{\left(\frac{1+T_m}{\omega_2} + \frac{1}{\omega_A}\right)^2 - \frac{4}{\omega_2 \omega_A}}\right]s\right)\left(1 + \left[\frac{1}{2}\left(\frac{1+T_m}{\omega_2} + \frac{1}{\omega_A}\right) - \frac{1}{2}\sqrt{\left(\frac{1+T_m}{\omega_2} + \frac{1}{\omega_A}\right)^2 - \frac{4}{\omega_2 \omega_A}}\right]s\right)}{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_A}\right)}$$

$$= \frac{\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)}{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_A}\right)} = \frac{\omega_2}{\omega_{z1}} \frac{\left(1+\frac{\omega_{z1}}{s}\right)\left(1+\frac{s}{\omega_{z2}}\right)}{\left(1+\frac{\omega_2}{s}\right)\left(1+\frac{s}{\omega_A}\right)}$$

This result gives no insight into the interpretation of the two zeros ω_{z1} and ω_{z2}, or into the midband value $F_m \equiv \frac{\omega_2}{\omega_{z1}}$.

However, a much simpler result is obtaine if the approximate real root form is used.

Check the value of $Q^2 = ac/b^2$ for the numerator quadratic of F:

$$Q^2 = \frac{1}{\omega_2 \omega_A \left(\frac{1+T_m}{\omega_2} + \frac{1}{\omega_A} \right)^2} = \frac{\omega_2}{\omega_A (1+T_m)^2 \left(1 + \cancel{\frac{\omega_2}{\omega_A (1+T_m)}} \right)^2} \ll (0.5)^2$$

Hence, the approximate factorization for well-separated real roots can be adopted:

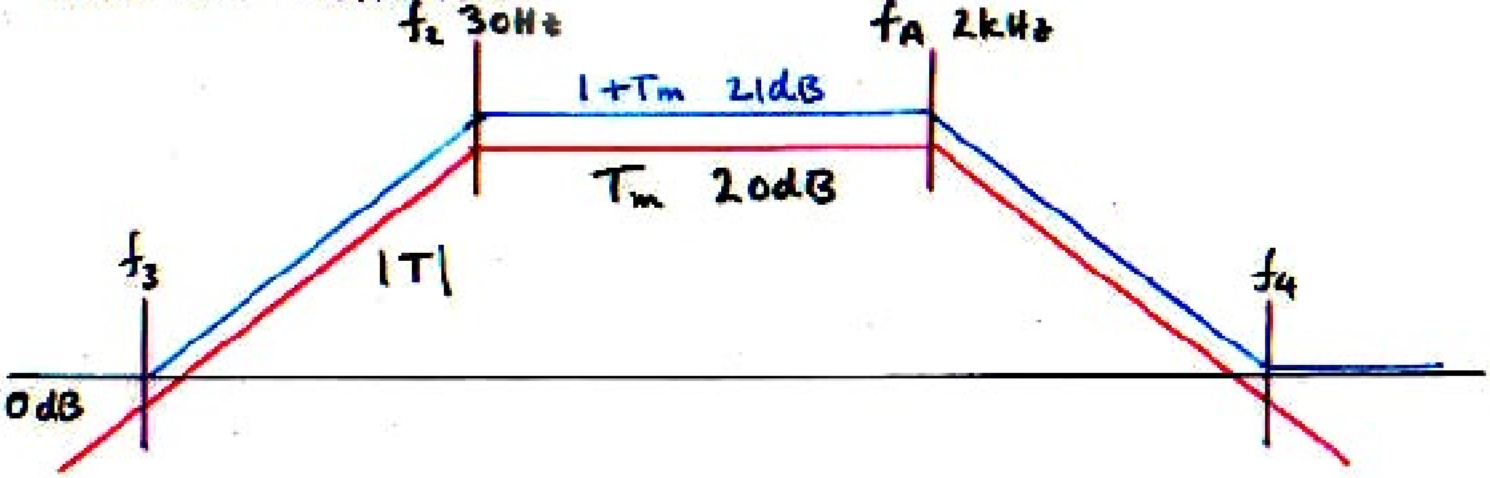
$$F \approx \frac{\left(1 + \left[\frac{1+T_m}{\omega_2} + \cancel{\frac{1}{\omega_A}} \right] s \right) \left(1 + \frac{s}{\omega_2 \omega_A \left[\frac{1+T_m}{\omega_2} + \cancel{\frac{1}{\omega_A}} \right]} \right)}{\left(1 + \frac{s}{\omega_2} \right) \left(1 + \frac{s}{\omega_A} \right)}$$

$$= \left(1 + T_m + \cancel{\frac{\omega_2}{\omega_A}} \right) \frac{\left(1 + \frac{\omega_2/(1+T_m)}{\left[1 + \cancel{\frac{\omega_2}{(1+T_m)\omega_A}} \right] s} \right) \left(1 + \frac{s}{\left[1 + \cancel{\frac{\omega_2}{(1+T_m)\omega_A}} \right] (1+T_m)\omega_A} \right)}{\left(1 + \frac{\omega_2}{s} \right) \left(1 + \frac{s}{\omega_A} \right)}$$

This is the same result obtained by Doing the Algebra on the Graph.

