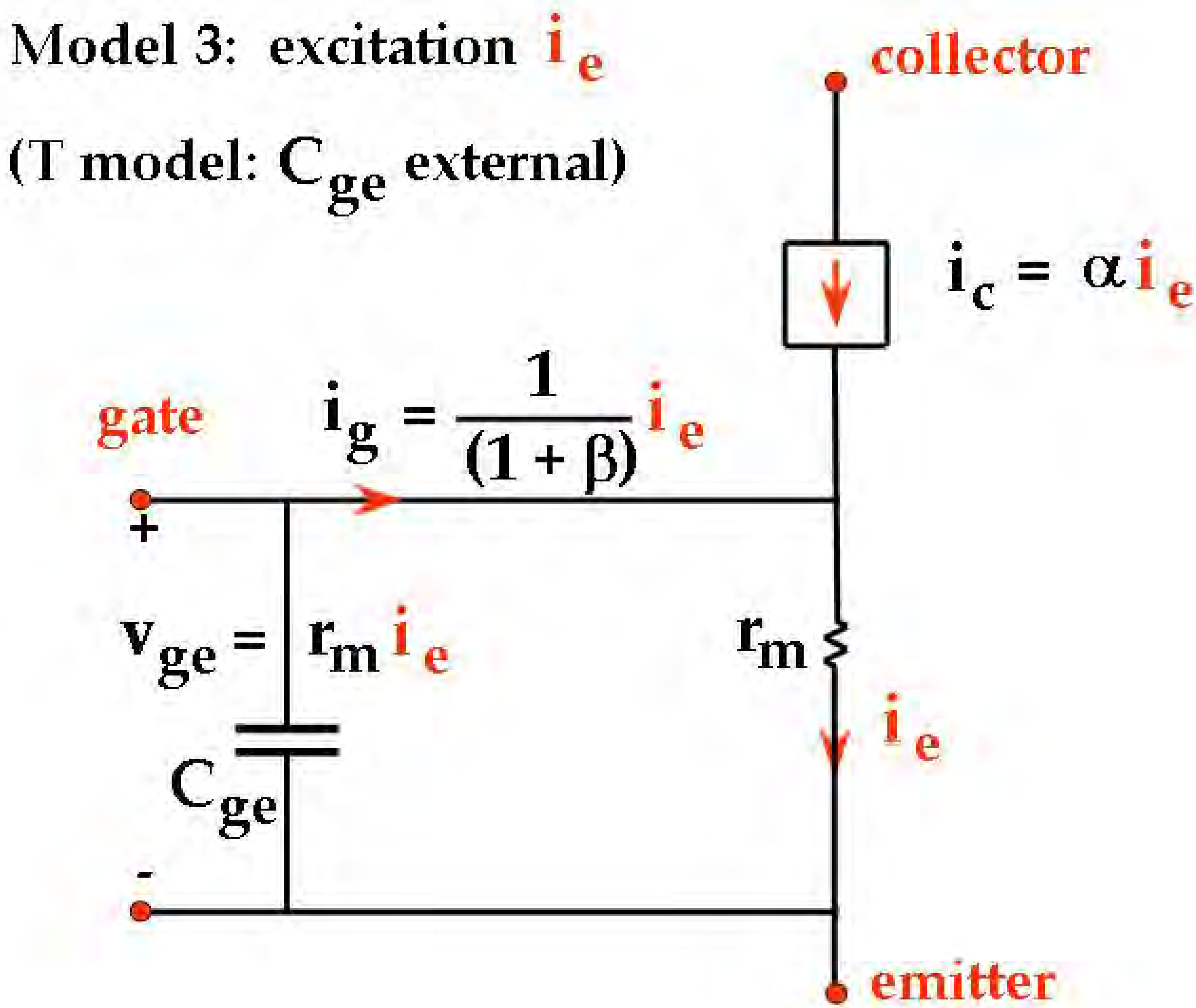
1A. FETs AND BJTs ARE CCDs

Field-Effect Transistors and Bipolar Junction Transistors are Charge-Controlled Devices

Both can be represented by the same small-signal equivalent circuit model

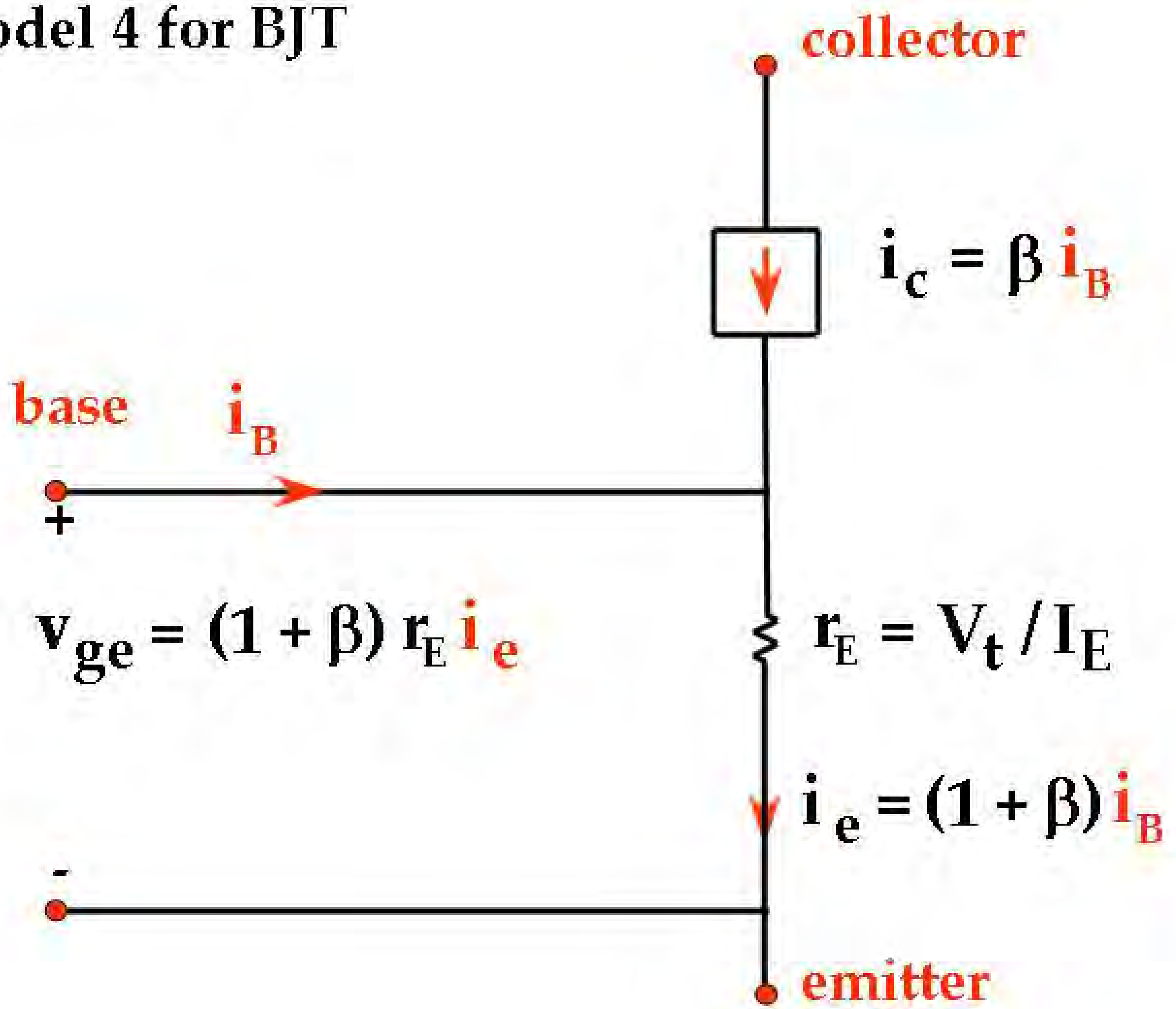
Recommended model for both the FET and the BJT:



For the FET, set α=1, β=∞

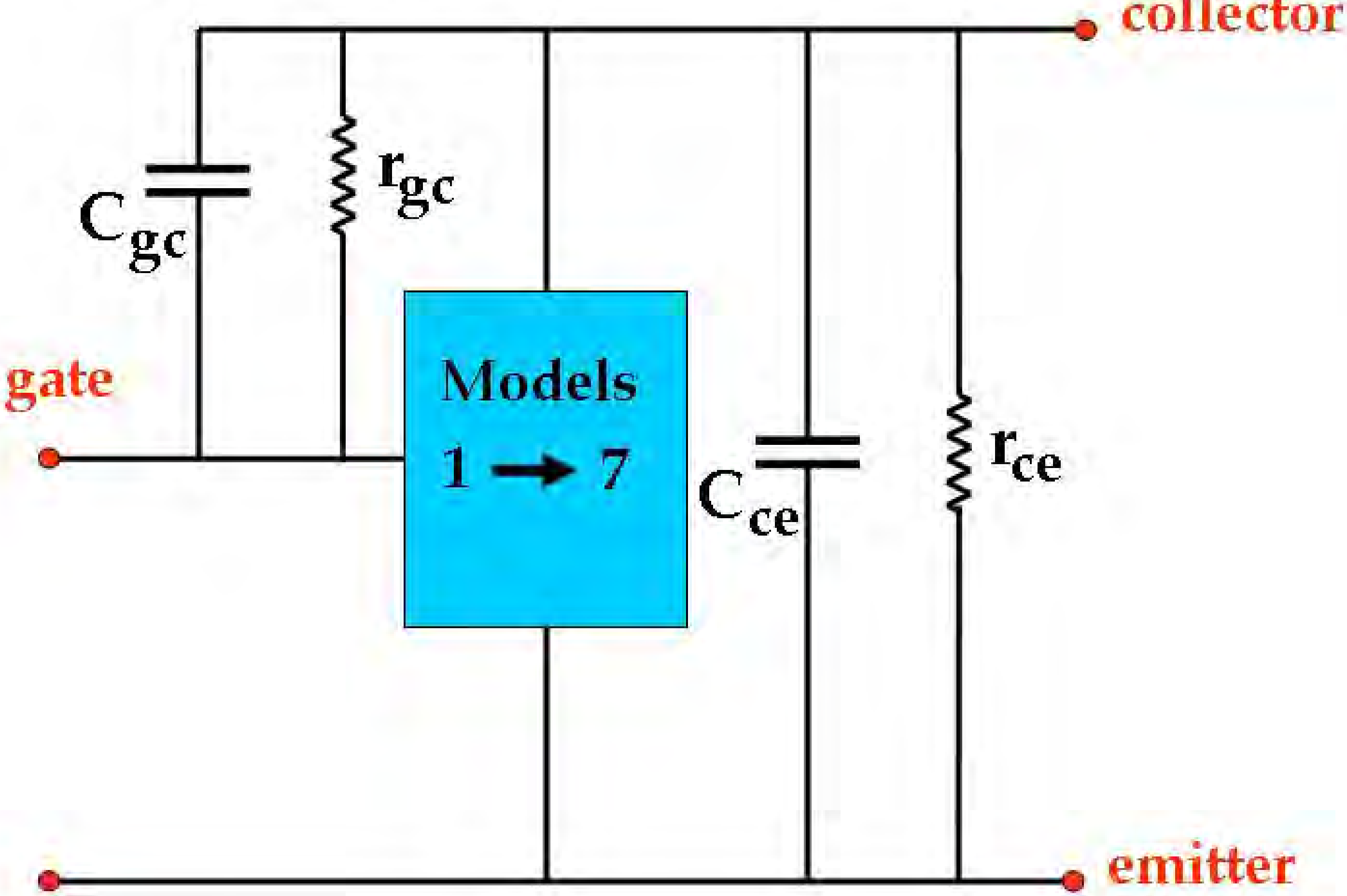
Conventional terminology:

Model 4 for BJT



This is the model used throughout the TT DVD

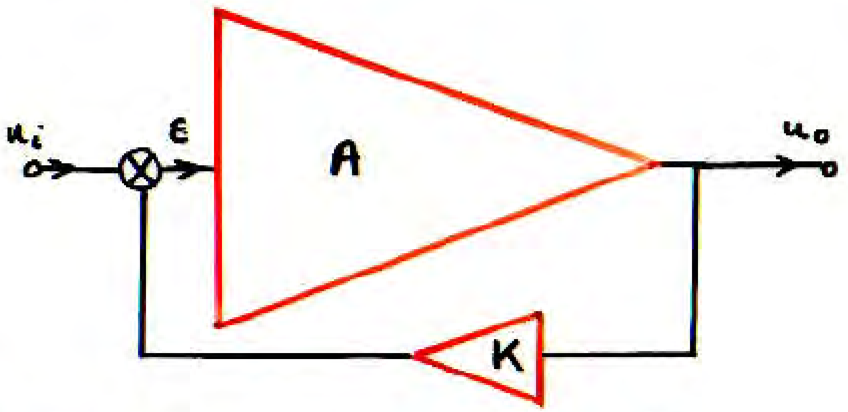
Addition of some "parasitic" elements: the Pi Model



10. BASIC FEEDBACK

An Improved Formula expresses the closed-loop gain G in terms of only two quantities, the Specification G_∞ and the Loop Gain T

Basic Properties



Solution for closed-loop gain $G \equiv \frac{u_o}{u_i}$:

$$\epsilon = u_i - K u_o \qquad\qquad u_o = A\epsilon$$

$$u_o = A(u_i - K u_o)$$

$$\frac{u_o}{u_i} \equiv G = \frac{A}{1+AK}$$

Other forms of the result:

internal gain
forward gain
open-loop gain

$$G = \frac{A}{1+AK} = \frac{A}{1+T} = \frac{A}{F}$$

closed-loop gain

feedback ratio

return ratio
loop gain

return difference
feedback factor

Other forms of the result:

internal gain / forward gain / open-loop gain

$$G = \frac{A}{1+AK} = \frac{A}{1+T} = \frac{A}{F}$$

closed-loop gain

feedback ratio

return ratio / loop gain

return difference / feedback factor

ideal closed-loop gain

$$= \left(\frac{1}{K}\right)\left(\frac{T}{1+T}\right) \equiv G_{\infty} D$$

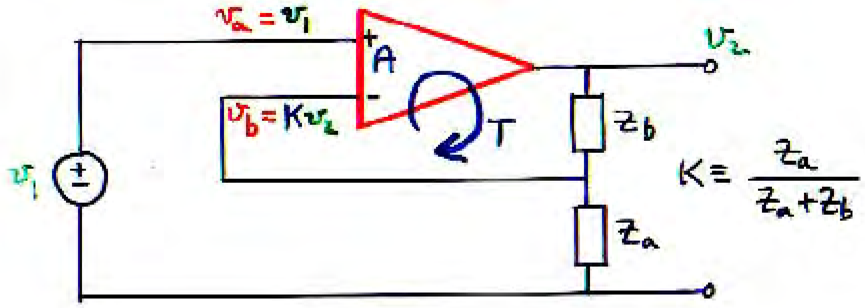
discrepancy factor

G_{∞} is given (the Specification)

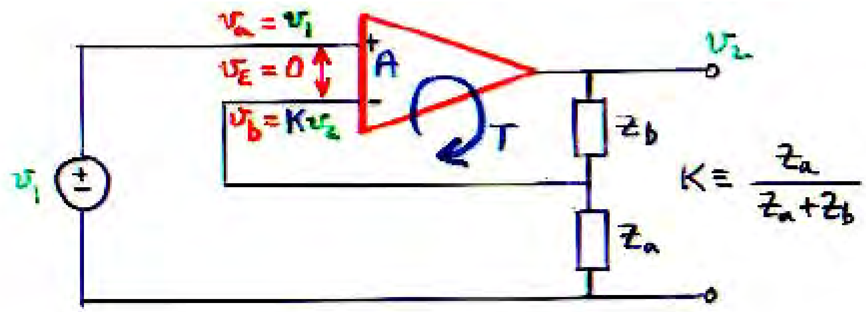
The problem of feedback design is how to make D close enough to 1, over the specified frequency range.

