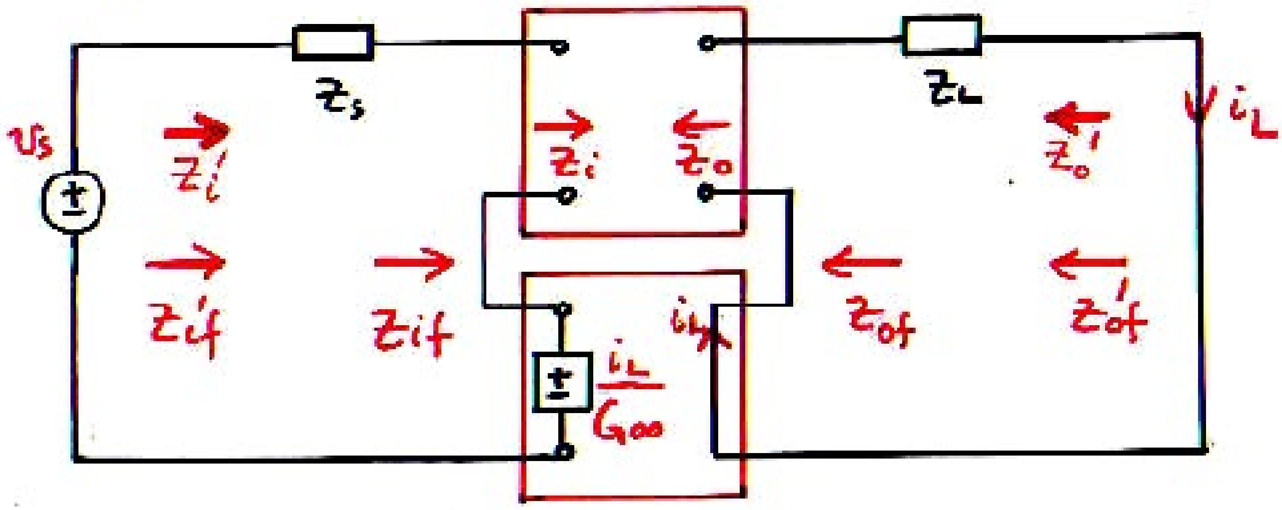
2. Voltage-to-current (Series current feedback)



$$Z_{of}' = \frac{Z_L G|_{Z_L\to\infty}}{G} = \frac{\dfrac{Z_L A|_{Z_L\to\infty}}{1+T|_{Z_L\to\infty}}}{\dfrac{A}{1+T}} = \frac{Z_L A|_{Z_L\to\infty}}{A}\,\frac{1+T}{1+\cancel{T|}_{Z_L\to\infty}} = Z_o'(1+T)$$

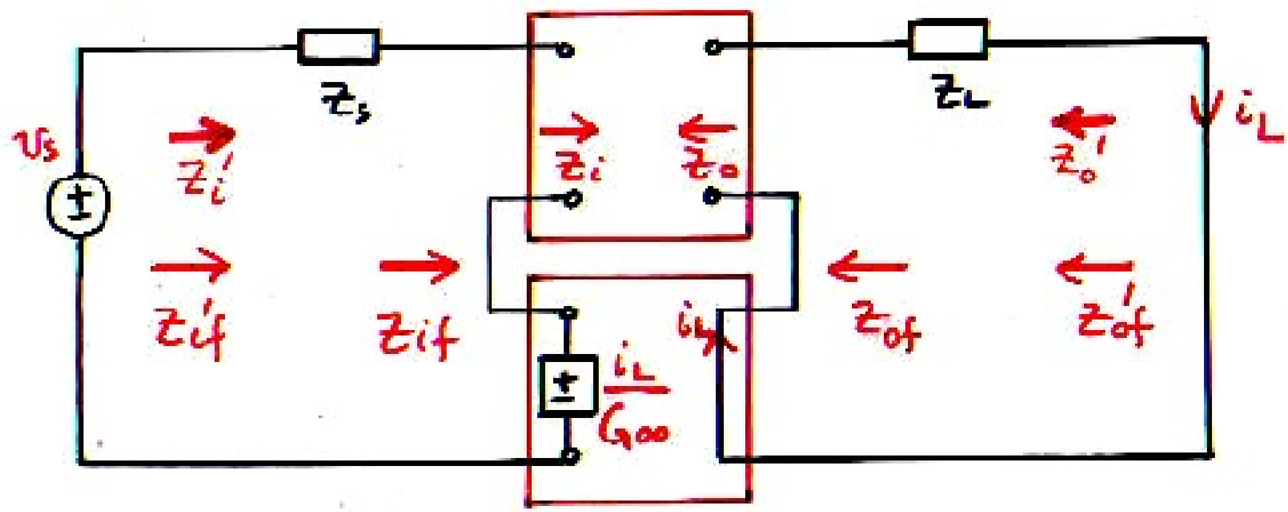
$$Z_{of} = Z_o(1+T|_{Z_L\to\cancel{\infty}\,0})$$

OR:

$$Z_{of}' = \frac{G_\infty \dfrac{Z_L T|_{Z_L\to\infty}}{1+\cancel{T}|_{Z_L\to\infty}}}{G_\infty \dfrac{T}{1+T}} = \frac{1+T}{T}\left[Z_L T|_{Z_L\to\infty}\right]$$

$$Z_{of} = \left.\frac{1+T}{T}\right|_{Z_L\to 0}\left[Z_L T|_{Z_L\to\infty}\right]$$

2. Voltage-to-current (Series current feedback)



$$Z_{if}' = \frac{Z_s G|_{Z_s\to\infty}}{G} = \frac{\dfrac{Z_s A|_{Z_s\to\infty}}{1+T|_{Z_s\to\infty}}}{\dfrac{A}{1+T}} = \frac{Z_s A|_{Z_s\to\infty}}{A}\,\frac{1+T}{\cancel{1+T|_{Z_s\to\infty}}} = Z_i'(1+T)$$

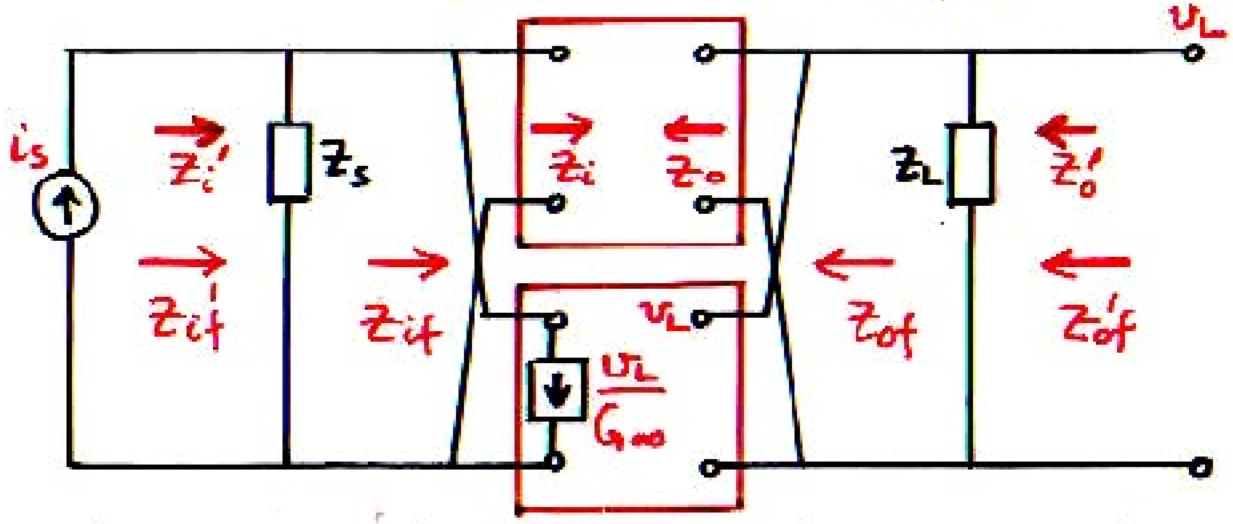
$$Z_{if} = Z_i(1+T|_{Z_s\to 0})$$

OR:

$$Z_{if}' = \frac{G_\infty \dfrac{Z_s T|_{Z_s\to\infty}}{\cancel{1+T|_{Z_s\to\infty}}}}{G_\infty \dfrac{T}{1+T}} = \frac{1+T}{T}\left[Z_s T|_{Z_s\to\infty}\right]$$

$$Z_{if} = \frac{1+T}{T}\bigg|_{Z_s\to 0}\left[Z_s T|_{Z_s\to\infty}\right]$$

3. Current-to-voltage (shunt voltage feedback)



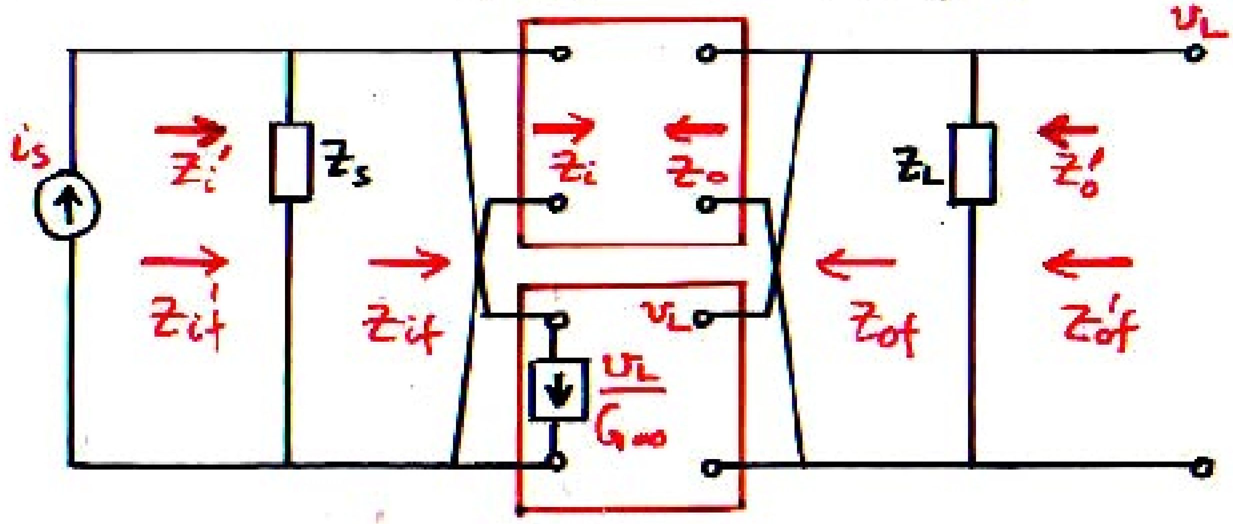
Ideal feedback path must "sense" output voltage and convert it to a fedback current. Current summing is done in shunt.

$$G = \frac{v_L}{i_s} = \frac{\text{voltage}}{\text{current}} = \frac{A}{1+T} = G_\infty \frac{T}{1+T}$$

where

$$A = \left.\frac{v_L}{i_s}\right|_{K = 1/G_\infty = 0}$$

3. Current-to-voltage (shunt voltage feedback)



$$Z_{of}' = \frac{G}{\left.\frac{G}{Z_L}\right|_{Z_L\to 0}} = \frac{\frac{A}{1+T}}{\left.\frac{A}{Z_L}\right|_{Z_L\to 0}\frac{1}{1+T|_{Z_L\to 0}}} = \frac{A}{\left.\frac{A}{Z_L}\right|_{Z_L\to 0}}\;\frac{1+T\cancel{|_{Z_L\to 0}}}{1+T} = \frac{Z_o'}{1+T}$$