The algebraic factorization could not have been done at all if T had three or more poles.

Exercise Solution



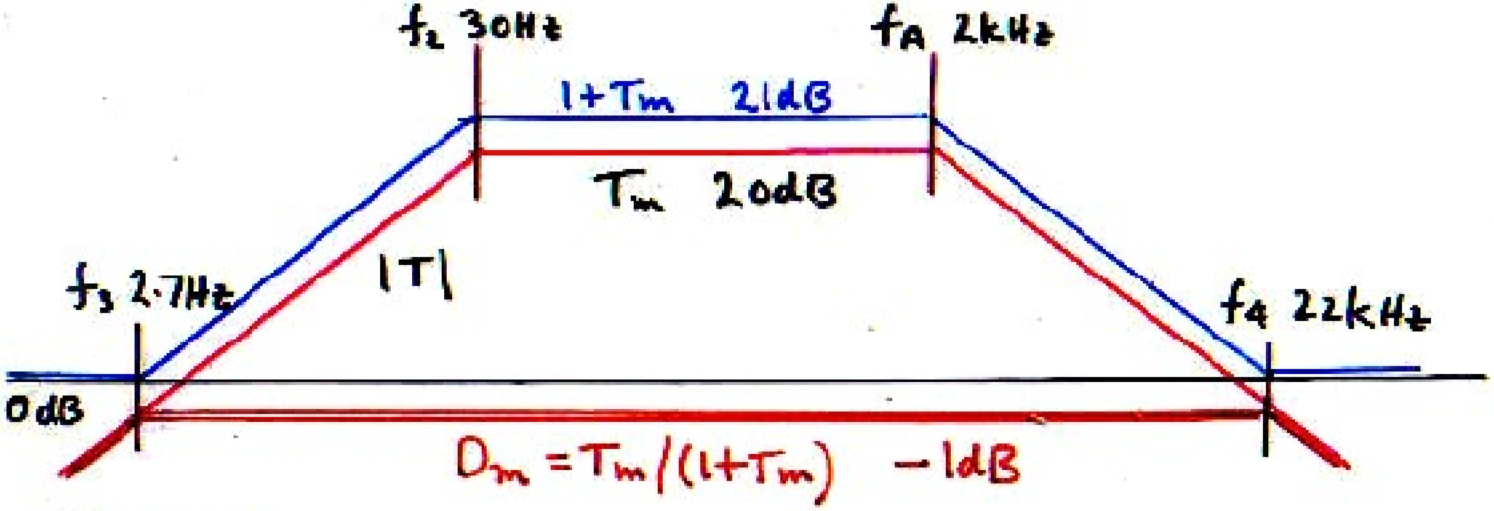
Exercise

By "doing the algebra on the picture," find the analytic pole-zero forms for $F = 1+T$ and $D = T/(1+T)$

$$F = (1+T_m)\,\frac{\left(1+\frac{\omega_3}{s}\right)\left(1+\frac{s}{\omega_4}\right)}{\left(1+\frac{\omega_2}{s}\right)\left(1+\frac{s}{\omega_A}\right)}$$

where $\omega_3 \equiv \dfrac{\omega_2}{1+T_m}$ $\qquad f_3 = \dfrac{30}{1+10} = 2.7\text{Hz}$

$\omega_4 \equiv (1+T_m)\omega_A$ $\qquad f_4 = 11 \times 2 = 22\text{ kHz}$



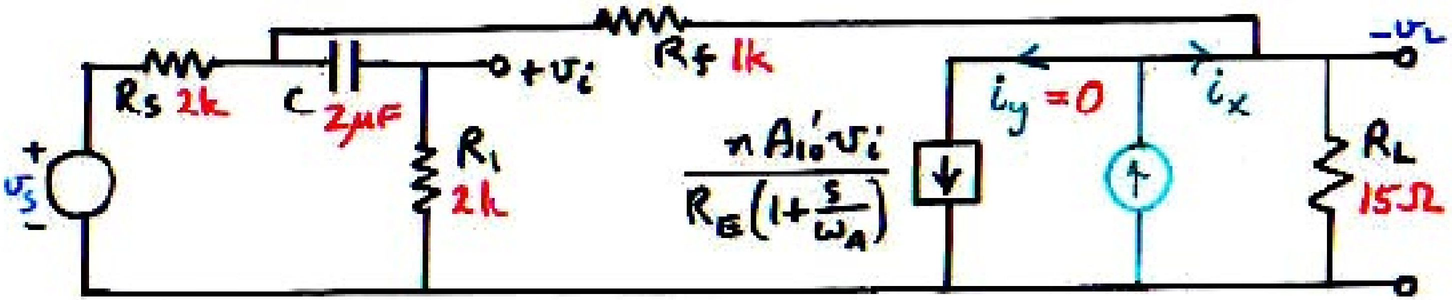
Exercise

By "doing the algebra on the picture," find the analytic pole-zero forms for $F = 1+T$ and $D = T/(1+T)$

$$D = \frac{T_m}{1+T_m} \frac{1}{\left(1+\frac{\omega_3}{s}\right)\left(1+\frac{s}{\omega_A}\right)}$$