G_∞ is the gain conventionally calculated

$$\text{on the assumption of} \begin{cases} \text{infinite loop gain } T \\ \text{infinite forward gain } A \\ \text{zero error signal } v_E \end{cases}$$



$v_a = v_1$
A
$v_b = Kv_2$
T
v_2
Z_b
v_1
Z_a
$K \equiv \frac{Z_a}{Z_a+Z_b}$

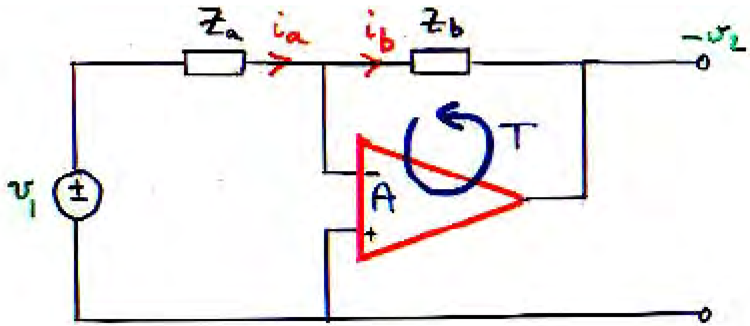


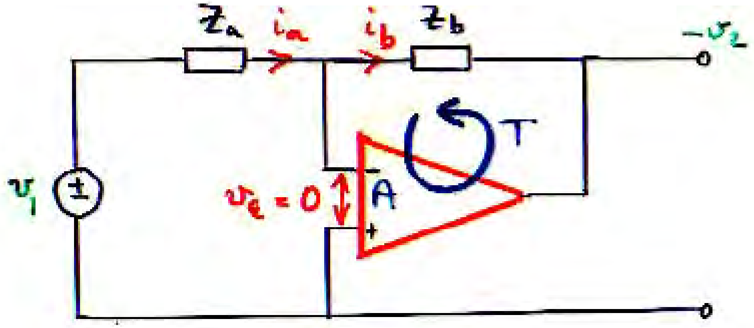
For zero error signal $v_\epsilon = 0$:

$$v_b = v_a$$

$$Kv_2 = v_1$$

$$\frac{v_2}{v_1} = G_{\infty} = \frac{1}{K} = \frac{Z_a + Z_b}{Z_a}$$


Za
ia
ib
Zb
-v2
T
A
v1



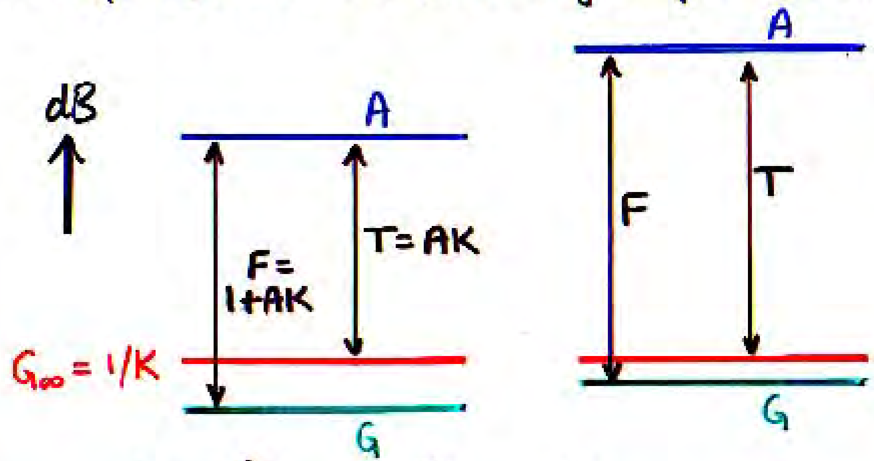
For zero error signal $v_\epsilon = 0$:

$$i_b = i_a$$

$$\frac{v_2}{Z_b} = \frac{v_1}{Z_a}$$

$$\frac{v_2}{v_1} = G_\infty = \frac{Z_b}{Z_a}$$

Relationships between the various gain quantities:



$$G = \frac{A}{1+AK} = \frac{A}{1+T} = \frac{A}{F}$$

Note: to get the same output u_o closed-loop as open-loop, must increase input u_i from u_o/A to $F u_o/A$.

Principal effect of feedback:

$$G \xrightarrow{T\to 0} A \qquad G \xrightarrow{T\to\infty} \frac{1}{K}$$

loop gain:
$T \equiv AK$

Feedback transfers sensitivity from A to K:

$$G = \frac{A}{1+AK}$$

$$\ln G = \ln A - \ln(1+AK)$$

$$\frac{\Delta G}{G} = \frac{\Delta A}{A} - \frac{K\Delta A + A\Delta K}{1+AK}$$

$$= \frac{1}{1+T}\frac{\Delta A}{A} - \frac{T}{1+T}\frac{\Delta K}{K}$$

decreases   increases

for increasing T

Extension of bandwidth

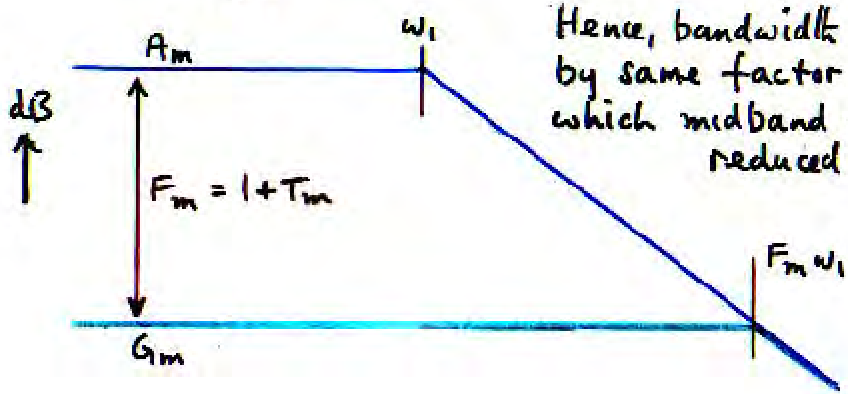
Consider the simplest high-frequency rolloff, a single pole:

$$A = A_m \frac{1}{1+\frac{s}{\omega_1}}$$

Then

$$G = \frac{A_m \frac{1}{1+\frac{s}{\omega_1}}}{1 + A_m K \frac{1}{1+\frac{s}{\omega_1}}} = \frac{A_m}{1 + A_m K + \frac{s}{\omega_1}} = \frac{A_m}{1+T_m} \frac{1}{1+\frac{s}{(1+T_m)\omega_1}}$$

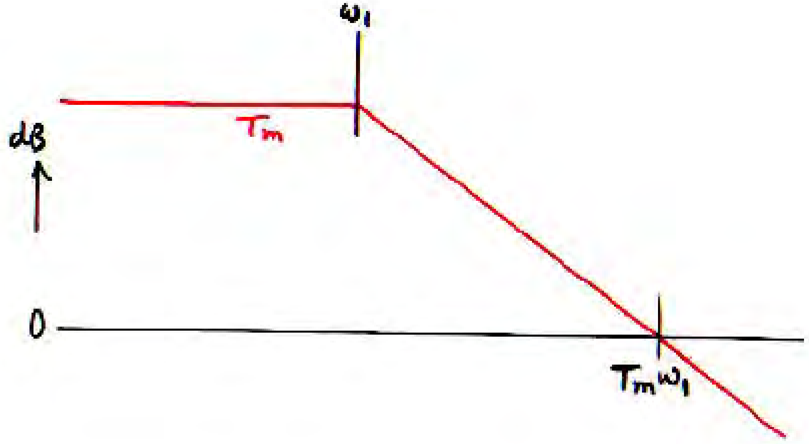
$$= G_m \frac{1}{1+\frac{s}{F_m \omega_1}} \quad \text{where} \quad G_m = \frac{A_m}{F_m}$$



Hence, bandwidth is increased by same factor F_m by which midband gain is reduced

An alternative approach emphasizes T:

$$T = A_m K \frac{1}{1+\frac{s}{\omega_1}} = T_m \frac{1}{1+\frac{s}{\omega_1}}$$



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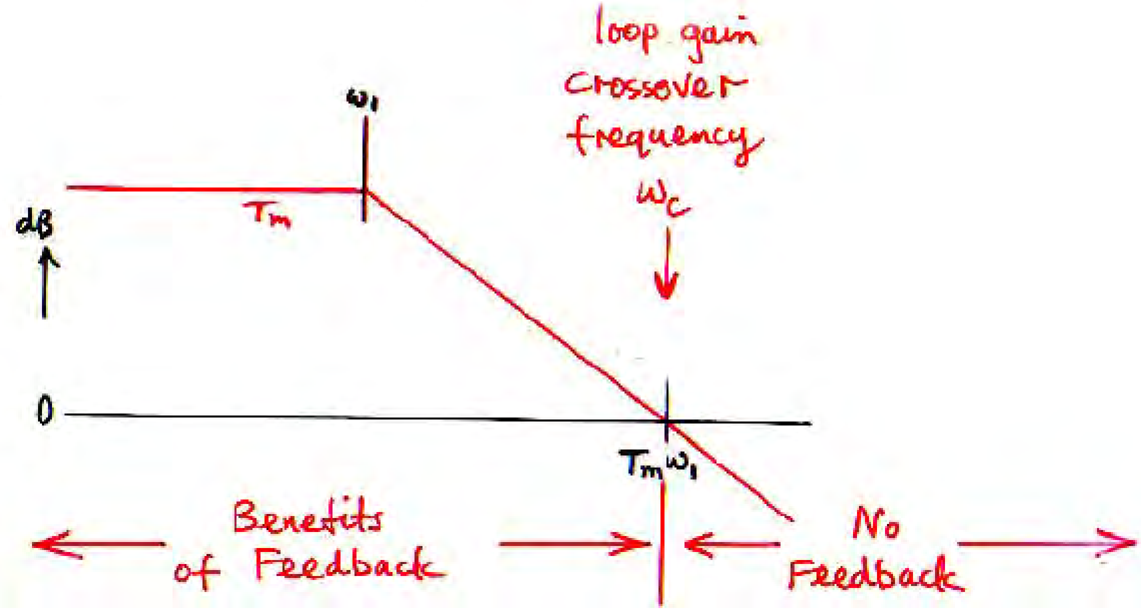
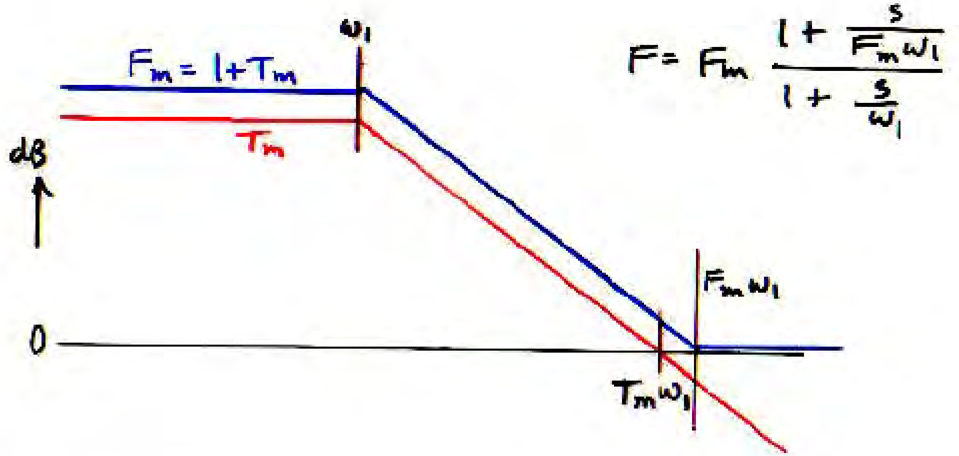


An alternative approach emphasizes T:

$$T = A_m K \frac{1}{1+\frac{s}{\omega_1}} = T_m \frac{1}{1+\frac{s}{\omega_1}}$$