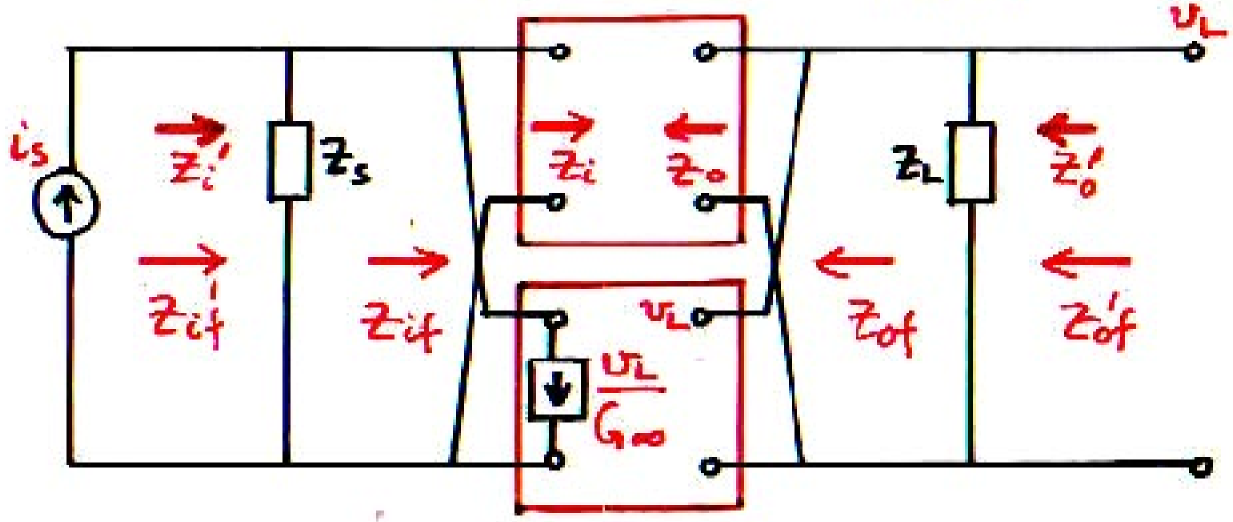
$$Z_{of} = \frac{Z_o}{1+T|_{Z_L\to\infty}}$$

OR:

$$Z_{of}' = \frac{G_\infty\frac{T}{1+T}}{G_\infty\left.\frac{T}{Z_L}\right|_{Z_L\to 0}\frac{1}{1+\cancel{T|_{Z_L\to 0}}}} = \frac{T}{1+T}\left[\left.\frac{Z_L}{T}\right|_{Z_L\to 0}\right]$$

$$Z_{of} = \left.\frac{T}{1+T}\right|_{Z_L\to\infty}\left[\left.\frac{Z_L}{T}\right|_{Z_L\to 0}\right]$$

3. Current-to-voltage (shunt voltage feedback)



$$Z_{if}' = \frac{G}{\left.\frac{G}{Z_s}\right|_{Z_s\to 0}} = \frac{\frac{A}{1+T}}{\left.\frac{A}{Z_s}\right|_{Z_s\to 0}\frac{1}{1+T|_{Z_s\to 0}}} = \frac{A}{\left.\frac{A}{Z_s}\right|_{Z_s\to 0}}\frac{1+\cancel{T|_{Z_s\to 0}}}{1+T} = \frac{Z_i'}{1+T}$$

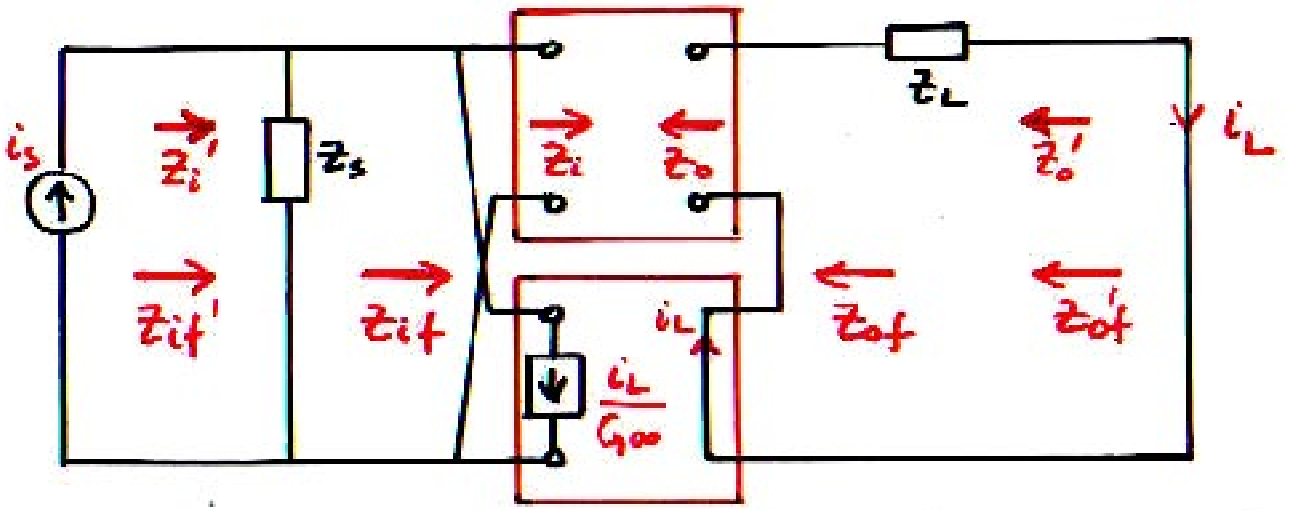
$$Z_{if} = \frac{Z_i}{1+T|_{Z_s\to\infty}}$$

OR:

$$Z_{if}' = \frac{G_\infty\frac{T}{1+T}}{\left.G_\infty\frac{T}{Z_s}\right|_{Z_s\to 0}\frac{1}{1+\cancel{T}|_{Z_s\to 0}}} = \frac{T}{1+T}\left[\left.\frac{Z_s}{T}\right|_{Z_s\to 0}\right]$$

$$Z_{if} = \left.\frac{T}{1+T}\right|_{Z_s\to\infty}\left[\left.\frac{Z_s}{T}\right|_{Z_s\to 0}\right]$$

4. Current-to-current (shunt current feedback)



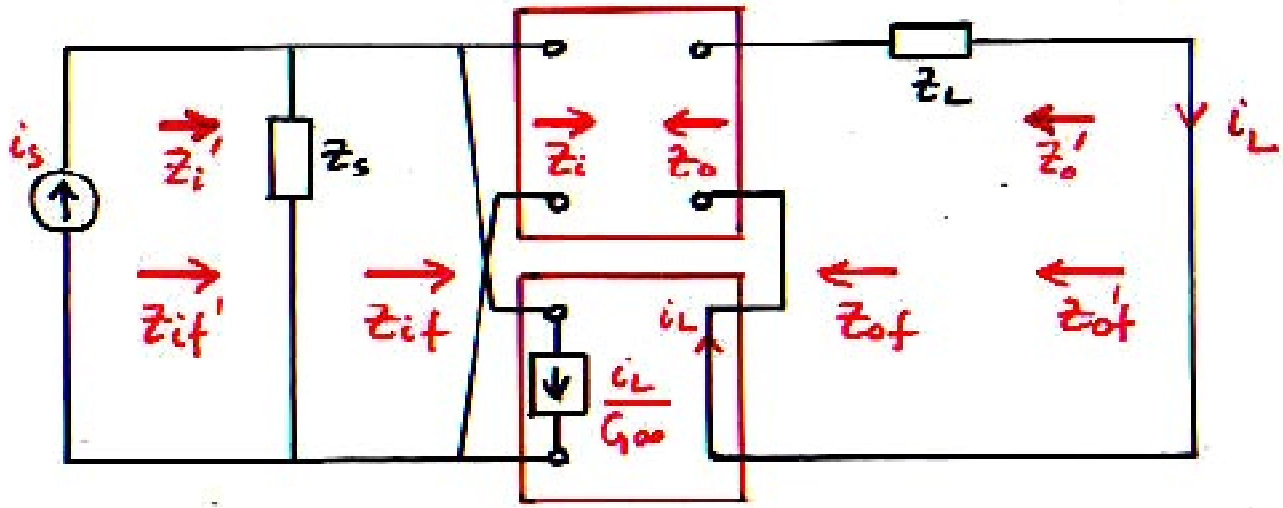
Ideal feedback path must "sense" output current and convert it to a fedback current. Current summing is done in shunt.

$$G = \frac{i_L}{i_s} = \frac{\text{current}}{\text{current}} = \frac{A}{1+T} = G_{\infty}\frac{T}{1+T}$$

where

$$A = \left.\frac{i_L}{i_s}\right|_{K = 1/G_{\infty} = 0}$$

4. Current-to-current (shunt current feedback)



$$Z_{of}' = \frac{Z_L G|_{Z_L\to\infty}}{G} = \frac{\dfrac{Z_L A|_{Z_L\to\infty}}{1+T|_{Z_L\to\infty}}}{\dfrac{A}{1+T}} = \frac{Z_L A|_{Z_L\to\infty}}{A}\,\frac{1+T}{\cancel{1+T|}_{Z_L\to\infty}} = Z_o'(1+T)$$

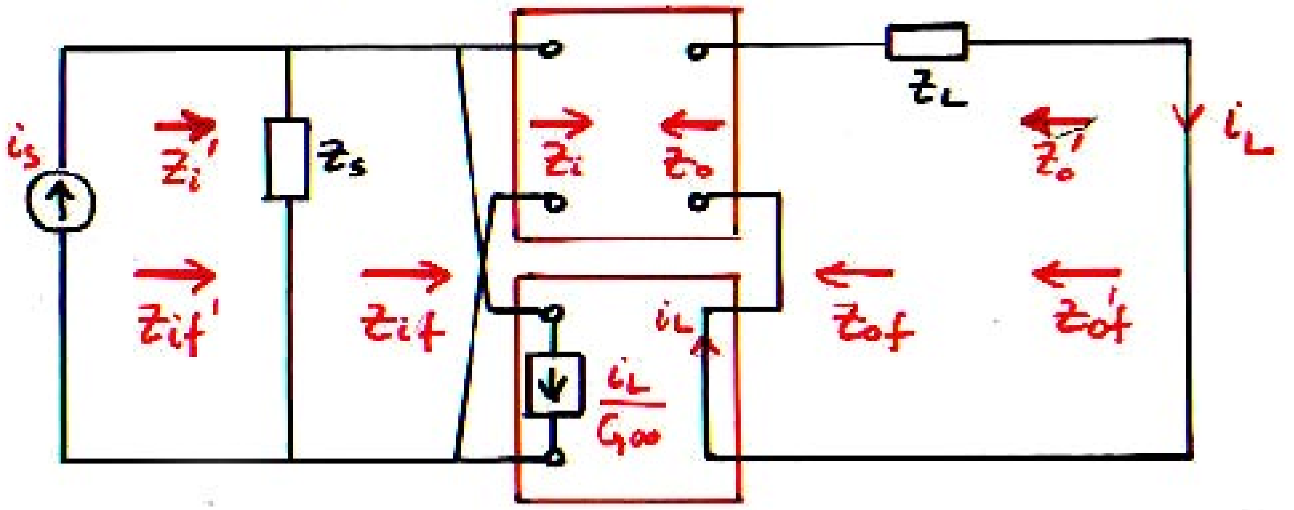
$$Z_{of} = Z_o(1+T|_{Z_L\to\cancel{\infty}\,0})$$