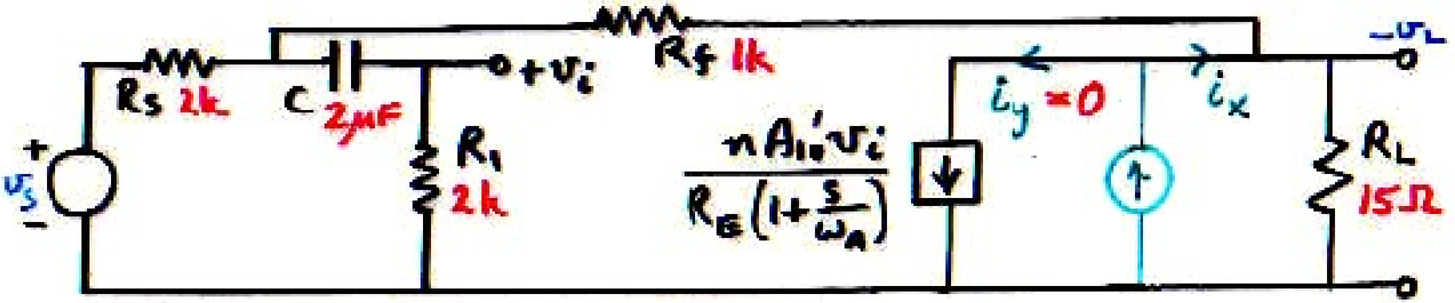
Ac model
Exercise

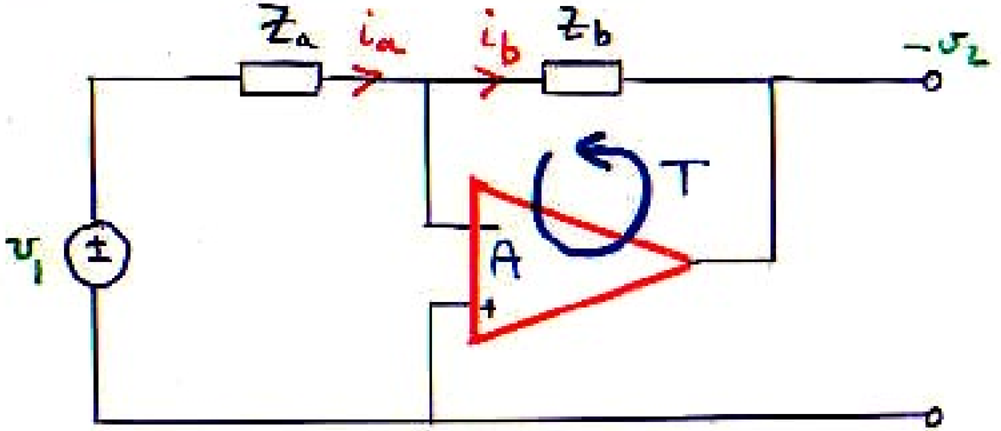
Find G_∞



Ac model
Exercise

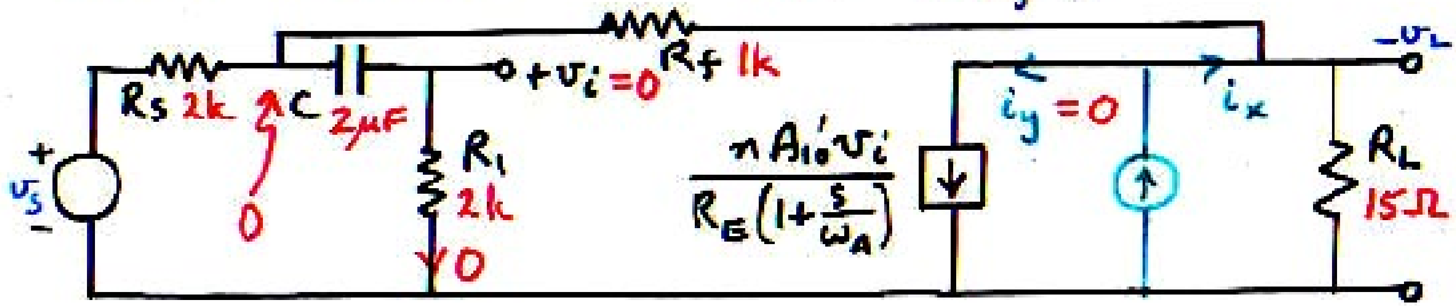
Find G_∞





Ac model
Exercise Solution

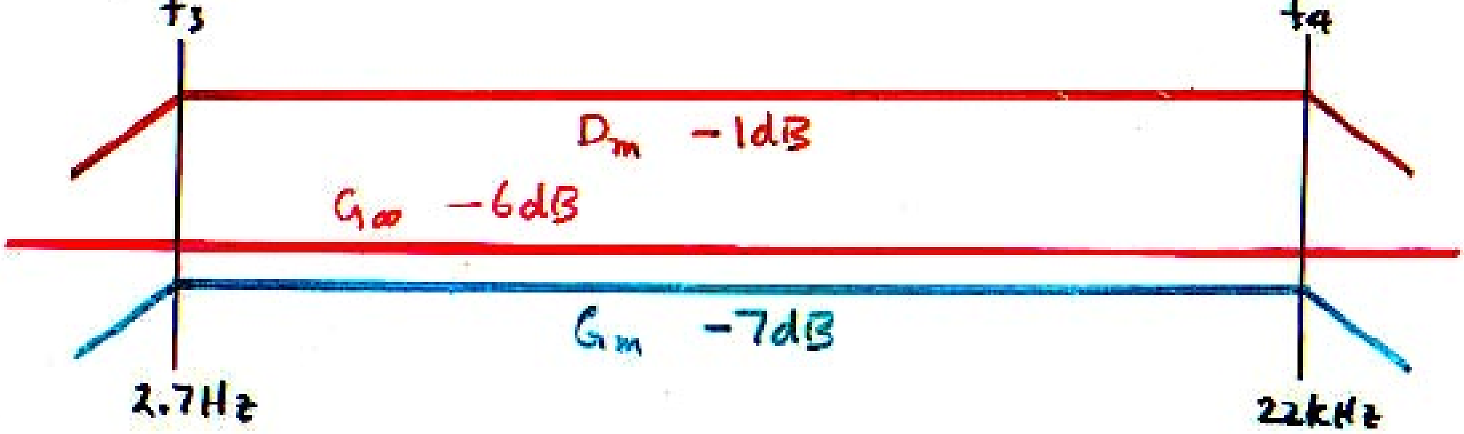
$$G_\infty = \frac{v_L}{v_S}\Big|_{i_y=0}$$



$$\frac{v_s}{R_s} = \frac{v_L}{R_f}$$

$$G_\infty = \frac{R_f}{R_s} = \frac{1}{2} \Rightarrow -6dB$$

Hence the closed-loop gain G is



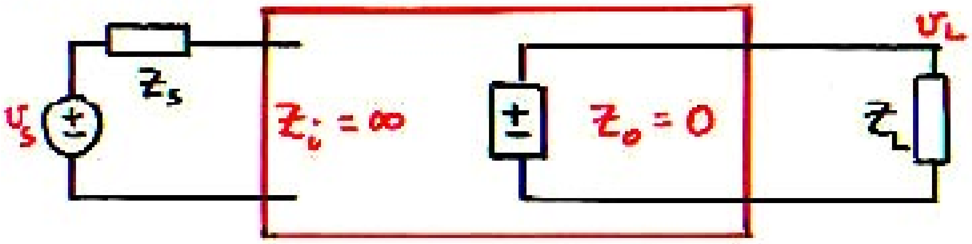
$$G = G_m \frac{1}{\left(1+\frac{\omega_3}{s}\right)\left(1+\frac{s}{\omega_4}\right)}$$

Generalization: Two Conditions for Injection of a Test Signal into a Closed Loop

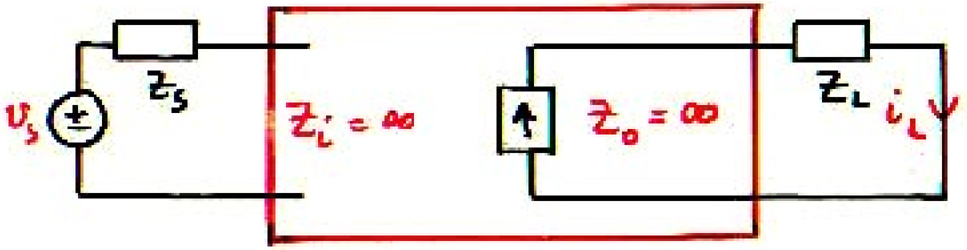
1. Must be inside the feedback loop
2. Injected signal must add to the forward signal without affecting the impedance loading

Condition 2 can be met by injection of a voltage in series with a dependent voltage source, or by injection of a current in parallel with a current source.