Construct $F = 1 + T$

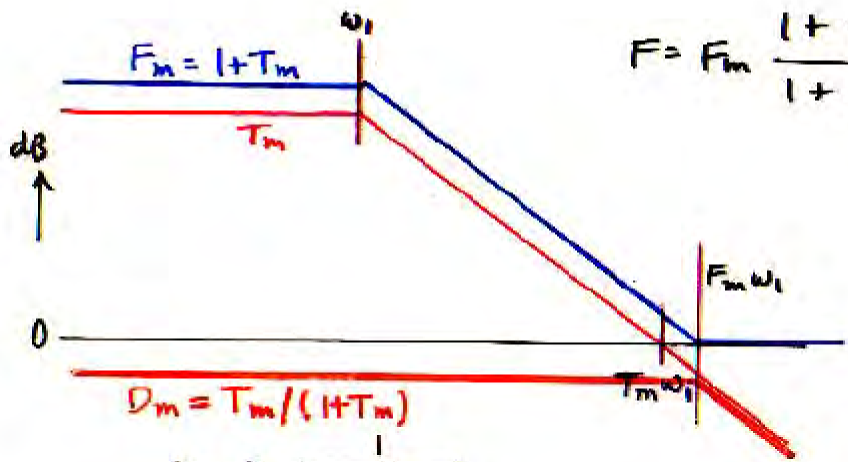


An alternative approach emphasizes T:

$$T = A_m K \frac{1}{1+\frac{s}{\omega_1}} = T_m \frac{1}{1+\frac{s}{\omega_1}}$$

Construct $F = 1 + T$

Construct $D = \frac{T}{1+T} = \frac{T}{F}$

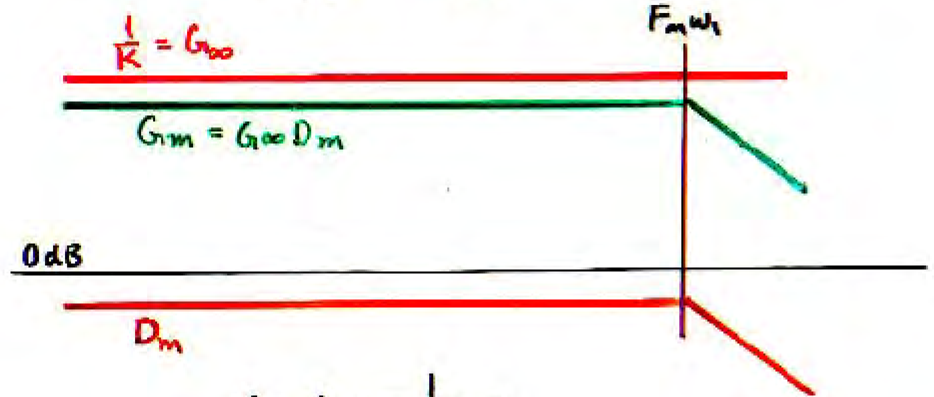


$$F = F_m \frac{1+\frac{s}{F_m\omega_1}}{1+\frac{s}{\omega_1}}$$

$$D = D_m \frac{1}{1+\frac{s}{F_m\omega_1}}$$

Hence the closed-loop gain G can be obtained from

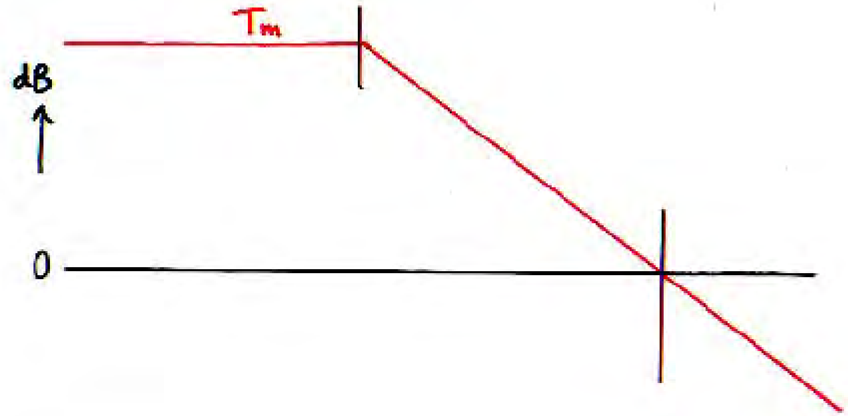
$$G = \frac{1}{K} D = G_{\infty} D$$



$$G = G_m \frac{1}{1 + \frac{s}{F_m \omega_1}}$$

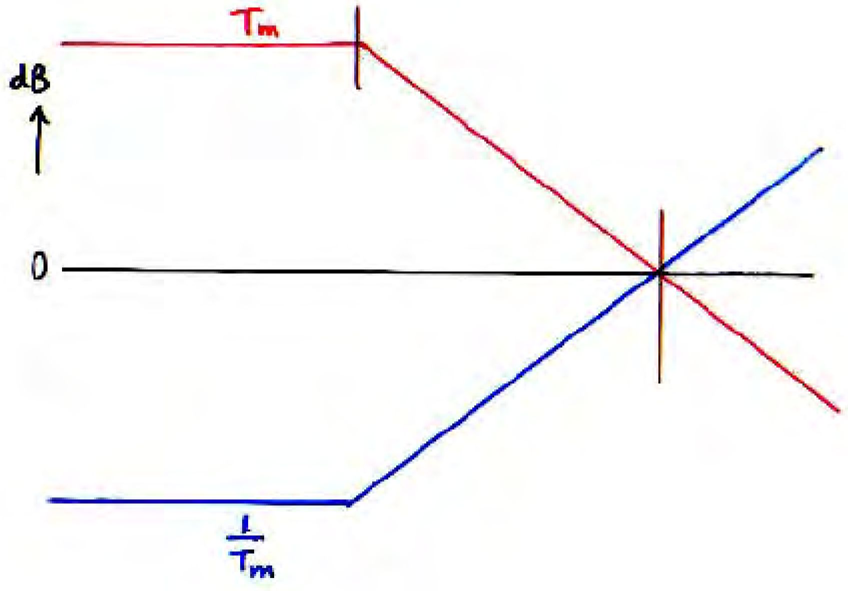
Another way to obtain D is directly from T as

$$D = \frac{1}{1 + \frac{1}{T}}$$



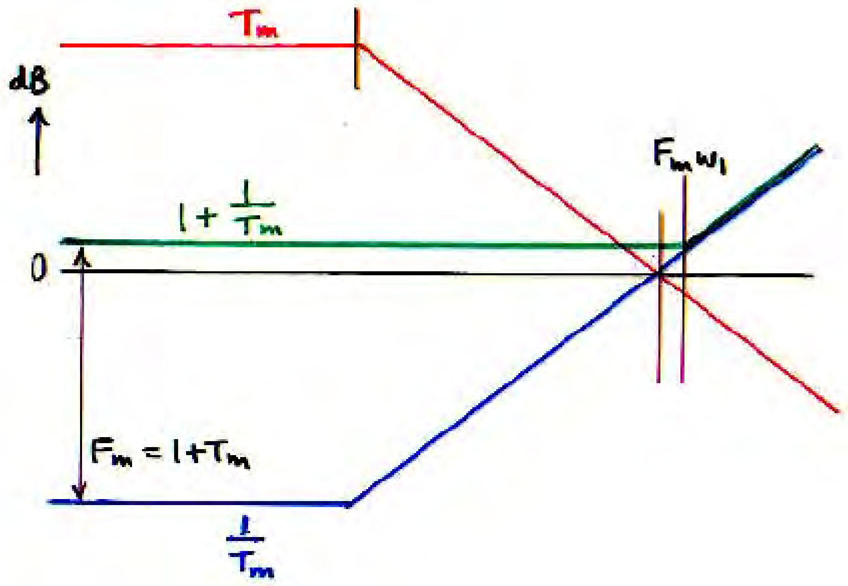
Another way to obtain D is directly from T as

$$D = \frac{1}{1+\frac{1}{T}}$$



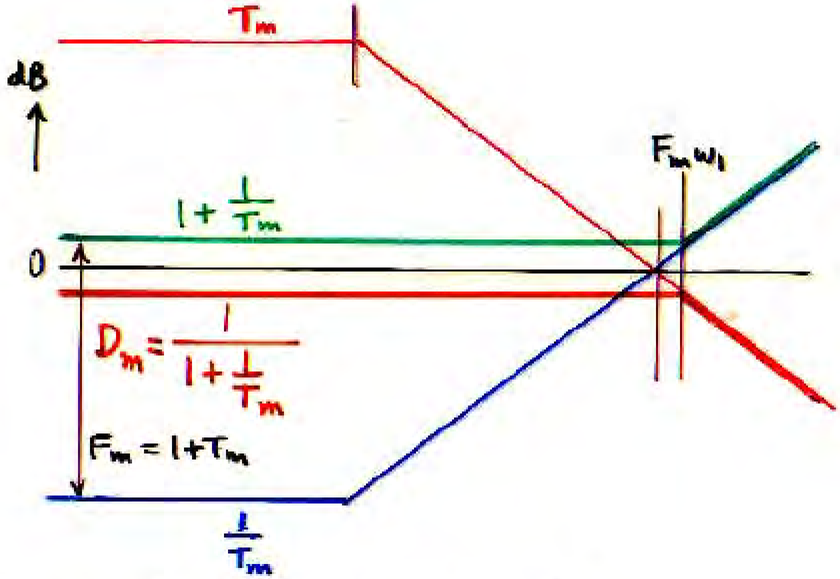
Another way to obtain D is directly from T as

$$D = \frac{1}{1+\frac{1}{T}}$$



Another way to obtain D is directly from T as

$$D = \frac{1}{1+\frac{1}{T}}$$



which gives the same result for D.

Although the factored pole-zero forms for F and D could easily have been obtained analytically in this case, the above graphical procedure saves much algebra in more complicated cases because suitable approximations can be seen immediately.

Notice that in finding $F = 1+T$ and $1/D = 1 + 1/T$ a sum (or in general a difference) is determined from the asymptotes on log scales. This is an example of the powerful technique of doing the algebra on the graph.

Generalization: Doing the Algebra on the Graph

The log-log scales of dB vs. log frequency graphs permit determination of

Not only:

Exact combinations of products and quotients of constituent factors

But also:

Approximate combinations of sums and differences of constituent factors: whichever is the larger dominates. This technique permits approximate analytic results to be obtained, in which algebraic approximations are replaced by graphical approximations.

Examples: Analytic determination from T of $F = 1+T$ and $D = T/F$.

Determination of Feedback System Parameters — General

The method of Loop Gain T determination T by injection of a test signal into the closed loop can be generalized and, by inclusion of the Null Double Injection technique, also leads to determination of the Ideal Closed-Loop Gain G_∞.

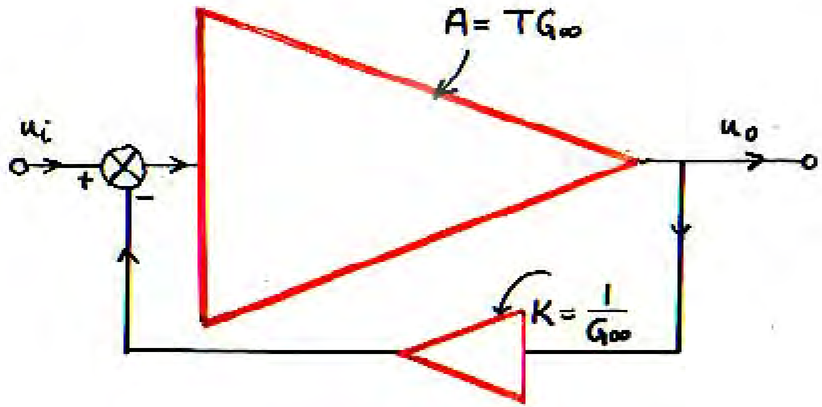
Basic relations:

$$G = \frac{A}{1+AK} = \frac{A}{1+T}$$

$$= \frac{1}{K}\frac{T}{1+T}$$

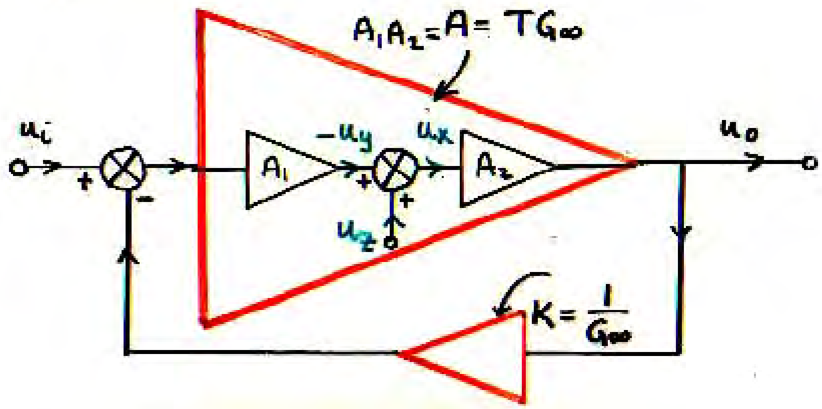
$$= G_\infty D$$

Note that $A = TG_\infty$ and $K = 1/G_\infty$:



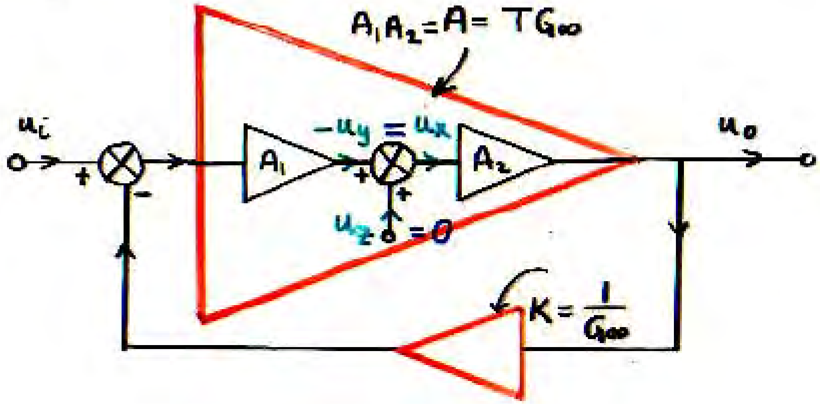
Note that $A = TG_\infty$ and $K = 1/G_\infty$:

Consider a second driving signal u_z injected into the forward path:



Note that $A = TG_\infty$ and $K = 1/G_\infty$:

Consider a second driving signal u_z injected into the forward path:

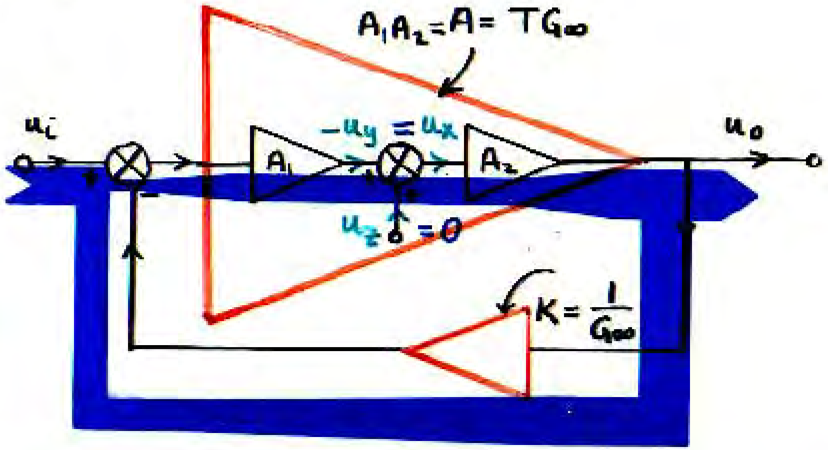


For single injection, u_i only (normal operation):

$$\left.\frac{u_o}{u_i}\right|_{u_z=0} \equiv G = G_\infty \frac{T}{1+T}$$ closed-loop gain

Note that $A = TG_\infty$ and $K = 1/G_\infty$:

Consider a second driving signal u_z injected into the forward path:



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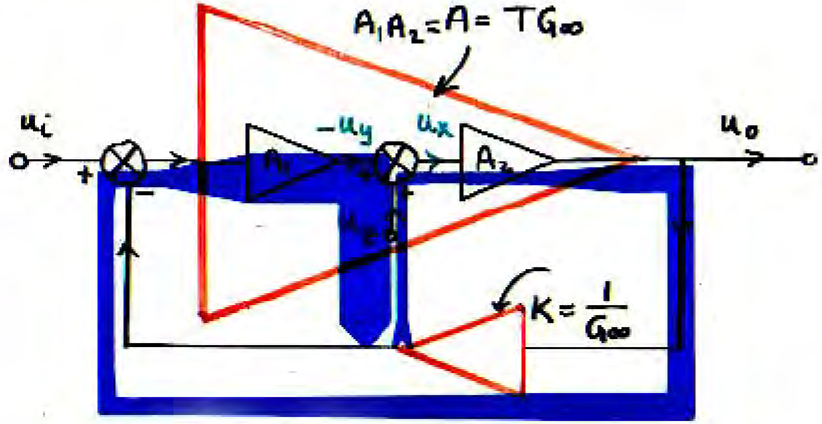

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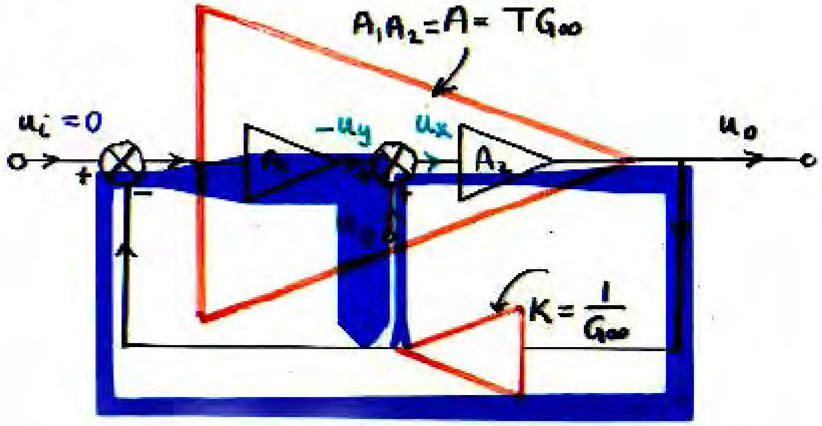
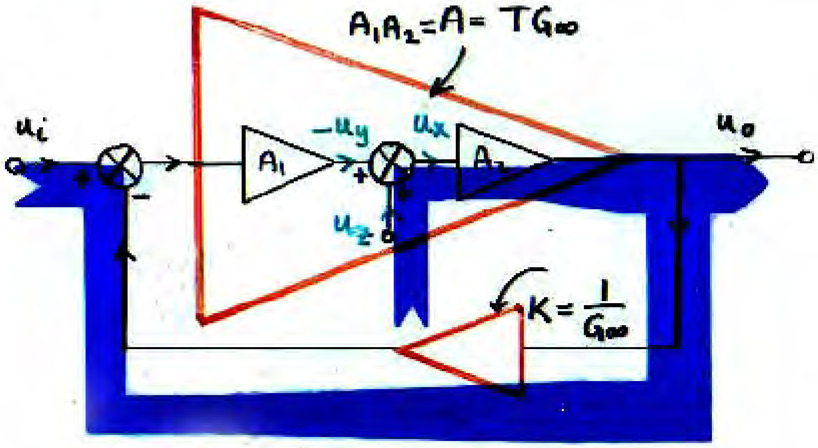


For single injection, u_z only,

$$\left.\frac{u_y}{u_x}\right|_{u_i=0} = A_2 K A_1 = AK \equiv T \quad \text{loop gain}$$

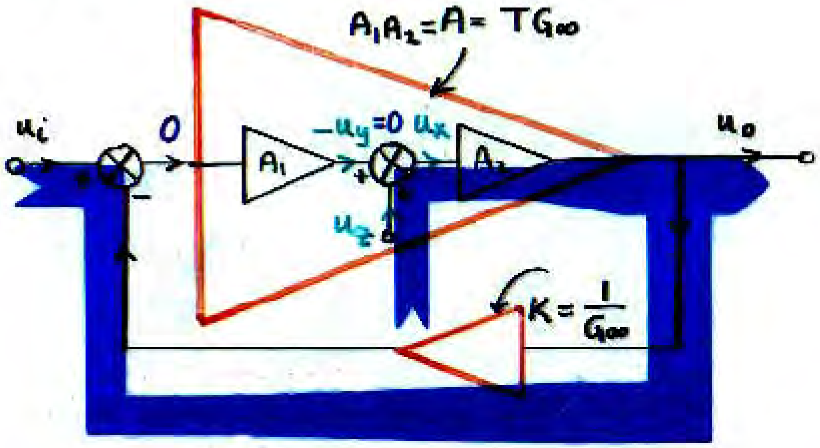
Note that $A = TG_{\infty}$ and $K = 1/G_{\infty}$:

Consider a second driving signal u_z injected into the forward path:



Note that $A = TG_\infty$ and $K = 1/G_\infty$:

Consider a second driving signal u_z injected into the forward path:



For double injection, u_i and u_z, adjusted to null u_y:

$$\left.\frac{u_o}{u_i}\right|_{u_y=0} = \frac{1}{K} \equiv G_\infty \quad \text{ideal closed-loop gain}$$

Generalization: The Feedback Theorem

The feedback path does several things:

1. Provides the feedback signal — ideal (desired)
2. Loads the output
3. Loads the input

$\Big\}$ nonideal (undesired)

Conventional form of the theorem:

$$G = \frac{A}{1+T} \quad \text{where} \quad T \equiv AK$$

Disadvantages:

Breaking the feedback path at the input or the output disturbs the loading effects.

Recommended form of the theorem:

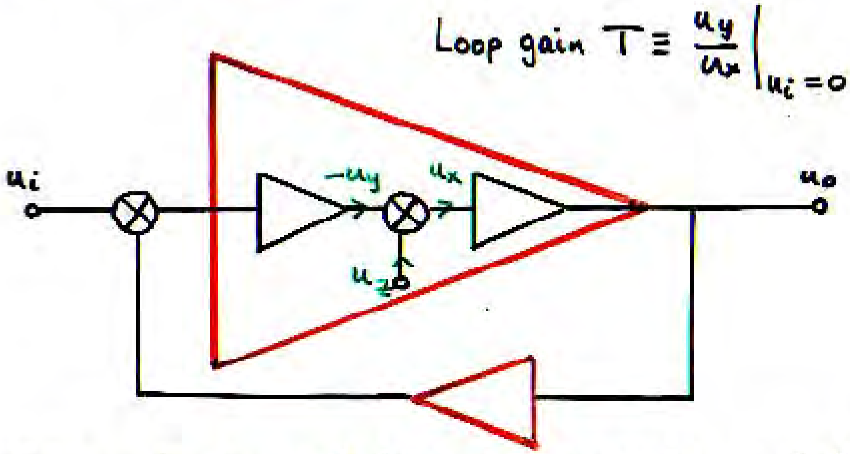
$$G = G_{\infty} D \quad \text{where} \quad G_{\infty} \equiv \frac{1}{K} \quad D \equiv \frac{T}{1+T}$$

Advantages:

1. G_{∞} and D are directly related to the important properties of the system:
   G_{∞}, the Ideal Loop Gain, is the design Specification;
   D, the Discrepancy Factor, must be designed to be close to unity over the specified frequency range.
2. T can be found by injection of a test signal into the closed loop, without disturbance of the feedback path loading effects, and G_{∞} can be found by Null Double Injection of both the input and a test signal.

NOTE: It is never necessary to know A, the open-loop forward gain: A is always embedded in T, which is why the nonideal loading effects of the feedback path are automatically accounted for.

Implementation of injection of second signal for loop gain determination



Conditions to be satisfied by injection point:

1. Must be inside the feedback loop
2. Injected signal must add to the forward signal without affecting the impedance loading

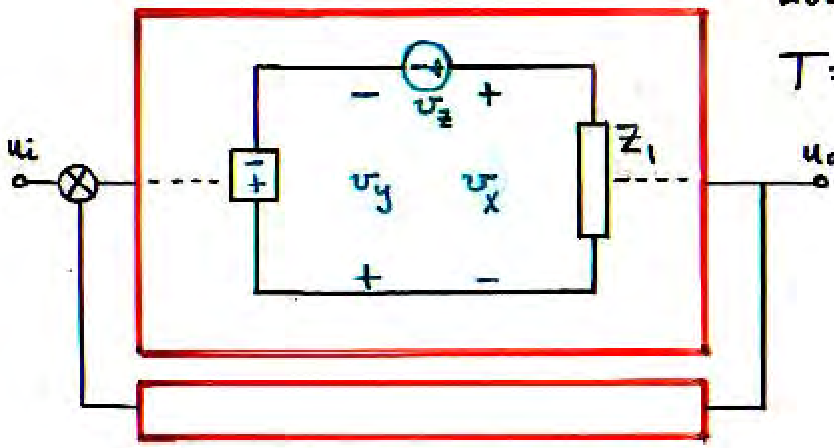
Two points satisfy these conditions

1. Inject a voltage in series with a controlled voltage generator:

$v_z = v_x + v_y$

Loop gain:

$$T = \frac{v_y}{v_x}\bigg|_{u_i = 0}$$



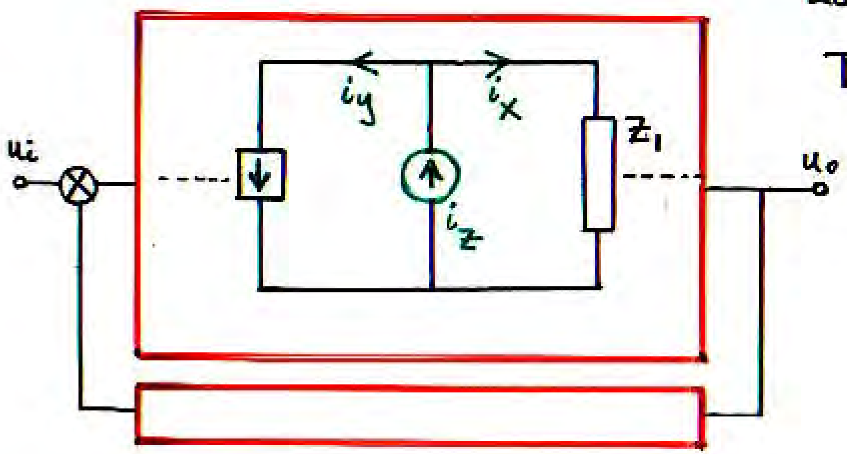
Two points satisfy these conditions

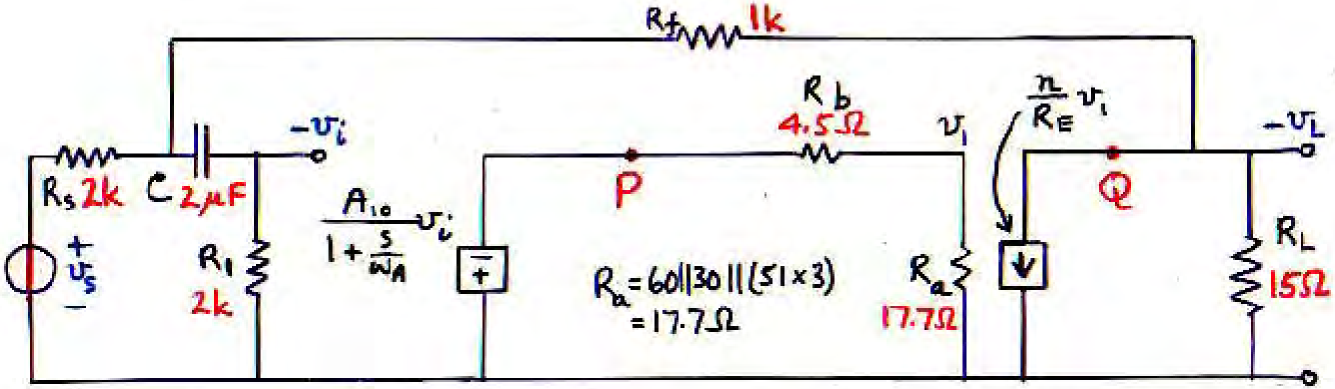
2. Inject a current in shunt with a controlled current generator:

$i_z = i_x + i_y$

Loop gain:

$$T = \left.\frac{i_y}{i_x}\right|_{u_i=0}$$



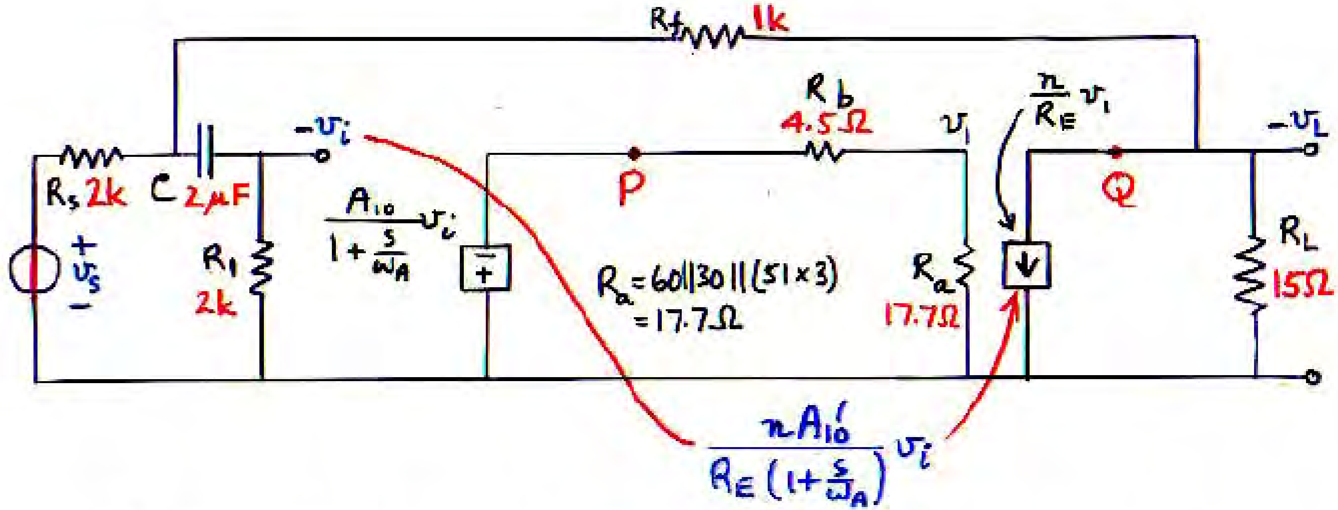


For all calculations concerning loop gain, the input voltage v_s is zero.

Suitable injection points:

Series voltage at point P

Shunt current at point Q



If current injection is chosen, the gain from v_i to the power stage output generator can be condensed into a single factor:

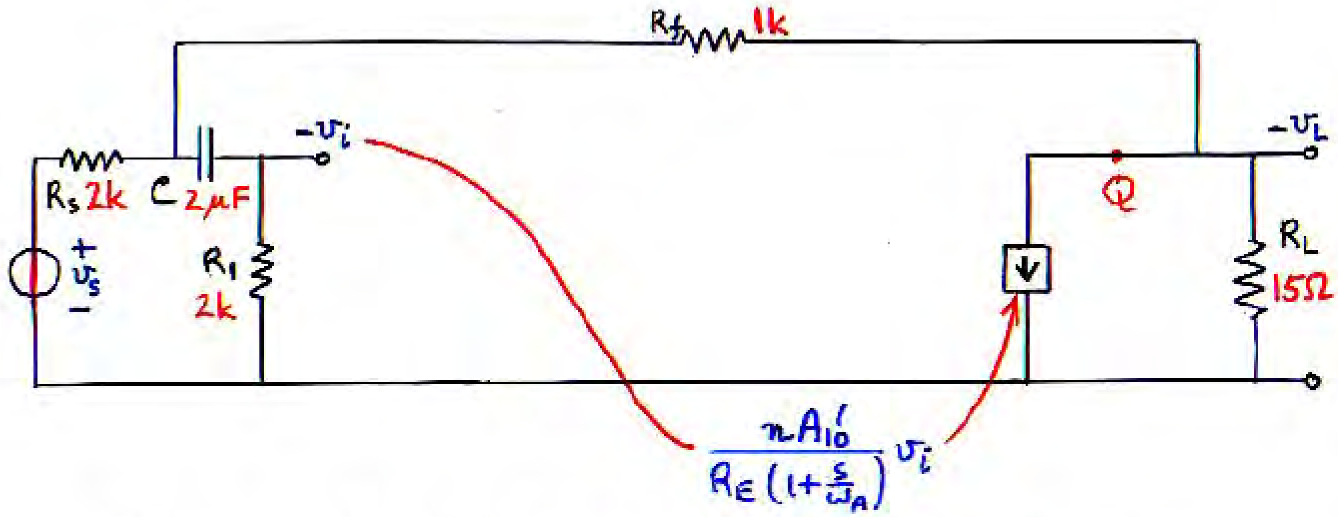
$$\frac{A_{10}}{1+\frac{s}{\omega_A}}\frac{R_a}{R_a+R_b}\frac{n}{R_E}v_i = \frac{nA_{10}'}{R_E(1+\frac{s}{\omega_A})}v_i$$

where

$$A_{10}' = A_{10}\frac{R_a}{R_a+R_b} = 8\,dB \times \frac{17.7}{17.7+4.5} = 2.51\times 0.80$$

$$= 2.0 \Rightarrow 6\,dB$$

is the loaded gain of the opamp.



If current injection is chosen, the gain from v_i to the power stage output generator can be condensed into a single factor:

$$\frac{A_{10}}{1+\frac{s}{\omega_A}}\frac{R_a}{R_a+R_b}\frac{n}{R_E}v_i = \frac{nA_{10}'}{R_E(1+\frac{s}{\omega_A})}v_i$$

where

$$A_{10}' \equiv A_{10}\frac{R_a}{R_a+R_b} = 8\,dB \times \frac{17.7}{17.7+4.5} = 2.51 \times 0.80$$

$$= 2.0 \Rightarrow 6\,dB$$

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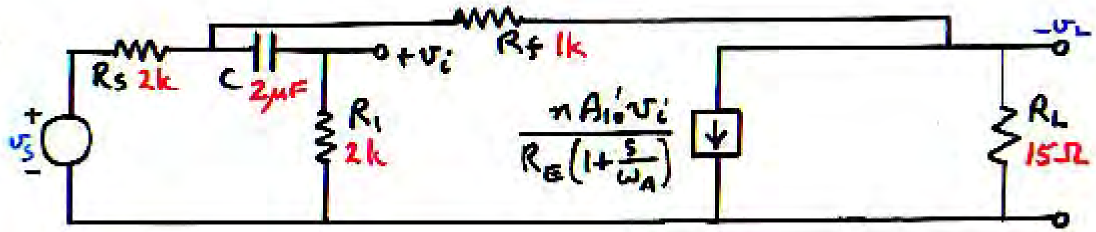
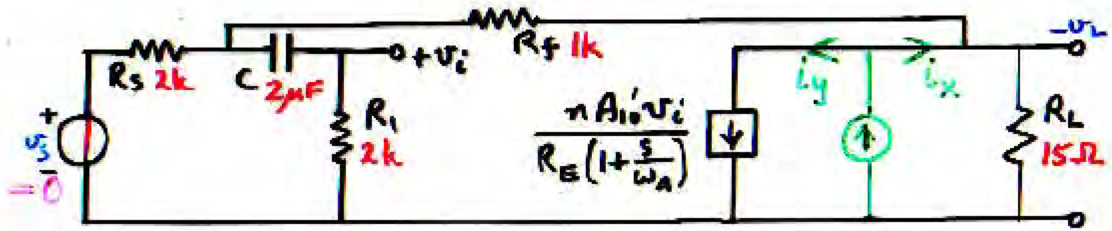
Ac model


Rs 2k
C 2μF
+vi
Rf 1k
-vL
R1 2k
vs
$\frac{n A_{10}' v_i}{R_E(1+\frac{s}{\omega_A})}$
RL 15Ω

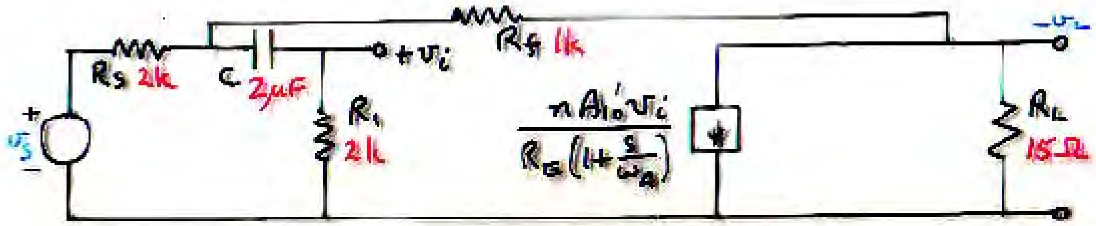
Ac model



$$T = \left.\frac{i_y}{i_x}\right|_{v_s=0} = \frac{R_L}{\cancel{R_L}+R_f+R_s\|R_1\left(1+\frac{\omega_1}{s}\right)} \; \frac{R_s}{R_s+R_1\left(1+\frac{\omega_1}{s}\right)} \; \frac{R_1 n A_{10}'}{R_E\left(1+\frac{s}{\omega_A}\right)}$$

where

$$\omega_1 \equiv \frac{1}{CR_1} \qquad f_1 = \frac{159}{2\times 2} = 40\text{Hz}$$



This method requires additional algebraic force to find the corner frequencies. Therefore, choose

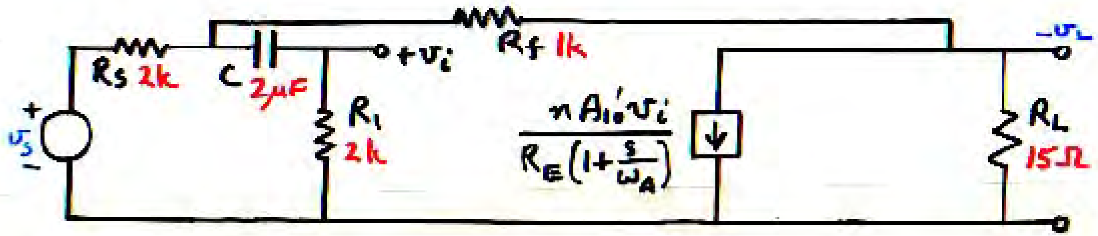
Better Method:

First, find T_m:

$$T_m = \left.\frac{i_y}{i_x}\right|_{v_s=0} = \frac{R_L}{\cancel{R_L} + R_f + R_s \| R_1}(R_s \| R_1)\frac{nA_{10}'}{R_E}$$

$$= \frac{(R_f \| R_s \| R_1)\, nA_{10}' R_L}{R_f R_E} = \frac{0.5 \times 2 \times 2 \times 15}{1 \times 3}$$

$$= 10 \Rightarrow 20\text{dB}$$



Second, find corner frequencies due to C by use of Extra Element Theorem.

Apply test signal u_z in place of C.

To find $Z_n = R_n$:

"input" signal for T ($v_s = 0$)

Adjust u_z in presence of i_x to null i_y

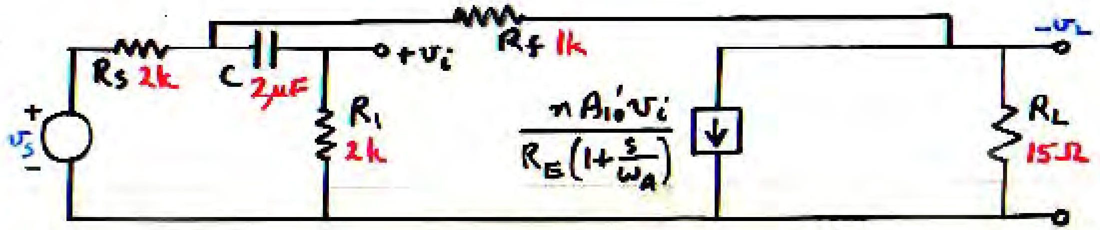
"output" signal for T

Hence: $v_i = 0$, and $R_n = \infty$

To find $Z_d = R_d$:

Apply u_z with $i_x = 0$

Hence: $R_d = (\cancel{R_L} + R_f) \| R_s + R_1$



Then:

∞

$$T = T_m \frac{1+\frac{Z}{Z_n}}{1+\frac{Z}{Z_d}} \frac{1}{1+\frac{s}{\omega_A}}$$

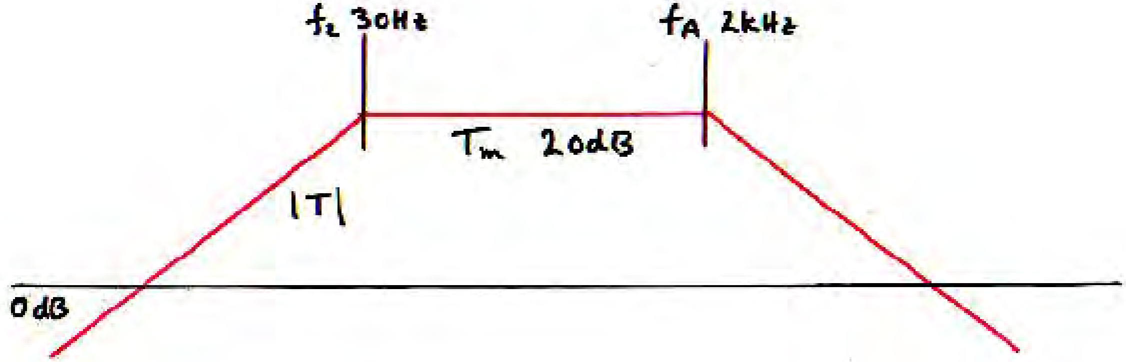
already explicit

$\frac{1}{sC}$ $\quad R_d$

$$= T_m \frac{1}{\left(1+\frac{\omega_z}{s}\right)\left(1+\frac{s}{\omega_A}\right)}$$

where

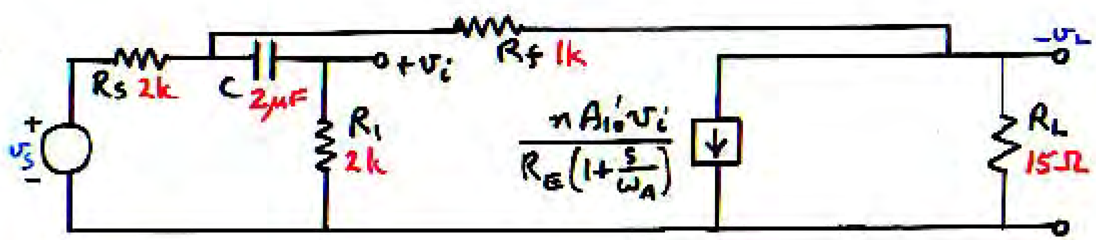
$$\omega_z \equiv \frac{1}{CR_d} = \frac{1}{C[R_f \| R_s + R_1]} \qquad f_z = \frac{159}{2[1\|2+2]} = 30\,\mathrm{Hz}$$