OR:

$$Z_{of}' = \frac{G_\infty \dfrac{Z_L T|_{Z_L\to\infty}}{\cancel{1+T|}_{Z_L\to\infty}}}{G_\infty \dfrac{T}{1+T}} = \frac{1+T}{T}\left[Z_L T|_{Z_L\to\infty}\right]$$

$$Z_{of} = \frac{1+T}{T}\Big|_{Z_L\to 0}\left[Z_L T|_{Z_L\to\infty}\right]$$

4. Current-to-current (shunt current feedback)



$$Z_{if}' = \frac{G}{\left.\frac{G}{Z_s}\right|_{Z_s\to 0}} = \frac{\frac{A}{1+T}}{\left.\frac{A}{Z_s}\right|_{Z_s\to 0}\frac{1}{1+T|_{Z_s\to 0}}} = \frac{A}{\left.\frac{A}{Z_s}\right|_{Z_s\to 0}}\,\frac{1+\cancel{T}|_{Z_s\to 0}}{1+T} = \frac{Z_i'}{1+T}$$

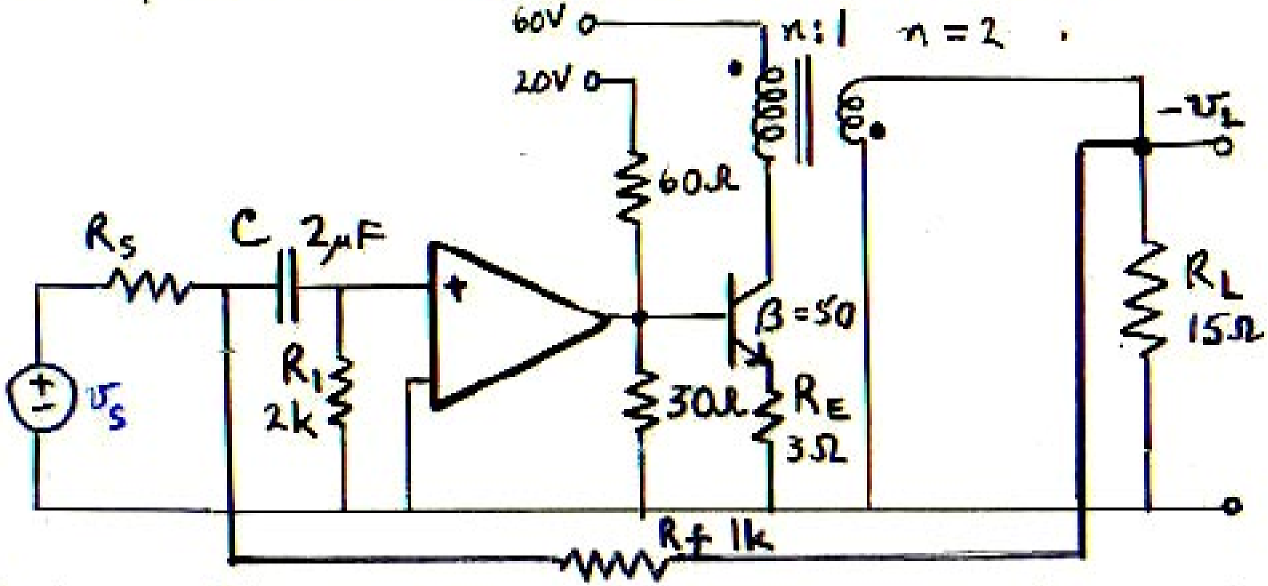
$$Z_{if} = \frac{Z_i}{1+T|_{Z_s\to\infty}}$$

OR:

$$Z_{if}' = \frac{G_\infty\frac{T}{1+T}}{G_\infty\left.\frac{T}{Z_s}\right|_{Z_s\to 0}\frac{1}{1+\cancel{T}|_{Z_s\to 0}}} = \frac{T}{1+T}\left[\left.\frac{Z_s}{T}\right|_{Z_s\to 0}\right]$$

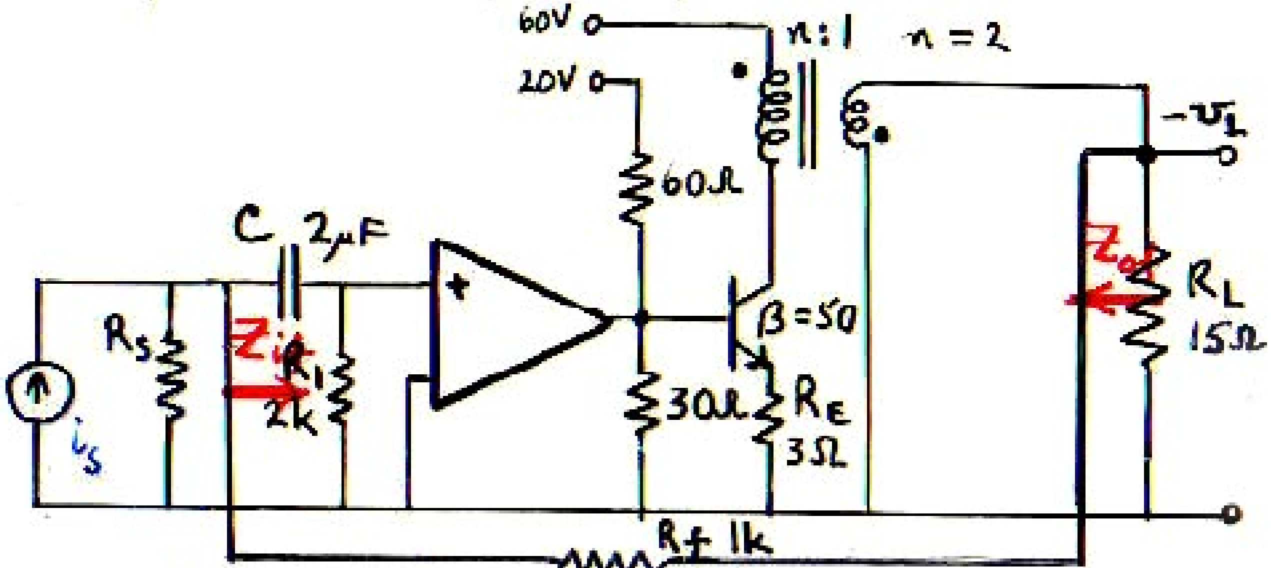
$$Z_{if} = \left.\frac{T}{1+T}\right|_{Z_s\to\infty}\left[\left.\frac{Z_s}{T}\right|_{Z_s\to 0}\right]$$

Single-ended Class A audio feedback power amplifier, based on the same power stage previously discussed. The driver opamp has a gain $A_1 = A_{10}/(1+s/\omega_A)$, where $A_{10} = 8\,\mathrm{dB}$ and $f_A = 2\,\mathrm{kHz}$, and an output impedance of $4.5\,\Omega$.



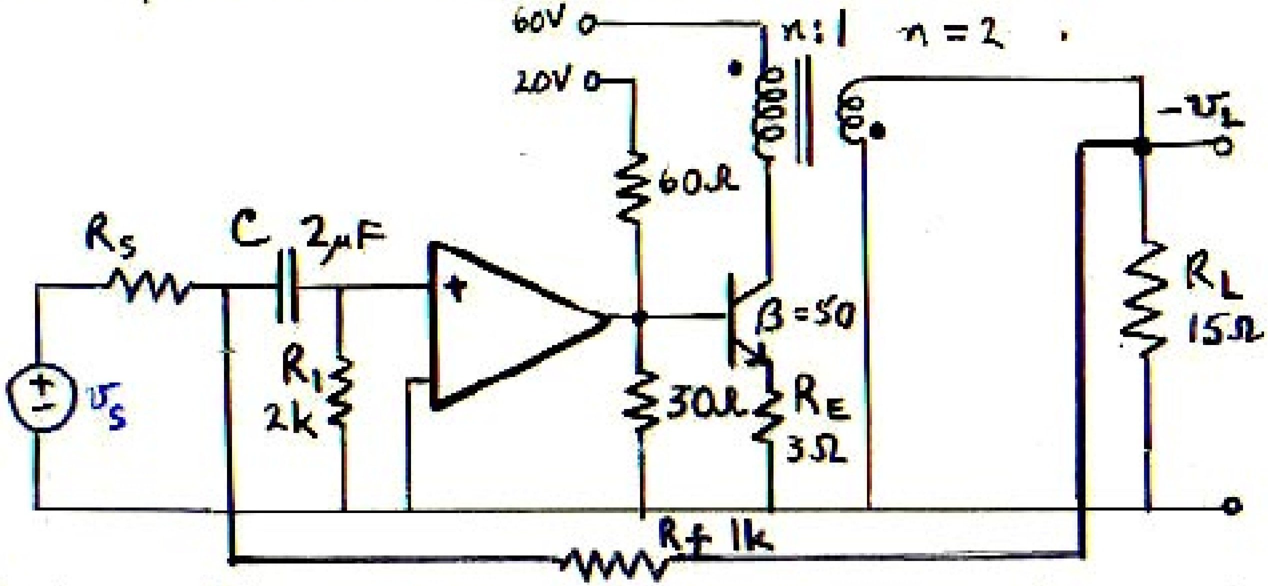
Although the gain is expressed as a voltage ratio, this is really a "Type 3" current-to-voltage feedback amplifier, because the feedback signal is connected at a shunt current-summing junction.

Single-ended Class A audio feedback power amplifier, based on the same power stage previously discussed. The driver opamp has a gain $A_1 = A_{10}/((1+s/\omega_A))$, where $A_{10} = 8dB$ and $f_A = 2kHz$, and an output impedance of 4.5Ω.



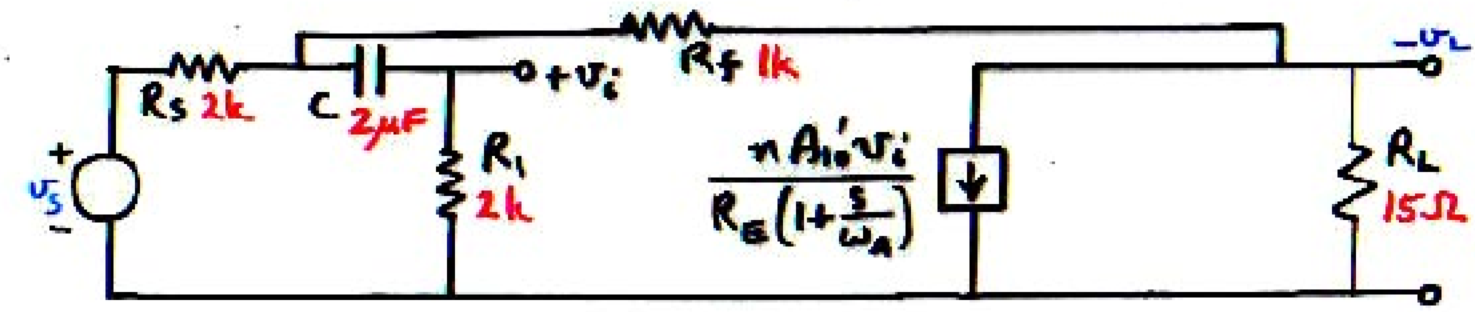
Hence, the "inside" output and input impedances Z_{of} and Z_{if} are both lowered by the feedback.

Single-ended Class A audio feedback power amplifier, based on the same power stage previously discussed. The driver opamp has a gain $A_1 = A_{10}/(1+s/\omega_A)$, where $A_{10} = 8dB$ and $f_A = 2kHz$, and an output impedance of 4.5Ω



Although the gain is expressed as a voltage ratio, this is really a "Type 3" current-to-voltage feedback amplifier, because the feedback signal is connected at a shunt current-summing junction.