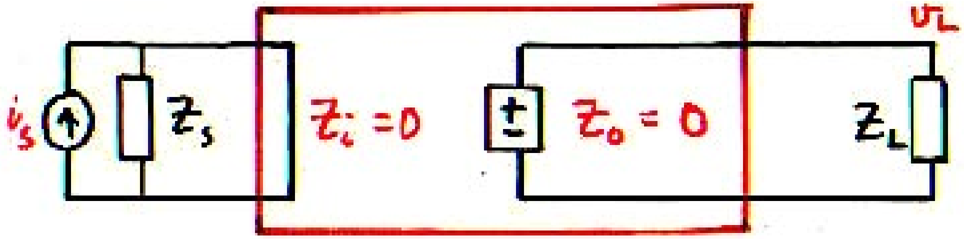
Ideal amplifiers



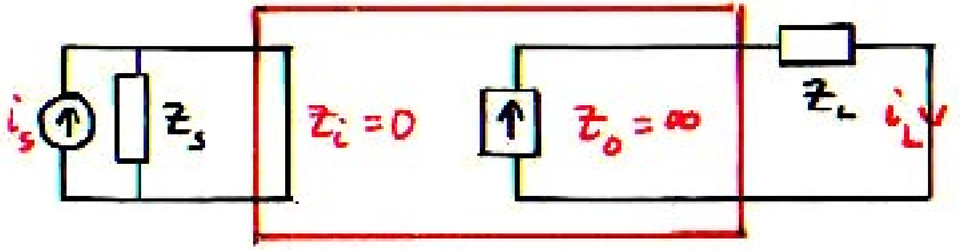
$$A = \frac{v_L}{v_s} = \frac{\text{voltage}}{\text{voltage}}$$



$$A = \frac{i_L}{v_s} = \frac{\text{current}}{\text{voltage}}$$



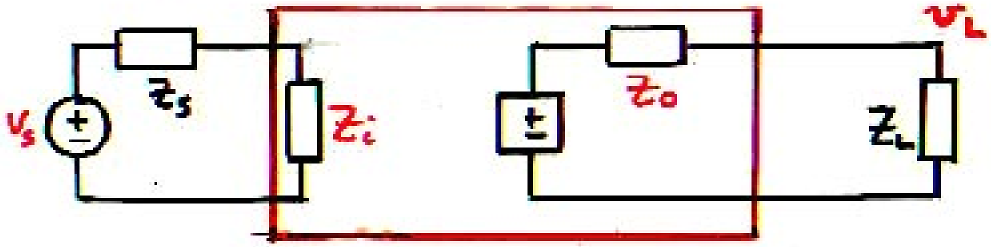
$$A = \frac{v_L}{i_s} = \frac{\text{voltage}}{\text{current}}$$



$$A = \frac{i_L}{i_s} = \frac{\text{current}}{\text{current}}$$

In the ideal cases, the gain is independent of Z_s and Z_L

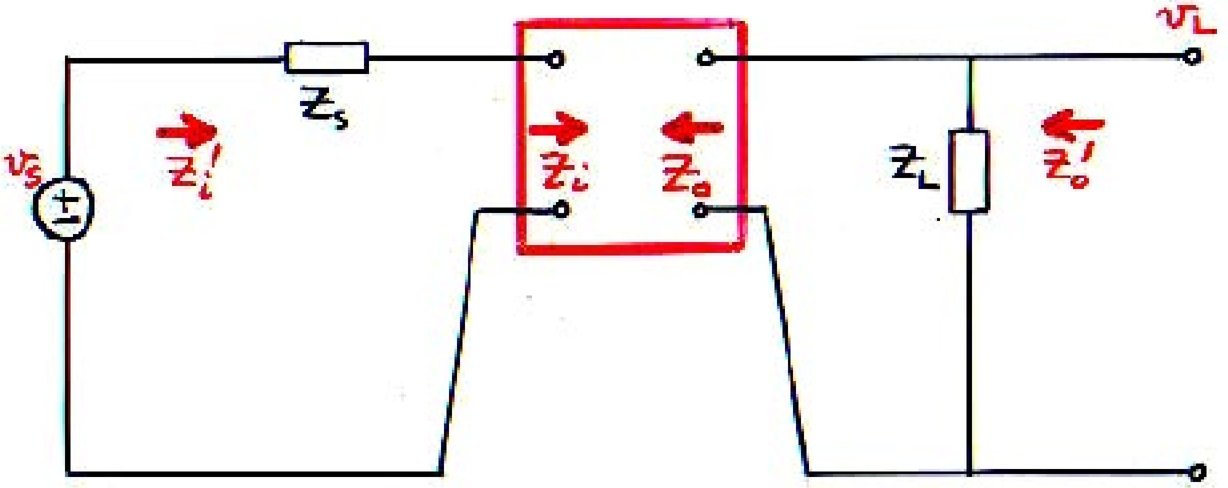
A practical amplifier is not ideal, and the gain does depend on Z_s and Z_L:

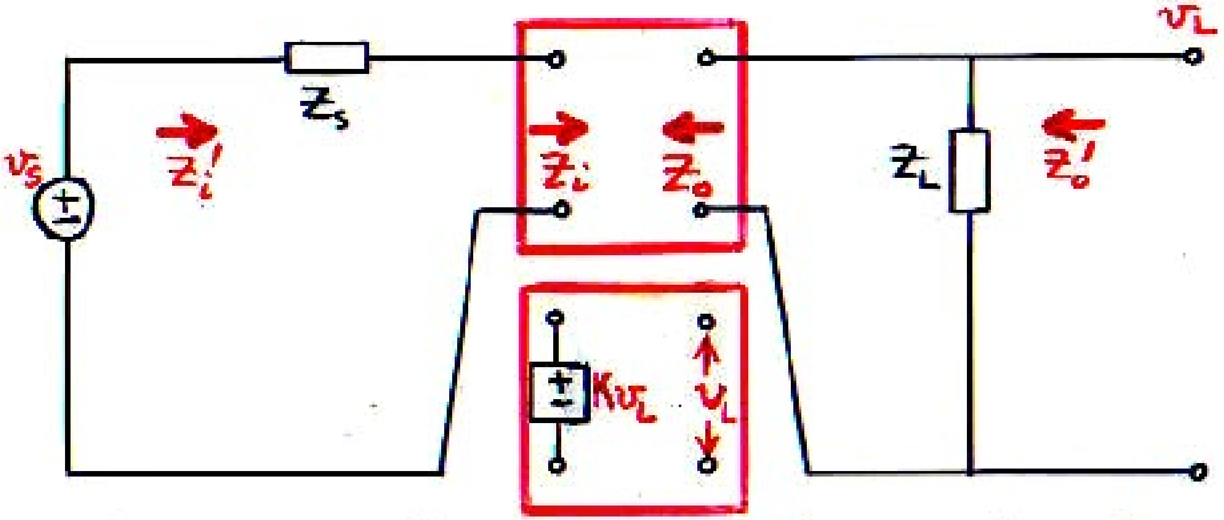


$A = A(Z_s, Z_L)$

However, an appropriate connection of feedback can make a nonideal amplifier approach more closely the properties of any one of the ideal amplifiers.

Feedback causes the closed-loop gain G to approach the reciprocal feedback ratio $1/K = G_\infty$. Thus G_∞ must be designed to have the same current or voltage transfer properties as the desired ideal amplifier.


v_S
Z_i'
Z_S
Z_i
Z_o
Z_L
Z_o'
v_L



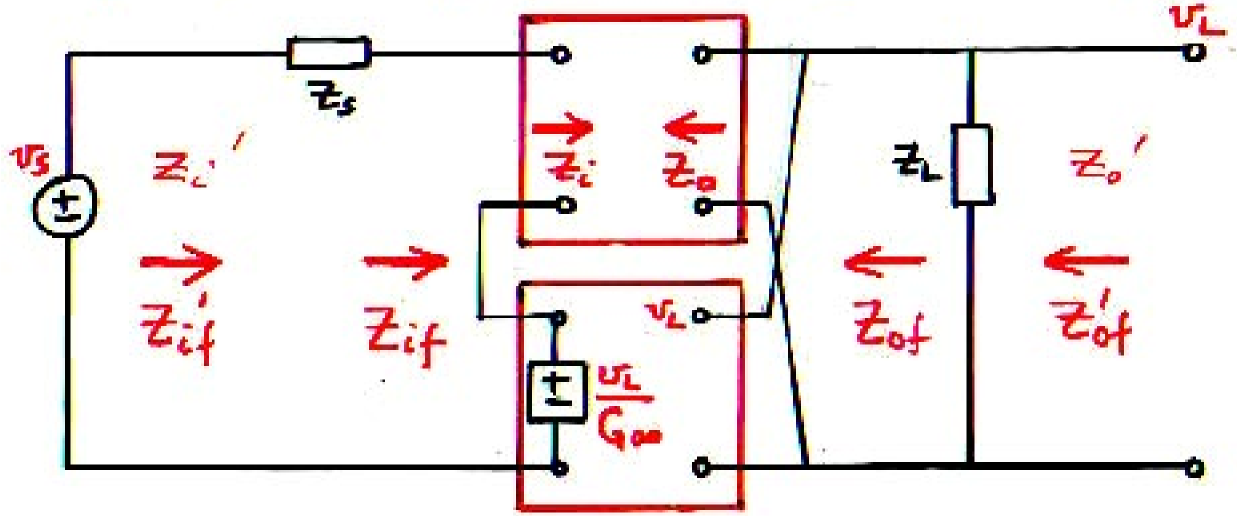
Ideal feedback path must "sense" output voltage and convert it to a fedback voltage. Voltage summing is done in series.

$$G = \frac{v_L}{v_s} = \frac{\text{voltage}}{\text{voltage}} = \frac{A}{1+T} = G_\infty \frac{T}{1+T}$$