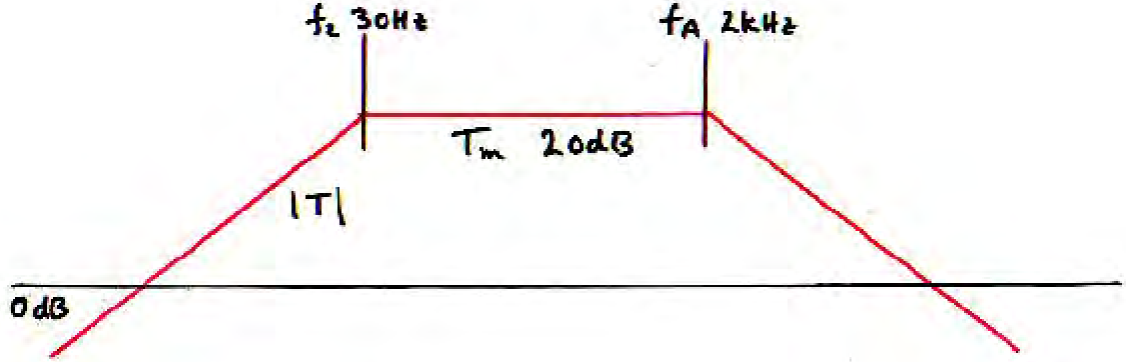


Exercise

By "doing the algebra on the picture," find the analytic pole-zero forms for $F = 1+T$ and $D = T/(1+T)$

Ac model





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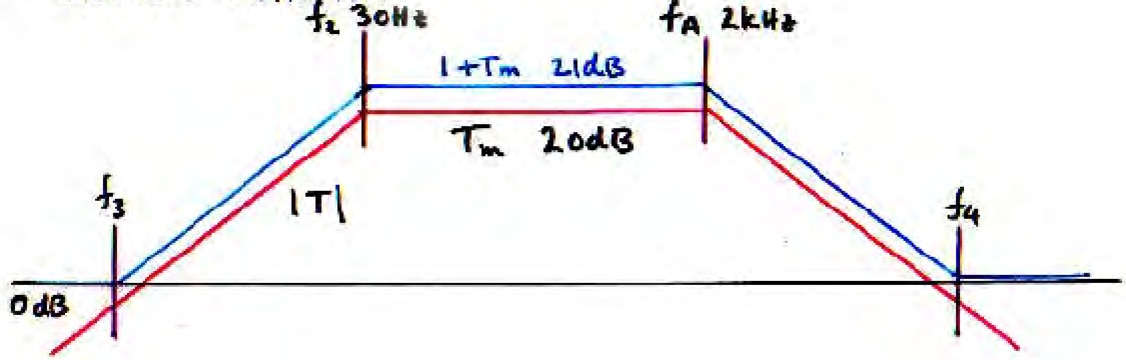
Exercise Solution



Exercise

By "doing the algebra on the picture," find the analytic pole-zero forms for $F = 1+T$ and $D = T/(1+T)$

$$F = (1+T_m)\,\frac{\left(1+\frac{\omega_3}{s}\right)\left(1+\frac{s}{\omega_4}\right)}{\left(1+\frac{\omega_2}{s}\right)\left(1+\frac{s}{\omega_A}\right)}$$

where $\omega_3 \equiv \dfrac{\omega_2}{1+T_m}$ $\qquad f_3 = \dfrac{30}{1+10} = 2.7\text{Hz}$

$\omega_4 \equiv (1+T_m)\omega_A \qquad f_4 = 11 \times 2 = 22\,\text{kHz}$

Calculation of $F = 1+T$ the Hard Way (by algebra):

$$F = 1+T = 1 + \frac{T_m \frac{s}{\omega_2}}{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_A}\right)} = \frac{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_A}\right) + T_m \frac{s}{\omega_2}}{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_A}\right)}$$

$$= \frac{1 + \left(\frac{1+T_m}{\omega_2} + \frac{1}{\omega_A}\right)s + \left(\frac{1}{\omega_2\omega_A}\right)s^2}{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_A}\right)}$$

$$= \frac{\left(1 + \left[\frac{1}{2}\left(\frac{1+T_m}{\omega_2} + \frac{1}{\omega_A}\right) + \frac{1}{2}\sqrt{\left(\frac{1+T_m}{\omega_2} + \frac{1}{\omega_A}\right)^2 - \frac{4}{\omega_2\omega_A}}\right]s\right)\left(1 + \left[\frac{1}{2}\left(\frac{1+T_m}{\omega_2} + \frac{1}{\omega_A}\right) - \frac{1}{2}\sqrt{\left(\frac{1+T_m}{\omega_2} + \frac{1}{\omega_A}\right)^2 - \frac{4}{\omega_2\omega_A}}\right]s\right)}{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_A}\right)}$$

$$= \frac{\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)}{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_A}\right)} = \frac{\omega_2}{\omega_{z1}} \frac{\left(1+\frac{\omega_{z1}}{s}\right)\left(1+\frac{s}{\omega_{z2}}\right)}{\left(1+\frac{\omega_2}{s}\right)\left(1+\frac{s}{\omega_A}\right)}$$

This result gives no insight into the interpretation of the two zeros ω_{z1} and ω_{z2}, or into the midband value $F_m \equiv \frac{\omega_2}{\omega_{z1}}$.

However, a much simpler result is obtaine if the approximate real root form is used.

Check the value of $Q^2 = ac/b^2$ for the numerator quadratic of F:

$$Q^2 = \frac{1}{\omega_2\omega_A\left(\frac{1+T_m}{\omega_2}+\frac{1}{\omega_A}\right)^2} = \frac{\omega_2}{\omega_A(1+T_m)^2\left(1+\cancel{\frac{\omega_2}{\omega_A(1+T_m)}}\right)^2} \ll (0.5)^2$$

Hence, the approximate factorization for well-separated real roots can be adopted:

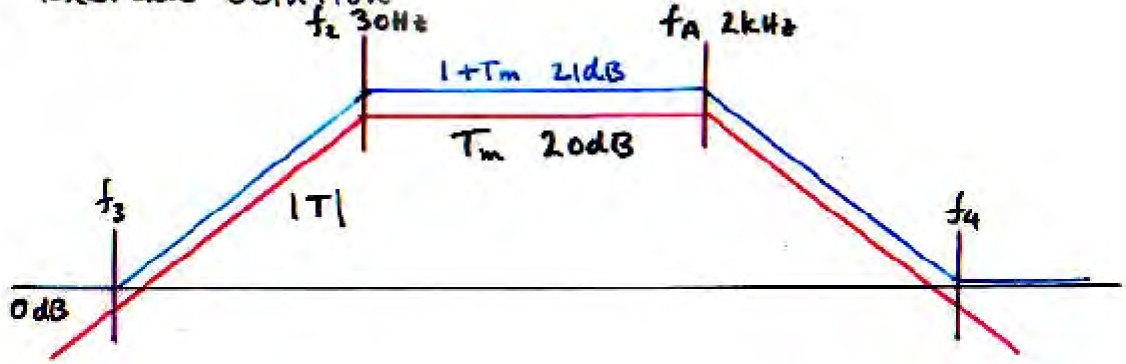
$$F \approx \frac{\left(1+\left[\frac{1+T_m}{\omega_2}+\cancel{\frac{1}{\omega_A}}\right]s\right)\left(1+\frac{s}{\omega_2\omega_A\left[\frac{1+T_m}{\omega_2}+\cancel{\frac{1}{\omega_A}}\right]}\right)}{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_A}\right)}$$

$$= \left(1+T_m+\cancel{\frac{\omega_2}{\omega_A}}\right)\frac{\left(1+\frac{\omega_2/(1+T_m)}{\left[1+\cancel{\frac{\omega_2}{(1+T_m)\omega_A}}\right]s}\right)\left(1+\frac{s}{\left[1+\cancel{\frac{\omega_2}{(1+T_m)\omega_A}}\right](1+T_m)\omega_A}\right)}{\left(1+\frac{\omega_2}{s}\right)\left(1+\frac{s}{\omega_A}\right)}$$

This is the same result obtained by Doing the Algebra on the Graph.

The algebraic factorization could not have been done at all if T had three or more poles.

Exercise Solution



Exercise

By "doing the algebra on the picture," find the analytic pole-zero forms for $F = 1+T$ and $D = T/(1+T)$

$$F = (1+T_m)\frac{\left(1+\frac{\omega_3}{s}\right)\left(1+\frac{s}{\omega_4}\right)}{\left(1+\frac{\omega_2}{s}\right)\left(1+\frac{s}{\omega_A}\right)}$$

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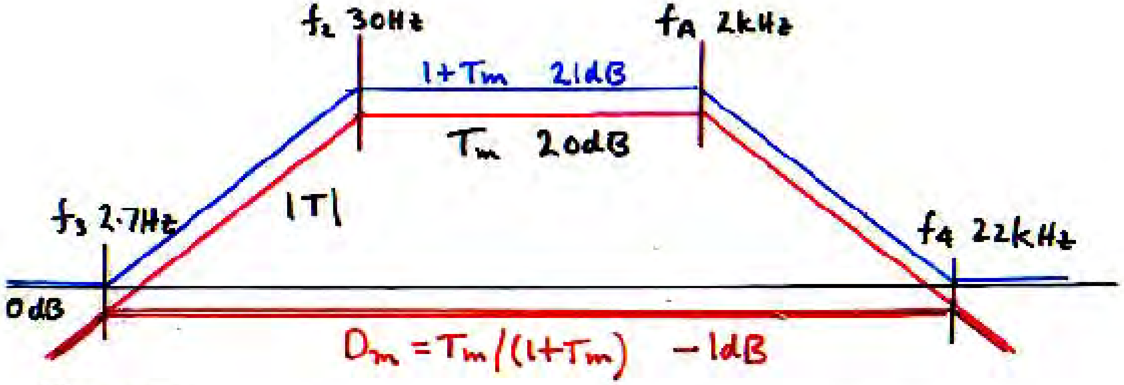
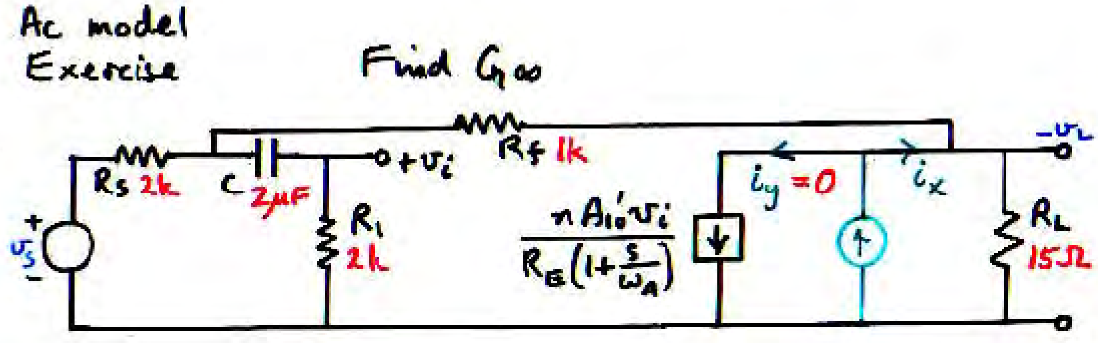


Exercise

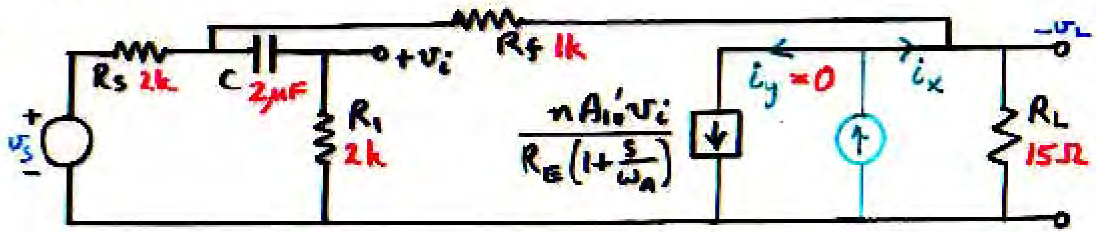
By "doing the algebra on the picture," find the analytic pole-zero forms for $F = 1+T$ and $D = T/(1+T)$

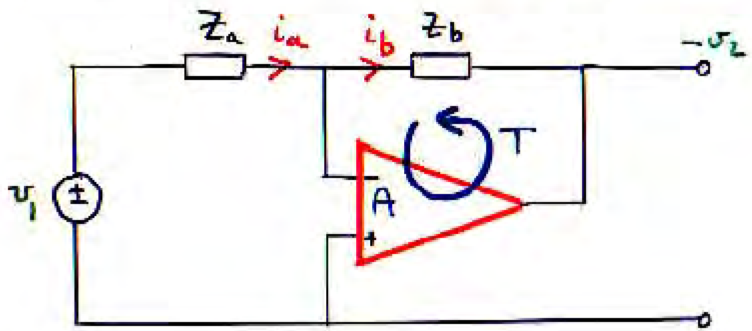
$$D = \frac{T_m}{1+T_m} \frac{1}{\left(1+\frac{\omega_3}{s}\right)\left(1+\frac{s}{\omega_A}\right)}$$