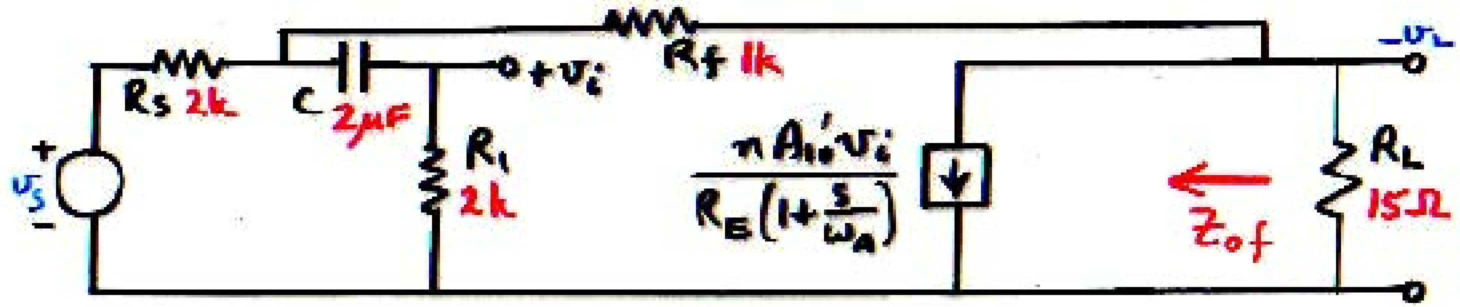
Ac model



+
v_S
−
R_S 2k
C 2μF
R_1 2k
+v_i
R_f 1k
$\dfrac{n A_{10}' v_i}{R_E\left(1+\frac{s}{\omega_A}\right)}$
R_L 15Ω
−v_L

Ac model

Find Z_{of}



Use $$Z_{of} = \frac{T}{1+T}\Big|_{R_L\to\infty}\left[\frac{R_L}{T}\Big|_{R_L\to 0}\right]$$

where

$$T = T_m \frac{1}{\left(1+\frac{\omega_2}{s}\right)\left(1+\frac{s}{\omega_A}\right)}$$

$$T_m = \frac{(R_f \| R_s \| R_1)\, n A'_{10} R_L}{R_f R_E}$$

$$\omega_2 \equiv \frac{1}{C[R_f \| R_s + R_1]}$$

$$\frac{R_L}{T}\bigg|_{R_L \to 0} = \frac{R_f R_E}{(R_f \| R_s \| R_1)\, n A_{io}'}\left(1+\frac{\omega_z}{s}\right)\left(1+\frac{s}{\omega_A}\right)$$

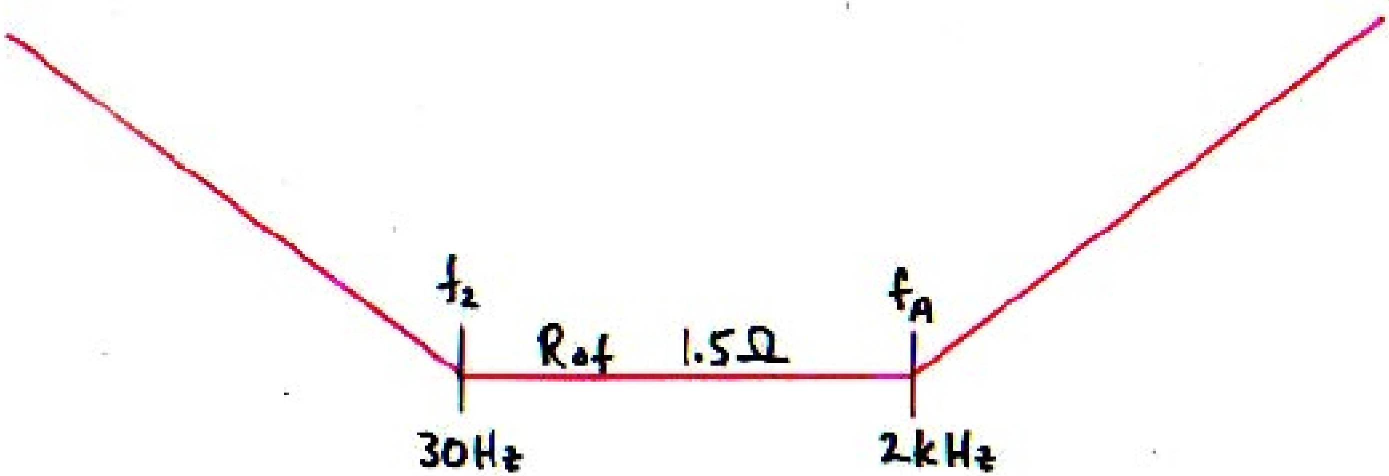
$$T\big|_{R_L \to \infty} = \infty, \quad \text{so} \quad \frac{T}{1+T}\bigg|_{R_L \to \infty} = 1$$

Hence

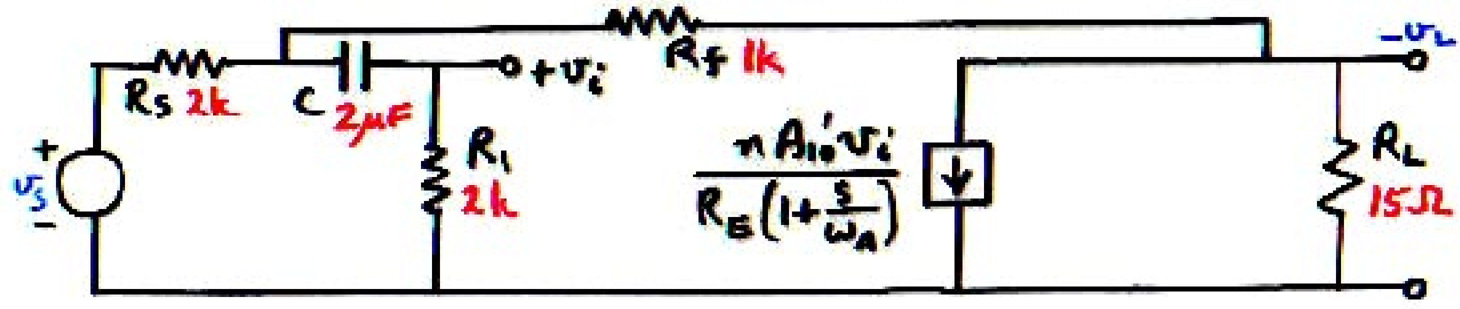
$$Z_{of} = R_{of}\left(1+\frac{\omega_z}{s}\right)\left(1+\frac{s}{\omega_A}\right)$$

where

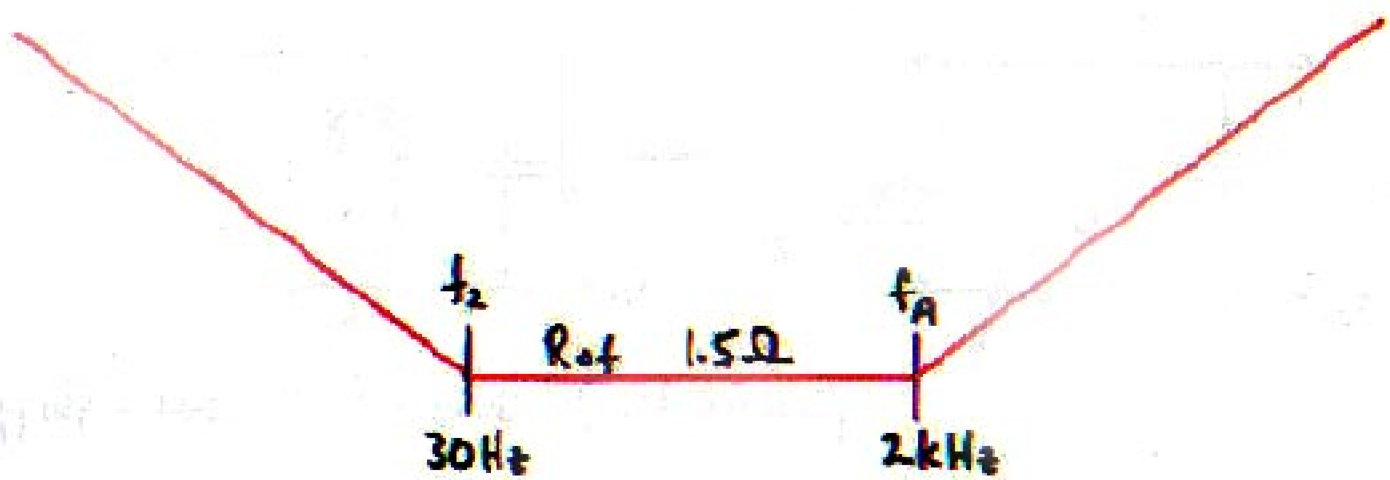
$$R_{of} \equiv \frac{R_f R_E}{(R_f \| R_s \| R_1)\, n A_{io}'} = \frac{1 \times 3}{0.5 \times 2 \times 2} = 1.5\,\Omega$$



Ac model

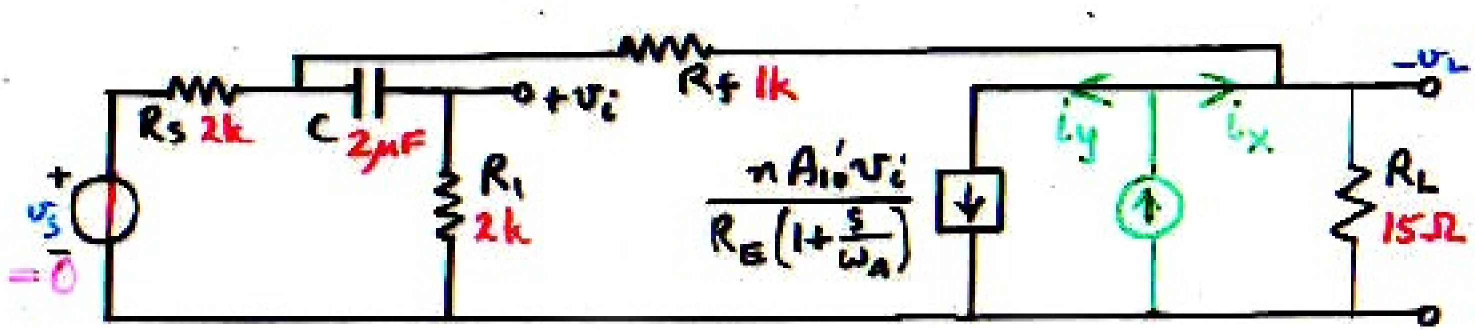


R_S 2k
C 2μF
$+v_i$
R_f 1k
$-v_L$
v_S
R_1 2k
$\frac{n A_{10}' v_i}{R_E\left(1+\frac{s}{\omega_A}\right)}$
R_L 15Ω



f_2
f_A
R_{of} 1.5Ω
30Hz
2kHz

Ac model



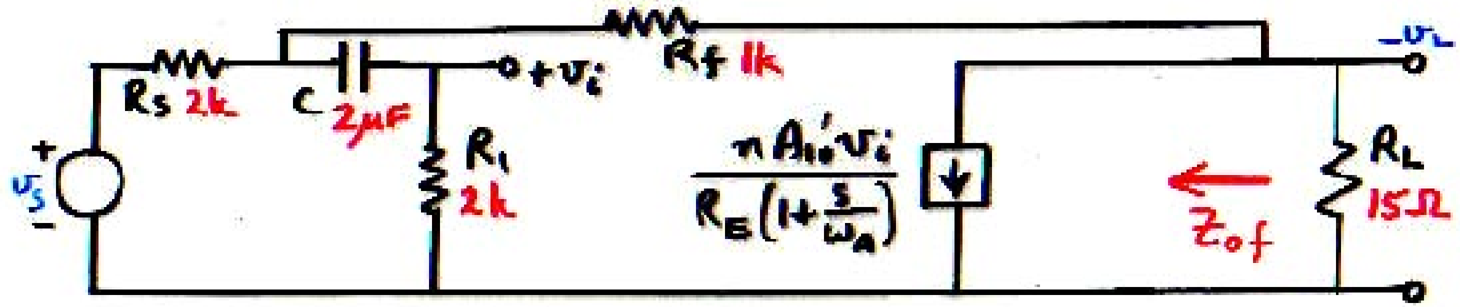
$$T = \left.\frac{i_y}{i_x}\right|_{v_s=0} = \frac{R_L}{\cancel{R_L} + R_f + R_s \| R_1\left(1+\frac{\omega_1}{s}\right)} \; \frac{R_s}{R_s + R_1\left(1+\frac{\omega_1}{s}\right)} \; \frac{R_1 n A_{10}'}{R_E\left(1+\frac{s}{\omega_A}\right)}$$

where

$$\omega_1 \equiv \frac{1}{CR_1} \qquad f_1 = \frac{159}{2\times 2} = 40\,\text{Hz}$$