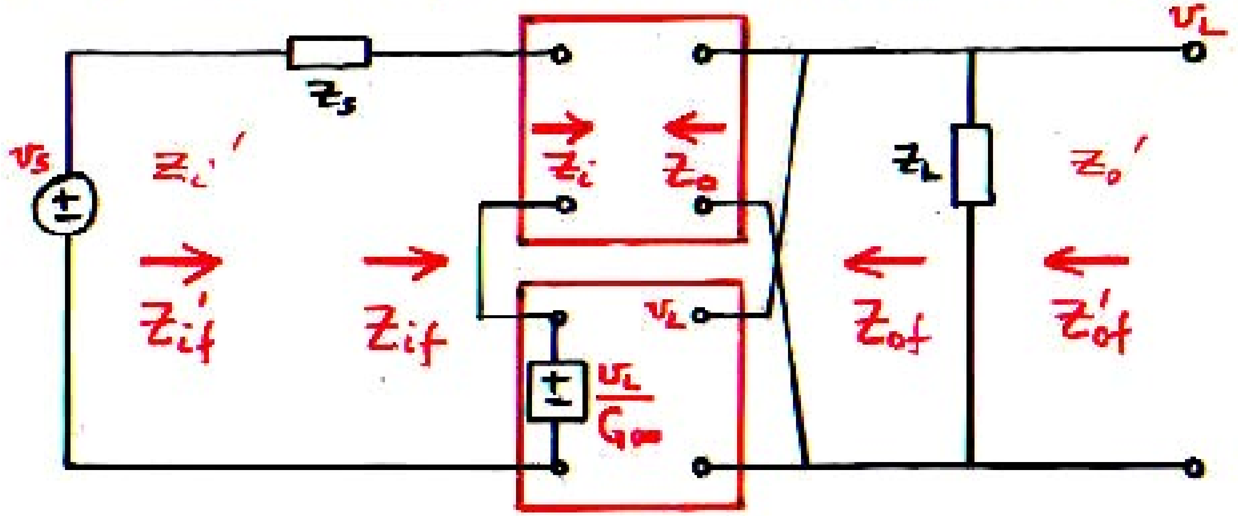
where

$$A = \left.\frac{v_L}{v_s}\right|_{K=1/G_\infty=0}$$

1. Voltage-to-voltage (series voltage feedback)



1. Voltage-to-voltage (series voltage feedback)



$$Z_{of}' = \frac{G}{\left.\frac{G}{Z_L}\right|_{Z_L\to 0}} = \frac{\frac{A}{1+T}}{\left.\frac{A}{Z_L}\right|_{Z_L\to 0}\frac{1}{1+T|_{Z_L\to 0}}} = \frac{A}{\left.\frac{A}{Z_L}\right|_{Z_L\to 0}}\ \frac{1+\cancel{T|_{Z_L\to 0}}}{1+T} = \frac{Z_o'}{1+T}$$

$$Z_{of} = \frac{Z_o}{1+T|_{Z_L\to\infty}}$$

OR:

$$Z_{of}' = \frac{G_\infty\frac{T}{1+T}}{G_\infty\left.\frac{T}{Z_L}\right|_{Z_L\to 0}\ \frac{1}{1+\cancel{T|_{Z_L\to 0}}}} = \frac{T}{1+T}\left[\left.\frac{Z_L}{T}\right|_{Z_L\to 0}\right]$$

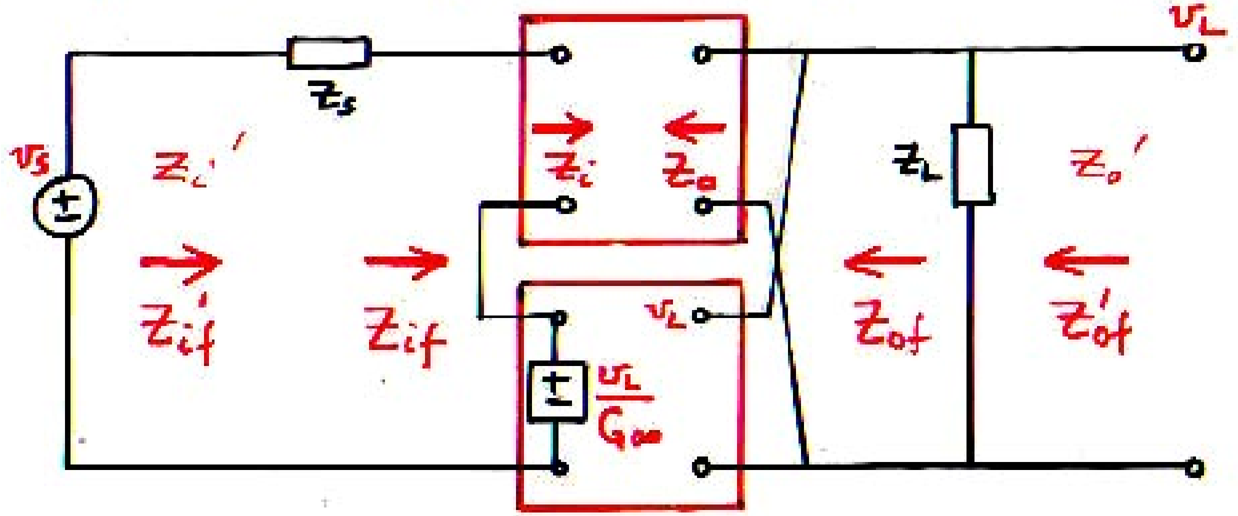
$$Z_{of} = \left.\frac{T}{1+T}\right|_{Z_L\to\infty}\left[\left.\frac{Z_L}{T}\right|_{Z_L\to 0}\right]$$

Outside output impedance is reduced by the factor $1+T$.