Ac model
Exercise
Find G_{∞}
R_s 2k
C 2μF
$+v_i$
R_f 1k
R_1
2k
$\frac{nA_{10}'v_i}{R_E(1+\frac{s}{\omega_A})}$
$i_y=0$
i_x
$-v_L$
R_L
15Ω
v_s
+
-

Ac model
Exercise

Find G_{∞}

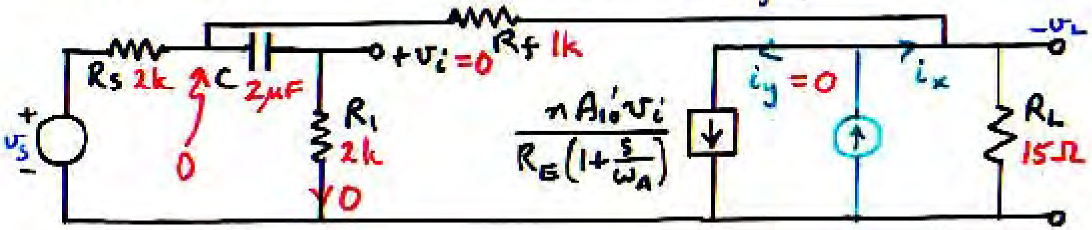




Ac model

Exercise Solution

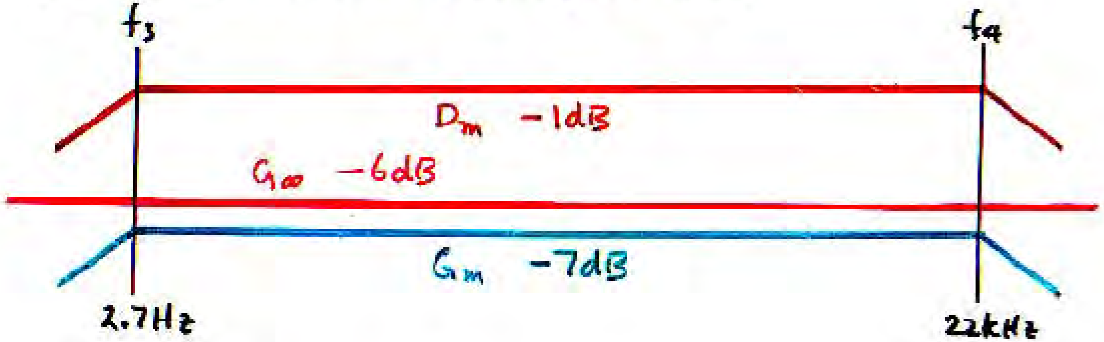
$$G_\infty = \left.\frac{v_L}{v_s}\right|_{i_y=0}$$



$$\frac{v_s}{R_s} = \frac{v_L}{R_f}$$

$$G_\infty = \frac{R_f}{R_s} = \frac{1}{2} \Rightarrow -6\text{dB}$$

Hence the closed-loop gain G is



$$G = G_m \frac{1}{\left(1 + \frac{\omega_3}{s}\right)\left(1 + \frac{s}{\omega_4}\right)}$$

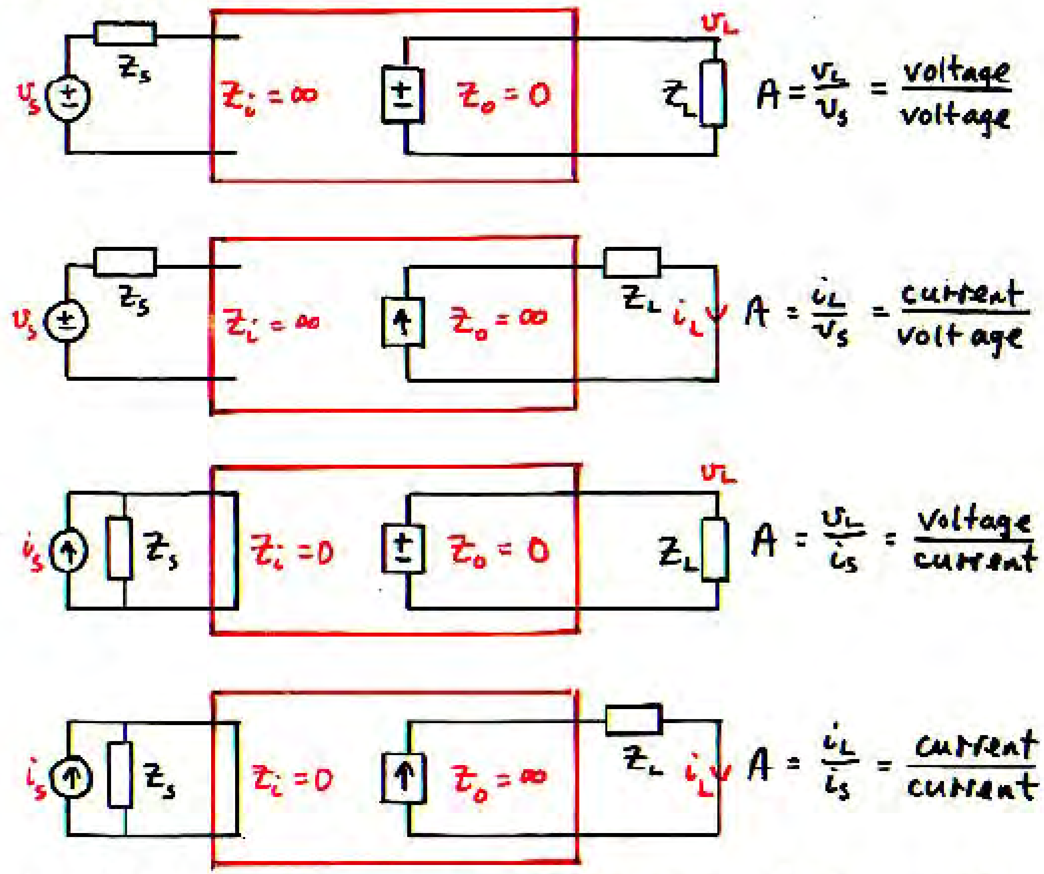
Generalization: Two Conditions for Injection of a Test Signal into a Closed Loop

1. Must be inside the feedback loop

2. Injected signal must add to the forward signal without affecting the impedance loading

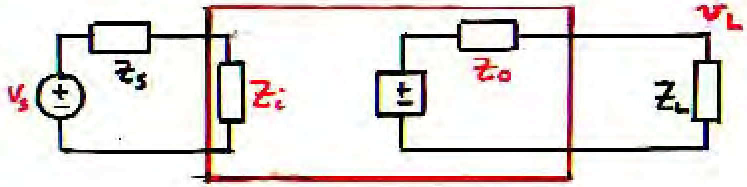
Condition 2 can be met by injection of a voltage in series with a dependent voltage source, or by injection of a current in parallel with a current source.

Ideal amplifiers



In the ideal cases, the gain is <u>independent</u> of Z_s and Z_L

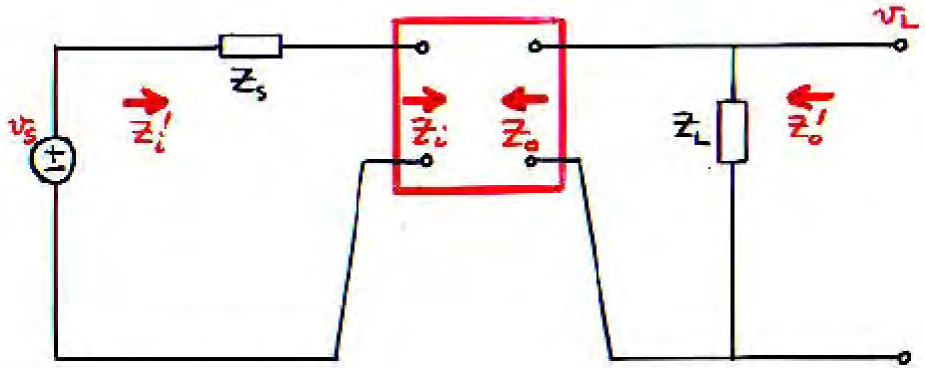
A practical amplifier is not ideal, and the gain does depend on Z_s and Z_L:



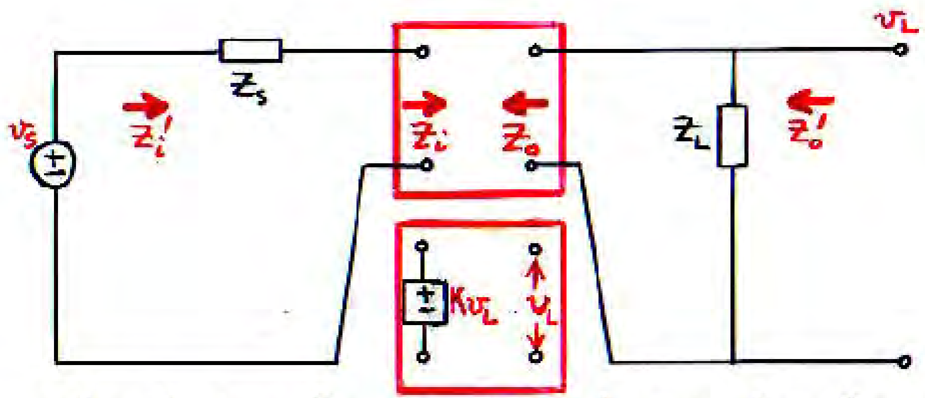
$A = A(Z_s, Z_L)$

However, an appropriate connection of feedback can make a nonideal amplifier approach more closely the properties of any one of the ideal amplifiers.

Feedback causes the closed-loop gain G to approach the reciprocal feedback ratio $1/K = G_\infty$. Thus G_∞ must be designed to have the same current or voltage transfer properties as the desired ideal amplifier.



v_L
Z_s
Z_i'
v_s
Z_i
Z_o
Z_L
Z_o'



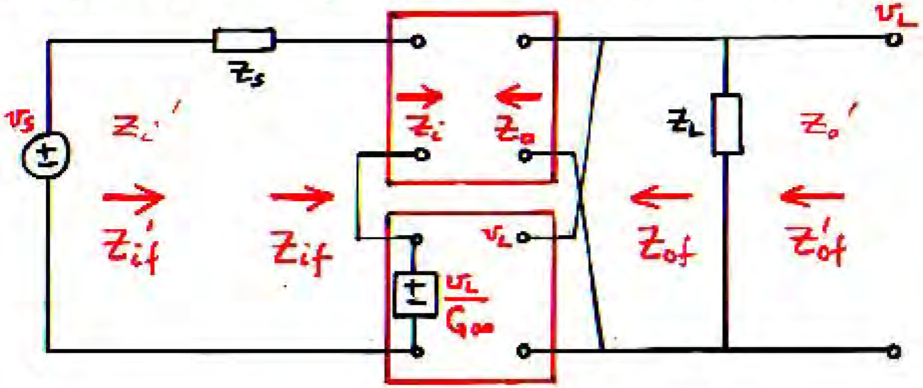
Ideal feedback path must "sense" output voltage and convert it to a fedback voltage. Voltage summing is done in series.

$$G = \frac{v_L}{v_S} = \frac{\text{voltage}}{\text{voltage}} = \frac{A}{1+T} = G_\infty \frac{T}{1+T}$$

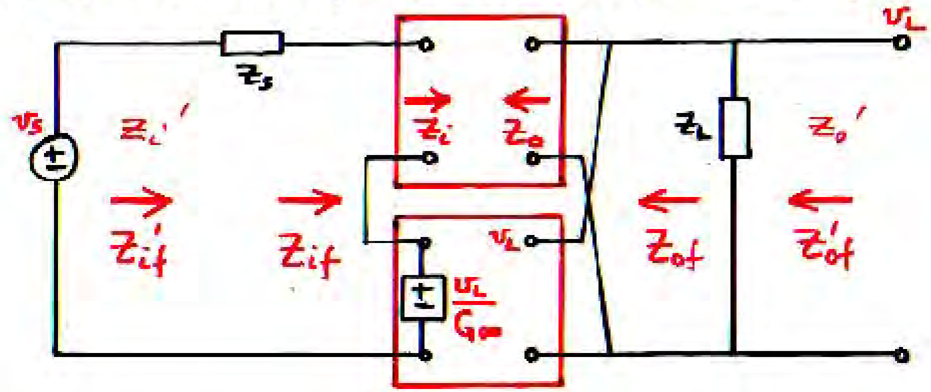
where

$$A = \left.\frac{v_L}{v_S}\right|_{K=1/G_\infty = 0}$$

1. Voltage-to-voltage (series voltage feedback)



1. Voltage-to-voltage (series voltage feedback)



$$Z'_{of} = \frac{G}{\left.\frac{G}{Z_L}\right|_{Z_L\to 0}} = \frac{\frac{A}{1+T}}{\left.\frac{A}{Z_L}\right|_{Z_L\to 0}\frac{1}{1+T|_{Z_L\to 0}}} = \frac{A}{\left.\frac{A}{Z_L}\right|_{Z_L\to 0}}\;\frac{1+T\cancel{|_{Z_L\to 0}}}{1+T} = \frac{Z'_o}{1+T}$$

$$Z_{of} = \frac{Z_o}{1+T|_{Z_L\to\infty}}$$

OR:

$$Z'_{of} = \frac{G_\infty\frac{T}{1+T}}{G_\infty\left.\frac{T}{Z_L}\right|_{Z_L\to 0}\;\frac{1}{1+T\cancel{|_{Z_L\to 0}}}} = \frac{T}{1+T}\left[\left.\frac{Z_L}{T}\right|_{Z_L\to 0}\right]$$

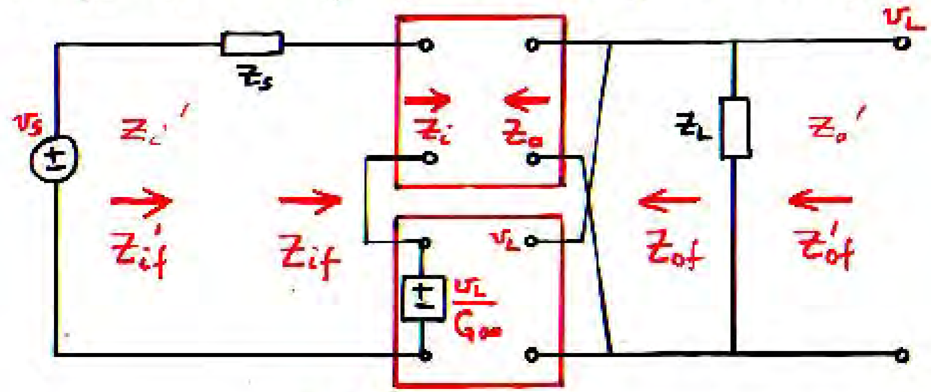
$$Z_{of} = \left.\frac{T}{1+T}\right|_{Z_L\to\infty}\left[\left.\frac{Z_L}{T}\right|_{Z_L\to 0}\right]$$

Outside output impedance is reduced by the factor $1+T$.