Ac model

Find Z_{of}



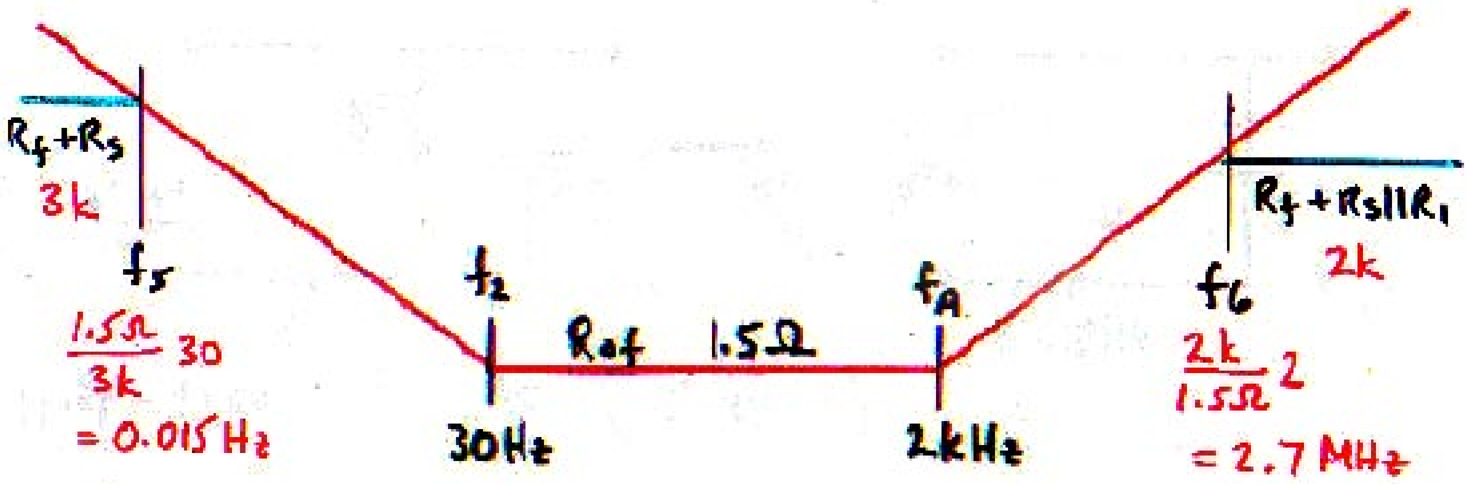
Use $Z_{of} = \left.\frac{T}{1+T}\right|_{R_L\to\infty}\left[\left.\frac{R_L}{T}\right|_{R_L\to 0}\right]$

where

$$T = T_m \frac{1}{\left(1+\frac{\omega_2}{s}\right)\left(1+\frac{s}{\omega_A}\right)}$$

$$T_m = \frac{(R_f \| R_s \| R_1)\, n A_{10}' R_L}{R_f R_E}$$

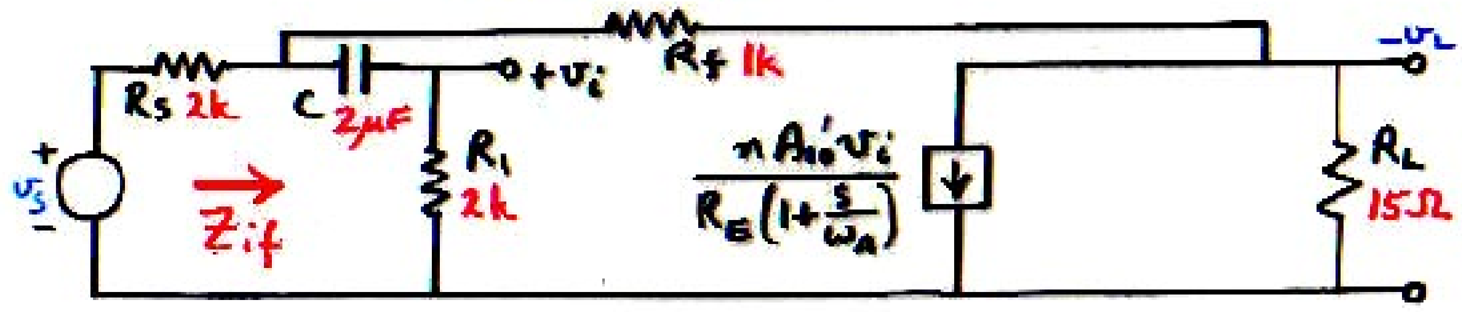
$$\omega_2 \equiv \frac{1}{C[R_f \| R_s + R_1]}$$



Note from the actual circuit that Z_{of} would limit at both low and high frequencies, because of the loading effect of the feedback path on R_L. This nonideality was ignored in the solution, but could have been included with further complication but no additional difficulty.

Ac model

Find Z_{if}



Use $$Z_{if} = \left.\frac{T}{1+T}\right|_{R_s\to\infty} \cdot \left.\frac{R_s}{T}\right|_{R_s\to 0}$$

where $$D = \frac{T}{1+T} = \frac{T_m}{1+T_m}\,\frac{1}{\left(1+\frac{\omega_3}{s}\right)\left(1+\frac{s}{\omega_4}\right)}$$

$$\omega_3 = \frac{\omega_2}{1+T_m} \qquad \omega_2 = \frac{1}{C\left[R_f \| R_s + R_1\right]}$$

$$\omega_4 = (1+T_m)\,\omega_A$$

$$\left.\frac{R_s}{T}\right|_{R_s\to 0} = \frac{R_f R_E}{n A_{10}' R_L}\left(1+\frac{\omega_7}{s}\right)\left(1+\frac{s}{\omega_A}\right)$$

where

$$\omega_7 \equiv \omega_2\big|_{R_s\to 0} = \frac{1}{CR_1} = \omega_1$$

$$D\big|_{R_s\to\infty} = \frac{(R_f \| R_1)\, n A_{10}' R_L}{R_f R_E \left(1+T_m\big|_{R_s\to\infty}\right)} \frac{1}{\left(1+\frac{\omega_8}{s}\right)\left(1+\frac{s}{\omega_9}\right)}$$

where

$$\omega_8 \equiv \omega_3\big|_{R_s\to\infty} = \frac{\omega_2\big|_{R_s\to\infty}}{1+T_m\big|_{R_s\to\infty}}$$

$$= \frac{1}{C(R_f+R_1)\left(1+T_m\big|_{R_s\to\infty}\right)}$$

$$T_m\big|_{R_s\to\infty} = \frac{(R_f \| R_1)\, n A_{10}' R_L}{R_f R_E} = \frac{(1\|2)\,2\times 2\times 15}{1\times 3} = 13$$

$$f_8 = \frac{159}{2(1+2)(1+13)} = 1.9\,Hz$$

$$\omega_9 \equiv \left(1+T_m\big|_{R_s\to\infty}\right)\omega_A$$

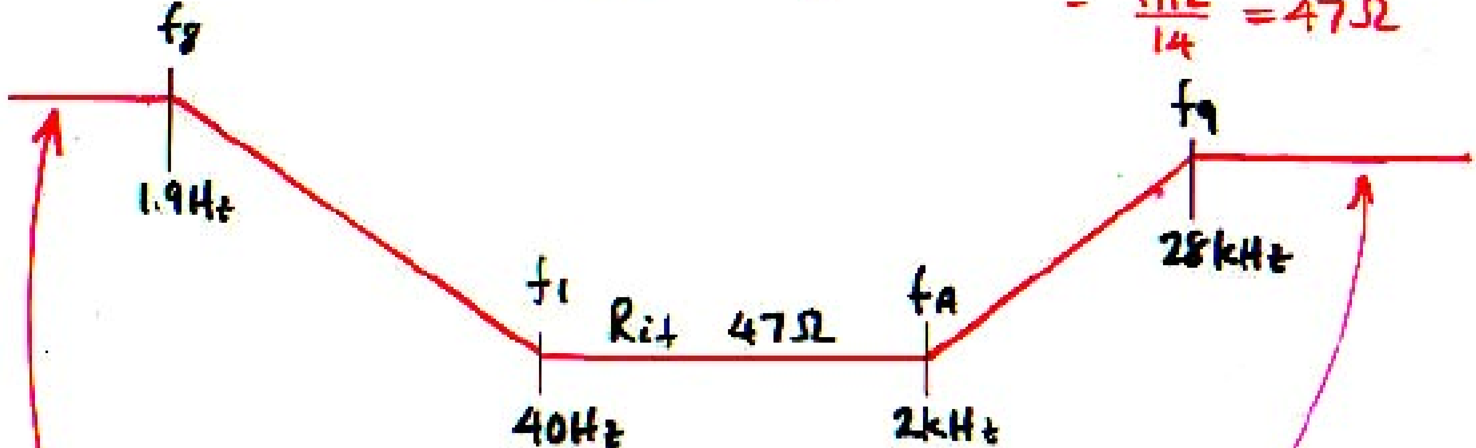
$$f_9 = 14\times 2 = 28\,kHz$$

Hence

$$Z_{if} = \frac{(R_f \| R_1)\, n A'_{io} R_L}{R_f R_E \left(1 + T_m|_{R_s \to \infty}\right)} \; \frac{R_f R_E}{n A_{io} R_L} \; \frac{\left(1+\frac{\omega_1}{s}\right)\left(1+\frac{s}{\omega_A}\right)}{\left(1+\frac{\omega_g}{s}\right)\left(1+\frac{s}{\omega_q}\right)}$$

$$= R_{if} \frac{\left(1+\frac{\omega_1}{s}\right)\left(1+\frac{s}{\omega_A}\right)}{\left(1+\frac{\omega_g}{s}\right)\left(1+\frac{s}{\omega_q}\right)} \qquad R_{if} = \frac{R_f \| R_1}{1 + T_m|_{R_s \to \infty}}$$

$$= \frac{1 \| 2}{14} = 47\,\Omega$$



$$R_{if} \frac{\omega_1}{\omega_g} = \frac{R_f \| R_1}{1 + T_m|_{R_s \to \infty}} \; \frac{C(R_f + R_1)\left(1 + T_m|_{R_s \to \infty}\right)}{C R_1}$$

$$= R_f$$

$$= 1\text{k}$$

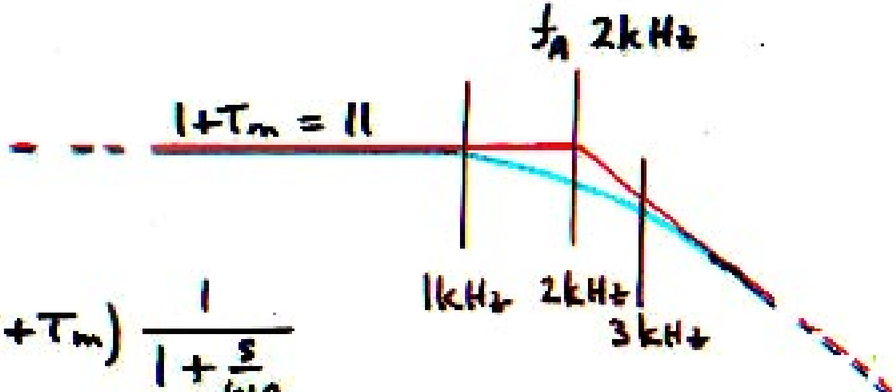
$$R_{if} \frac{\omega_q}{\omega_A} = \frac{R_f \| R_1}{1 + T_m|_{R_s \to \infty}} \left(1 + T_m|_{R_s \to \infty}\right)$$

$$= R_f \| R_1 = 0.67\text{k}$$

Example: Distortion

When the open-loop amplifier is delivering full output power at 1kHz, the output stage develops 3% 2nd harmonic and 5% 3rd harmonic distortion. Find the full output power closed-loop total harmonic distortion at 1kHz.