Inside output impedance is reduced by the factor $1+T|_{Z_L\to\infty}$, which is larger than $1+T$.

Outside and inside output impedances Z'_{of} and Z_{of} can each be found directly from the loop gain T. They are almost equal (for large T), hence both are much smaller than Z_L.

1. Voltage-to-voltage (series voltage feedback)



$$Z_{if}' = \frac{Z_s G|_{Z_s \to \infty}}{G} = \frac{\dfrac{Z_s A|_{Z_s\to\infty}}{1+T|_{Z_s\to\infty}}}{\dfrac{A}{1+T}} = \frac{Z_s A|_{Z_s\to\infty}}{A}\,\frac{1+T}{\cancel{1+T|_{Z_s\to\infty}}} = Z_i'(1+T)$$

$$Z_{if} = Z_i(1+T|_{Z_s\to 0})$$

OR:

$$Z_{if}' = \frac{G_\infty \dfrac{Z_s T|_{Z_s\to\infty}}{\cancel{1+T|_{Z_s\to\infty}}}}{G_\infty \dfrac{T}{1+T}} = \frac{1+T}{T}\left[Z_s T|_{Z_s\to\infty}\right]$$

$$Z_{if} = \frac{1+T}{T}\bigg|_{Z_s\to 0}\left[Z_s T|_{Z_s\to\infty}\right]$$

Outside input impedance is increased by the factor $1+T$.

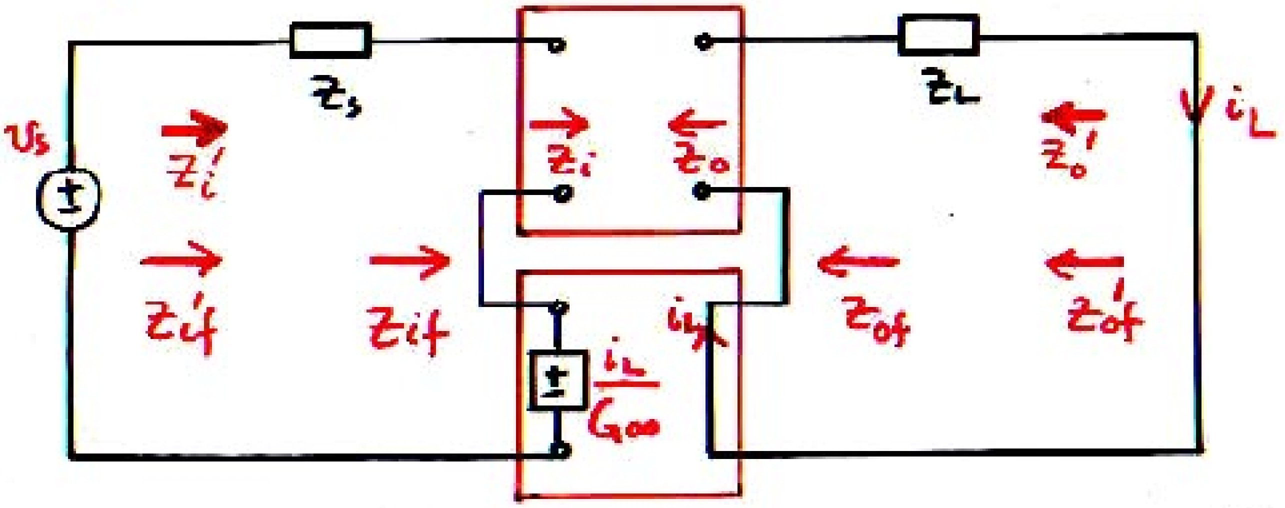
Inside input impedance is increased by the factor $1+T|_{Z_s \Rightarrow 0}$, which is larger than $1+T$.

Outside and inside input impedances Z'_{if} and Z_{if} can each be found directly from the loop gain T. They are almost equal (for large T), hence both are much larger than Z_s.

Bottom Line:

The input and output impedances of a feedback amplifier can be found from a knowledge solely of the loop gain, which further emphasizes the fact that the loop gain T is the single central, important property of a feedback amplifier.

2. Voltage-to-current (series current feedback)



Ideal feedback path must "sense" output current and convert it to a fedback voltage. Voltage summing is done in series.

$$G = \frac{i_L}{v_s} = \frac{\text{current}}{\text{voltage}} = \frac{A}{1+T} = G_\infty \frac{T}{1+T}$$

where

$$A = \left.\frac{i_L}{v_s}\right|_{k = 1/G_\infty = 0}$$