Percentage of each harmonic is reduced by $|1+T|$ at the harmonic frequency.



$$1+T = (1+T_m)\frac{1}{1+\frac{s}{\omega_A}}$$

$$|1+T|_{2kHz} = 11\frac{1}{\sqrt{1+\left(\frac{2}{2}\right)^2}} = \frac{11}{\sqrt{2}} = 7.8$$

$$|1+T|_{3kHz} = 11\frac{1}{\sqrt{1+\left(\frac{3}{2}\right)^2}} = \frac{11}{\sqrt{3.8}} = 6.1$$

$$\text{THD} = \sqrt{\left(\frac{3\%}{7.8}\right)^2 + \left(\frac{5\%}{6.1}\right)^2} = \sqrt{0.39^2 + 0.82^2}$$

$$= 0.91\%$$

Generalization: Effect of Feedback on Impedances

1. Output impedance is $\left\{\begin{matrix}\text{decreased}\\ \text{increased}\end{matrix}\right\}$ by $\left\{\begin{matrix}\text{voltage}\\ \text{current}\end{matrix}\right\}$
   feedback from the output.

   Input impedance is $\left\{\begin{matrix}\text{increased}\\ \text{decreased}\end{matrix}\right\}$ by $\left\{\begin{matrix}\text{series}\\ \text{shunt}\end{matrix}\right\}$
   feedback to the input.

2. The "outside" impedances are changed by
   a factor $(1+T)$

   The "inside" impedances are changed by
   a larger factor.

3. Alternatively, the input and output
   impedances can be found solely
   from the loop gain T.

Stability

If the open-loop gain A is stable, the closed-loop gain

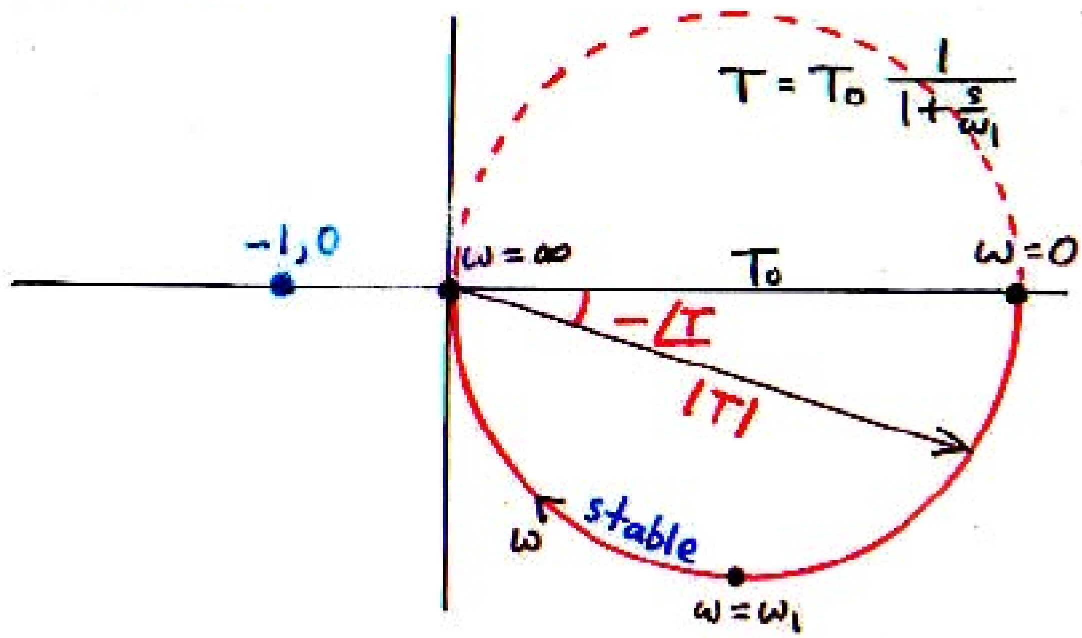
$$G = \frac{A}{1+T}$$

is stable if $1+T$ has no roots in the right half-plane (rhp).

By complex variable theory, this implies that a polar plot of $1+T$ must not encircle the origin; or, equivalently, that a polar plot of T must not encircle the $(-1, 0)$ point. (Nyquist Stability Criterion).

Simple cases of the Nyquist plot of loop gain T:

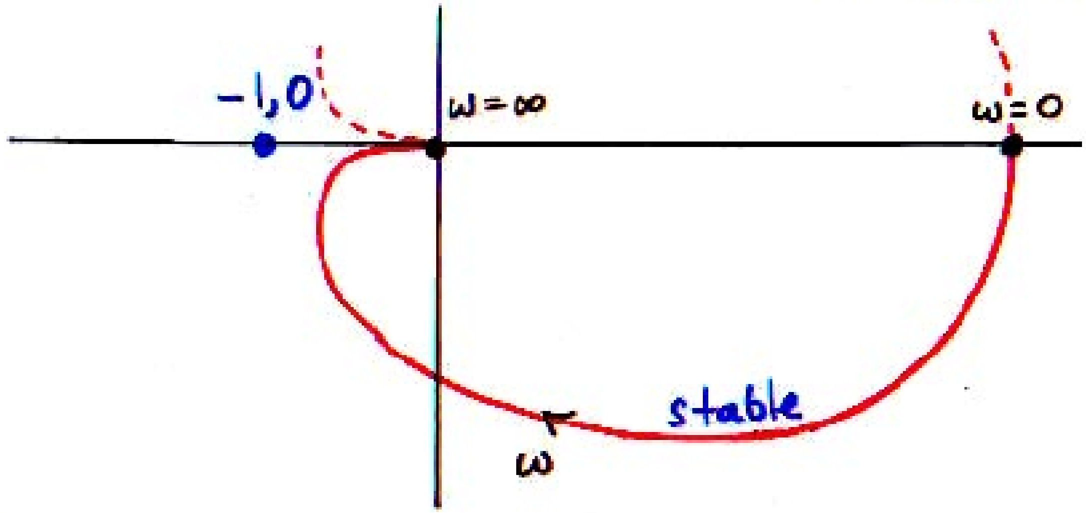
1-pole response:



Always stable

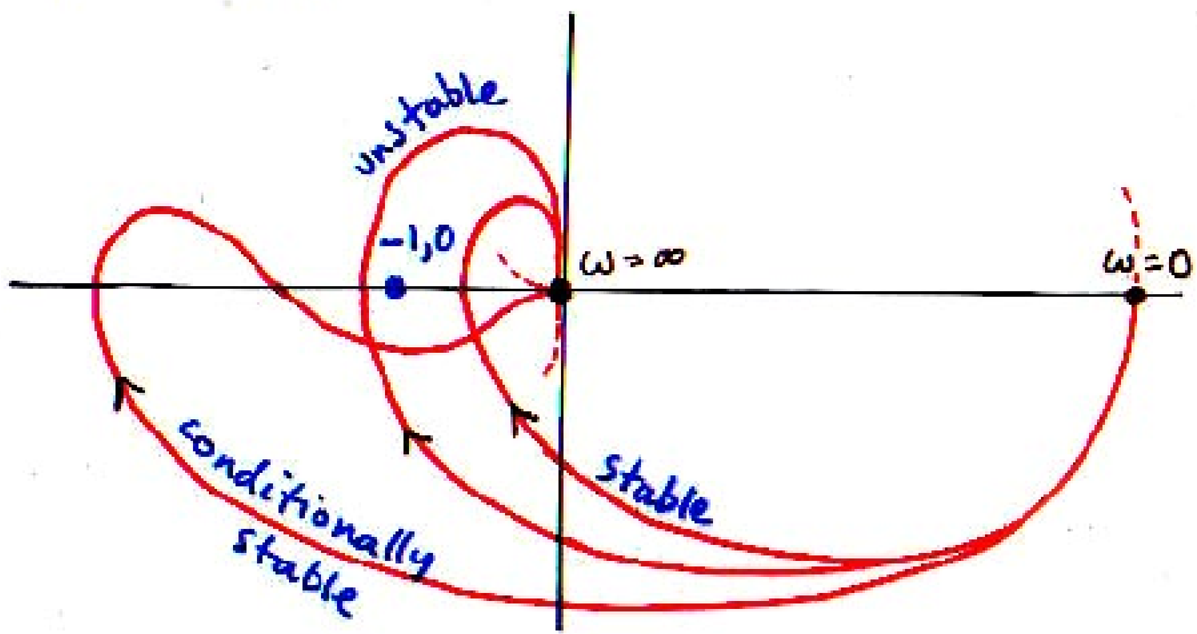
2-pole response

$$T = T_0 \frac{1}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)}$$



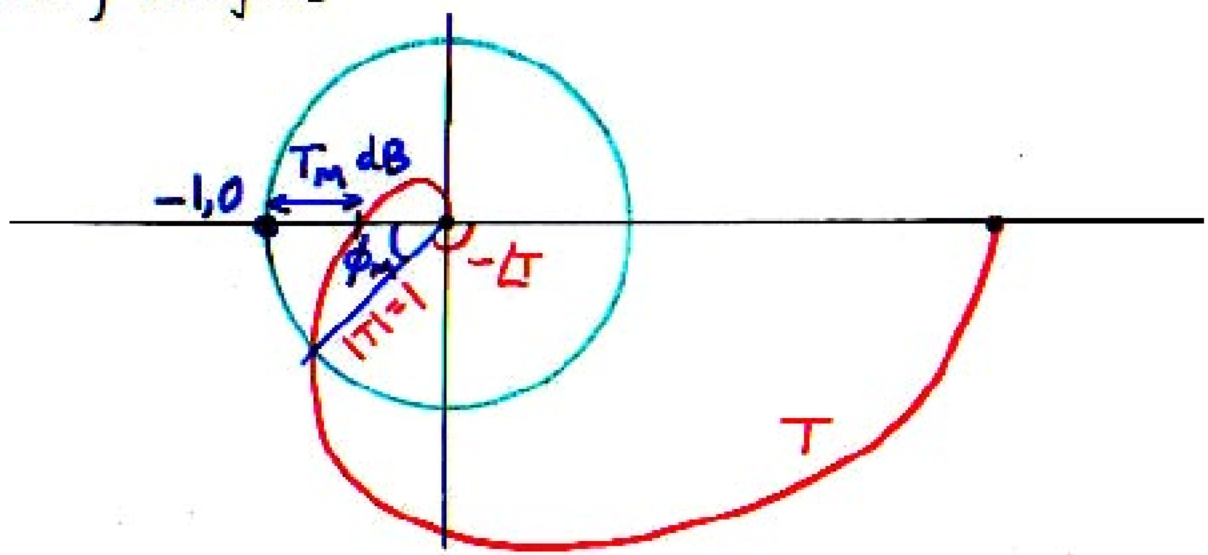
Always stable

3-pole response



Can be stable or unstable

Stability margins

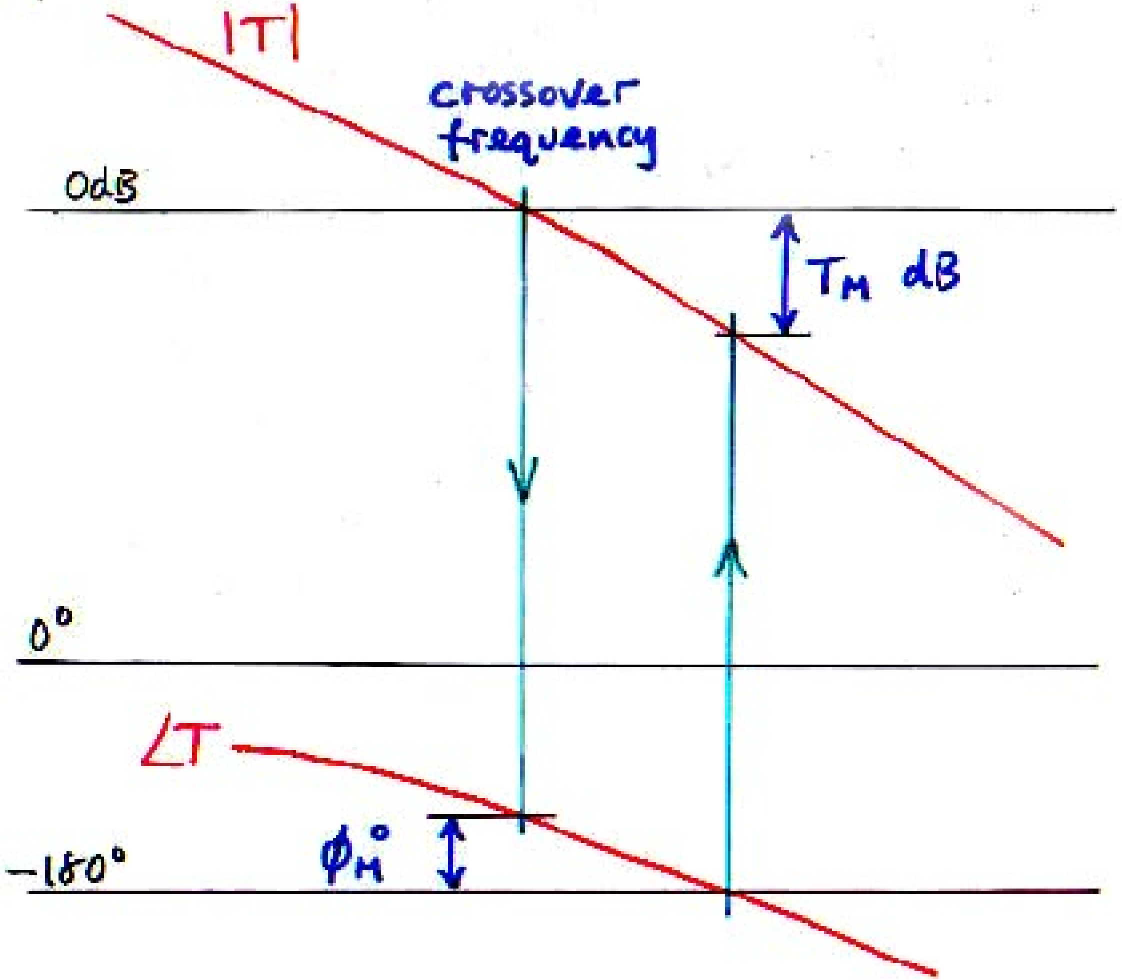


Phase margin $\phi_M \equiv 180° + \angle T$ when $|T| = 1$ (0dB)

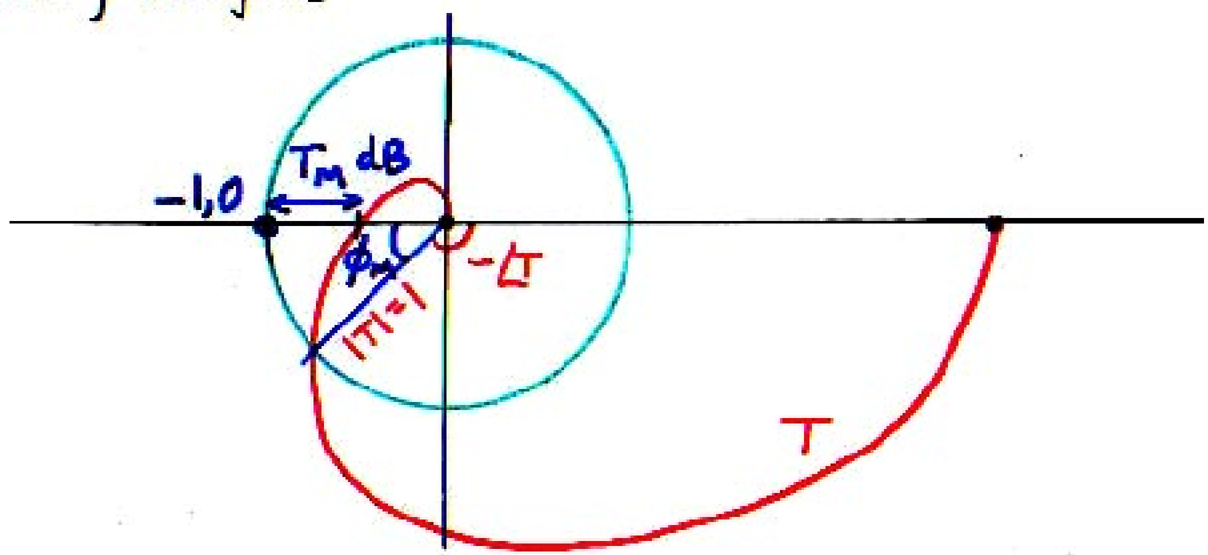
$\uparrow$ negative (lag)

Gain margin $T_M|_{dB} \equiv -T|_{dB}$ when $\angle T = -180°$

Bode plot:



Stability margins

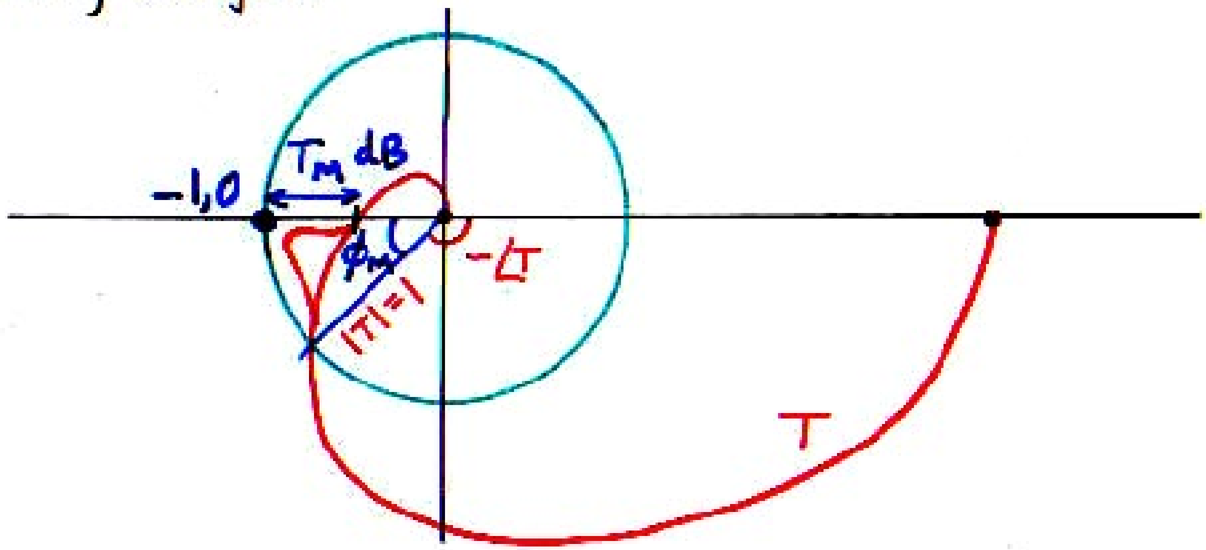


Phase margin $\phi_M \equiv 180° + \angle T$ when $|T| = 1$ (0dB)

$\angle T$ negative (lag)

Gain margin $T_M|_{dB} \equiv -T|_{dB}$ when $\angle T = -180°$

Stability margins



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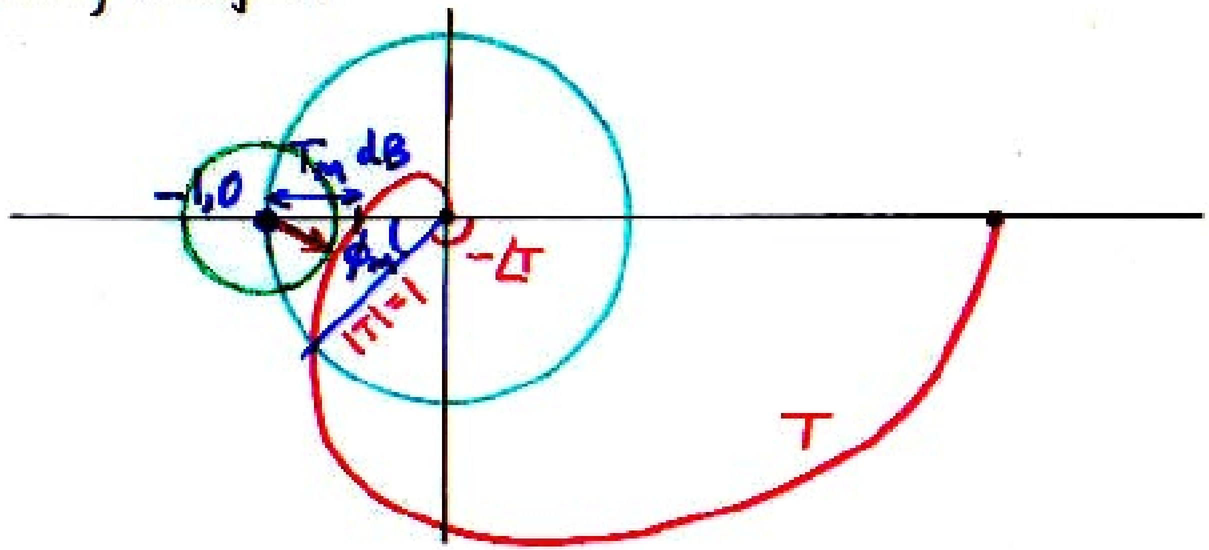

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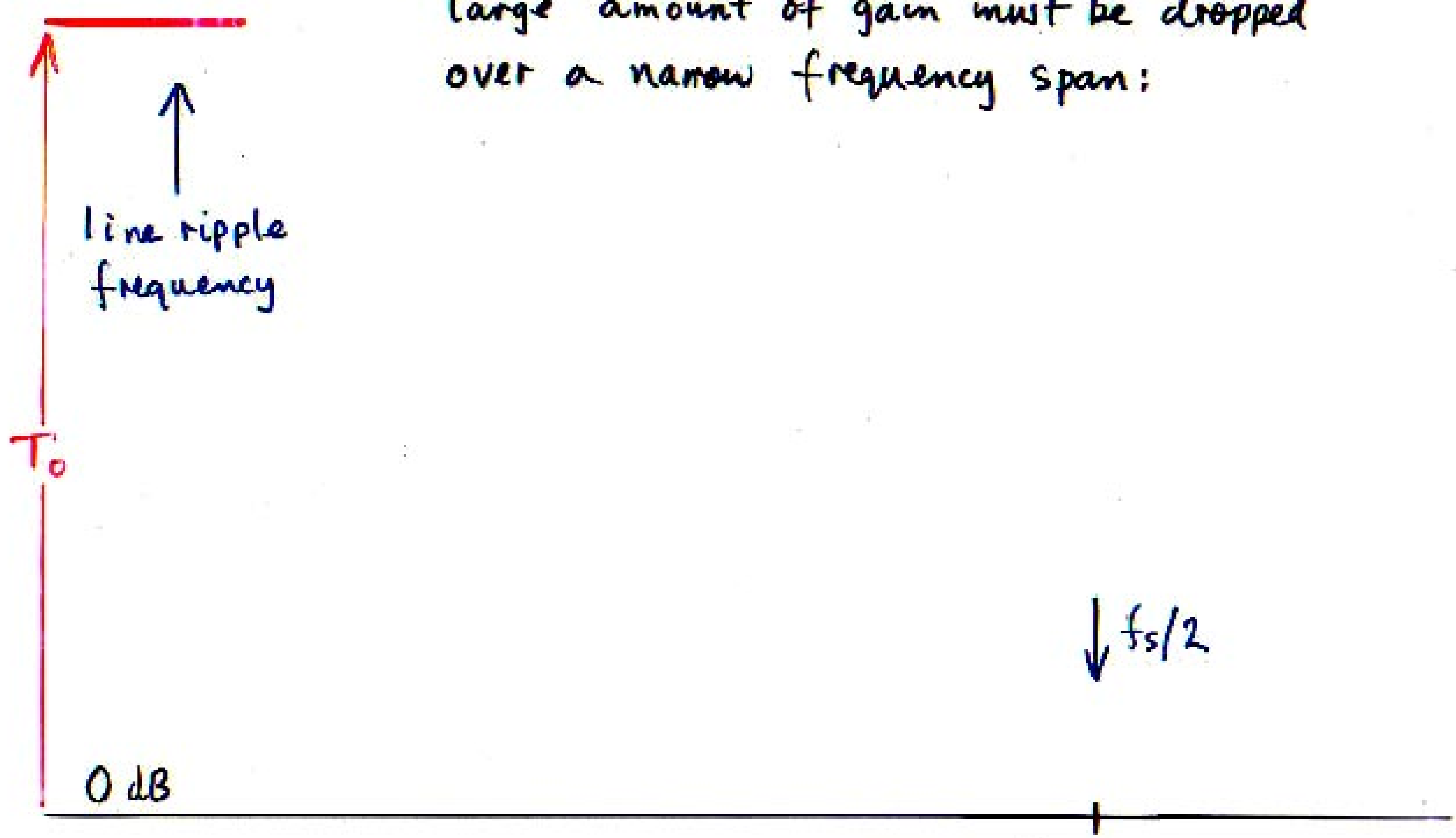
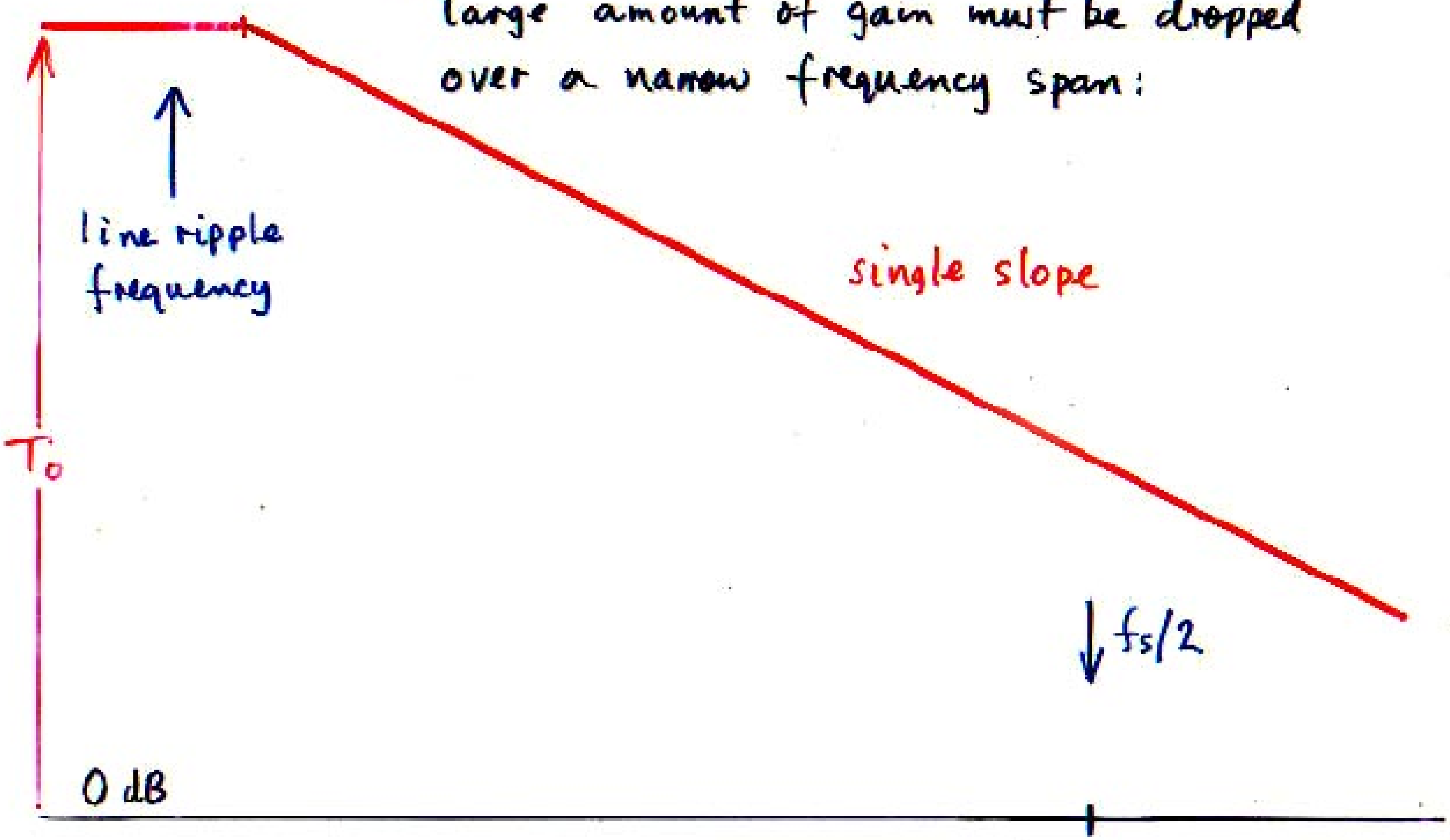
A conditionally stable system may be necessary when a large amount of gain must be dropped over a narrow frequency span:



A conditionally stable system may be necessary when a large amount of gain must be dropped over a narrow frequency span:



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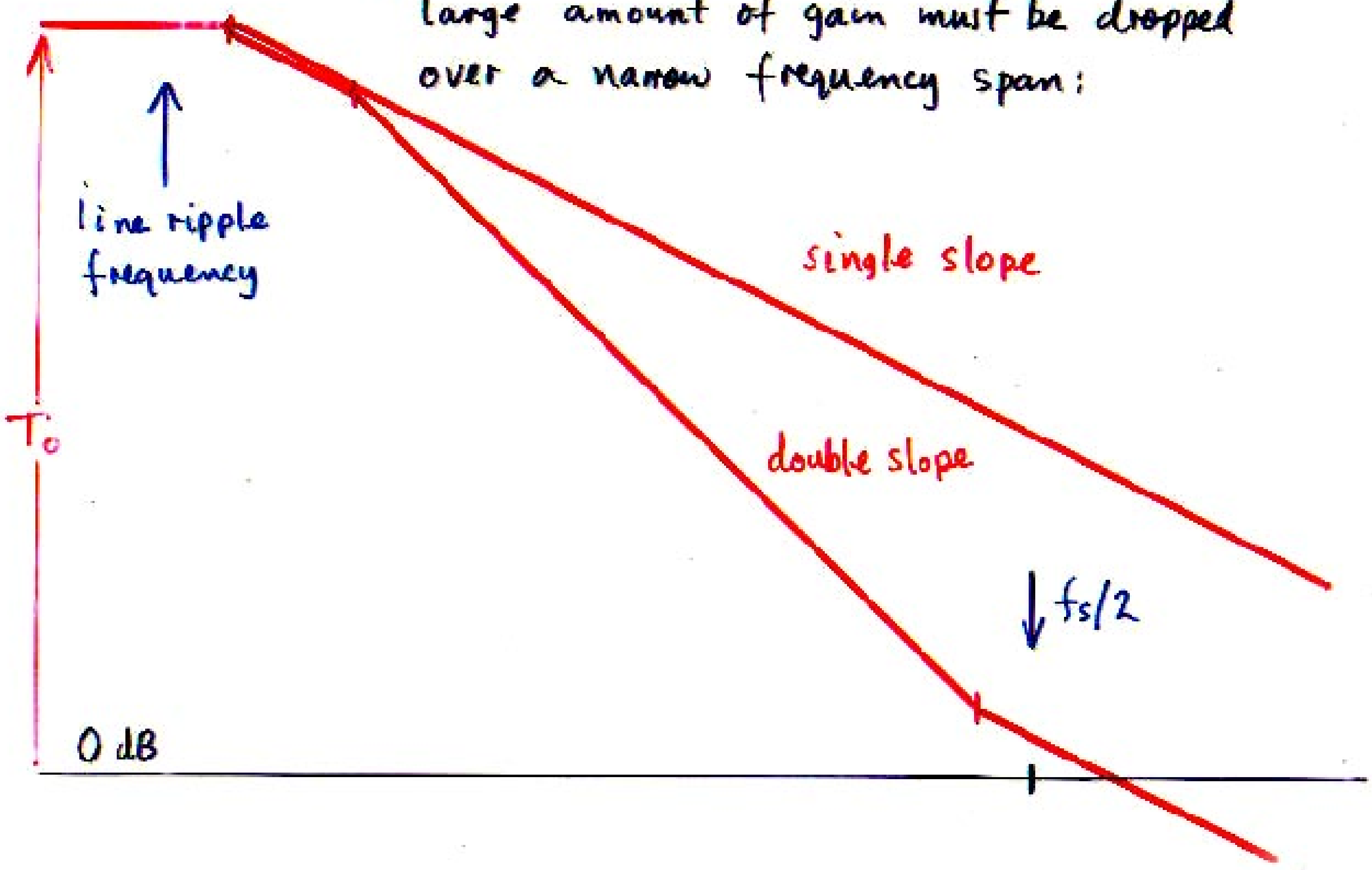
A conditionally stable system may be necessary when a large amount of gain must be dropped over a narrow frequency span:



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