Inside output impedance is reduced by the factor $1+T|_{Z_L \to \infty}$, which is larger than $1+T$.

Outside and inside output impedances Z'_{of} and Z_{of} can each be found directly from the loop gain T. They are almost equal (for large T), hence both are much smaller than Z_L.

1. Voltage-to-voltage (series voltage feedback)



$$Z_{if}' = \frac{Z_s G|_{Z_s\to\infty}}{G} = \frac{\frac{Z_s A|_{Z_s\to\infty}}{1+T|_{Z_s\to\infty}}}{\frac{A}{1+T}} = \frac{Z_s A|_{Z_s\to\infty}}{A}\,\frac{1+T}{\cancel{1+T|_{Z_s\to\infty}}} = Z_i'(1+T)$$

$$Z_{if} = Z_i(1+T|_{Z_s\to 0})$$

OR:

$$Z_{if}' = \frac{G_\infty \frac{Z_s T|_{Z_s\to\infty}}{\cancel{1+T|_{Z_s\to\infty}}}}{G_\infty \frac{T}{1+T}} = \frac{1+T}{T}\left[Z_s T|_{Z_s\to\infty}\right]$$

$$Z_{if} = \left.\frac{1+T}{T}\right|_{Z_s\to 0}\left[Z_s T|_{Z_s\to\infty}\right]$$

Outside input impedance is increased by the factor $1+T$.

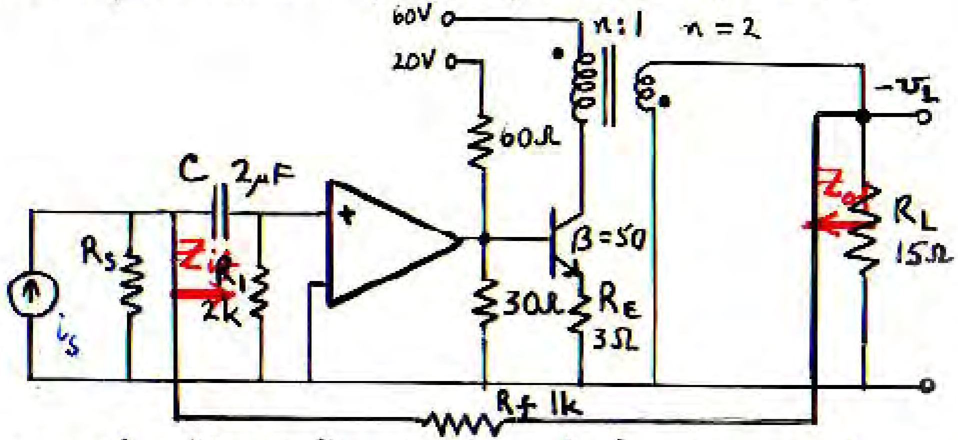
Inside input impedance is increased by the factor $1+T|_{Z_s \Rightarrow 0}$, which is larger than $1+T$.

Outside and inside input impedances Z'_{if} and Z_{if} can each be found directly from the loop gain T. They are almost equal (for large T), hence both are much larger than Z_s.

Bottom Line:

The input and output impedances of a feedback amplifier can be found from a knowledge solely of the loop gain, which further emphasizes the fact that the loop gain T is the single central, important property of a feedback amplifier.

Single-ended Class A audio feedback power amplifier, based on the same power stage previously discussed. The driver opamp has a gain $A_1 = A_{10}/(1+s/\omega_A)$, where $A_{10} = 8\,dB$ and $f_A = 2kHz$, and an output impedance of 4.5Ω.



Hence, the "inside" output and input impedances Z_{of} and Z_{if} are both lowered by the feedback.

Generalization: Effect of Feedback on Impedances

1. Output impedance is $\begin{Bmatrix}\text{decreased}\\\text{increased}\end{Bmatrix}$ by $\begin{Bmatrix}\text{voltage}\\\text{current}\end{Bmatrix}$ feedback from the output.

   Input impedance is $\begin{Bmatrix}\text{increased}\\\text{decreased}\end{Bmatrix}$ by $\begin{Bmatrix}\text{series}\\\text{shunt}\end{Bmatrix}$ feedback to the input.

2. The "outside" impedances are changed by a factor $(1+T)$

   The "inside" impedances are changed by a larger factor.

3. Alternatively, the input and output impedances can be found solely from the loop gain T.

Stability

If the open-loop gain A is stable, the closed-loop gain

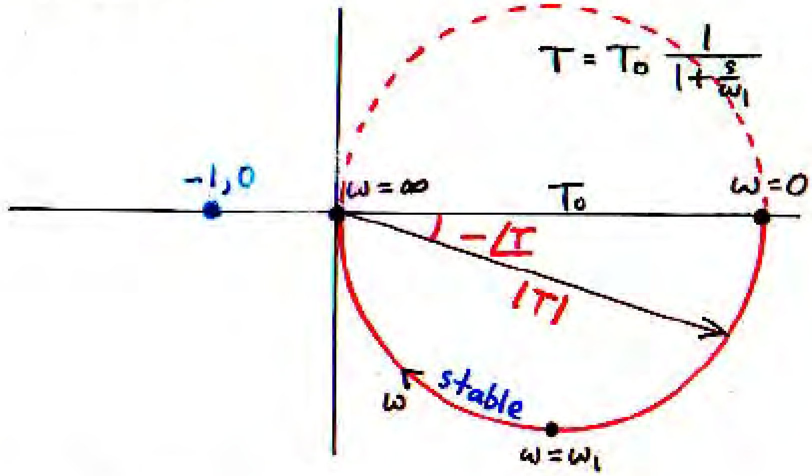
$$G = \frac{A}{1+T}$$

is stable if $1+T$ has no roots in the right half-plane (rhp).

By complex variable theory, this implies that a polar plot of $1+T$ must not encircle the origin; or, equivalently, that a polar plot of T must not encircle the $(-1, 0)$ point. (Nyquist Stability Criterion).

Simple cases of the Nyquist plot of loop gain T:

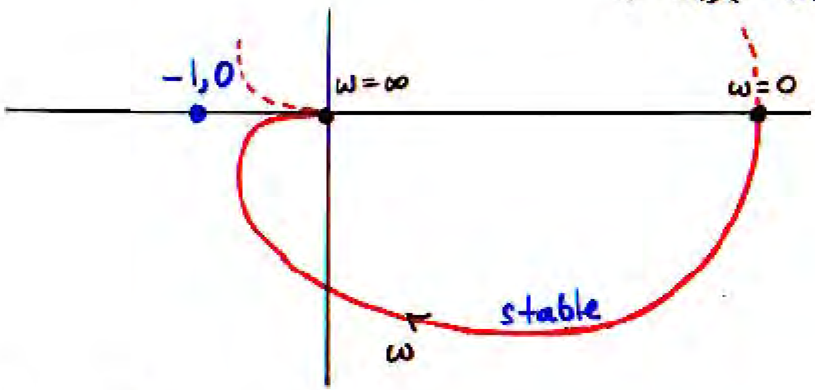
1-pole response:



Always stable

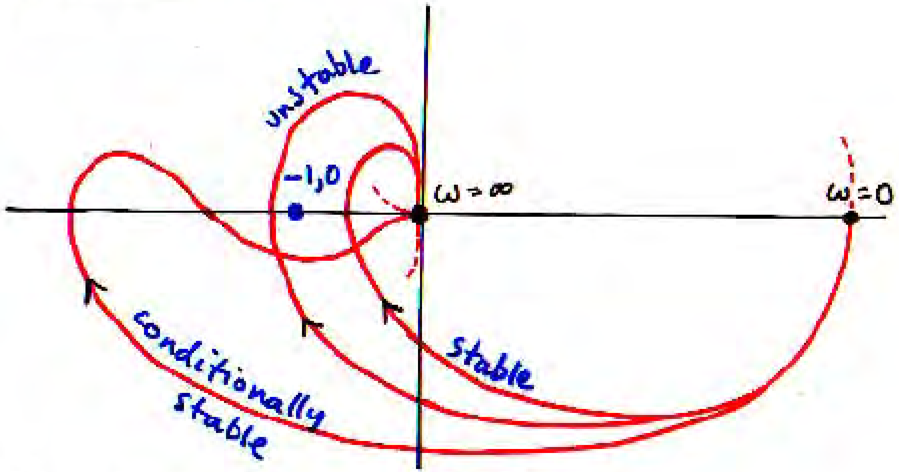
2-pole response

$$T = T_0 \frac{1}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)}$$



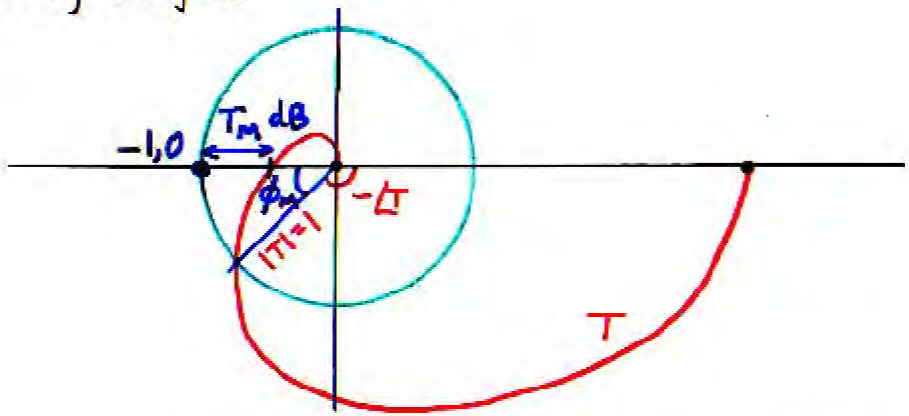
Always stable

3-pole response



Can be stable or unstable

Stability margins

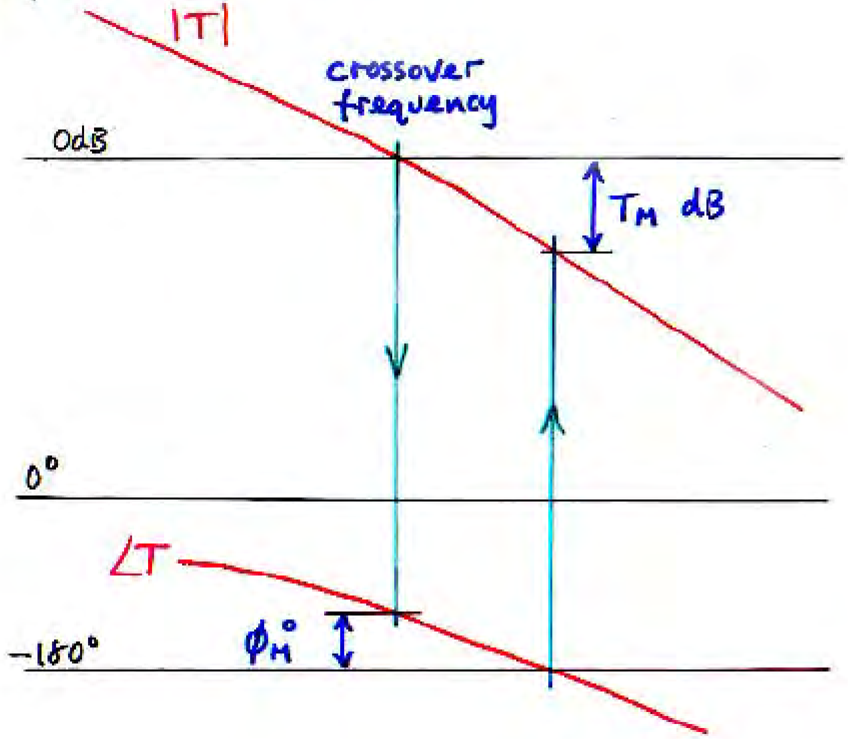


Phase margin $\phi_M \equiv 180^\circ + \angle T$ when $|T| = 1$ (0dB)

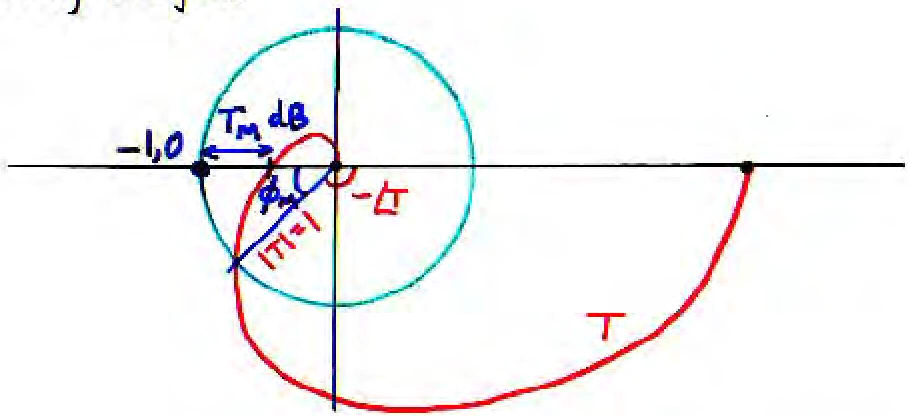
negative (lag)

Gain margin $T_M|_{dB} \equiv -T|_{dB}$ when $\angle T = -180^\circ$

Bode plot:



Stability margins

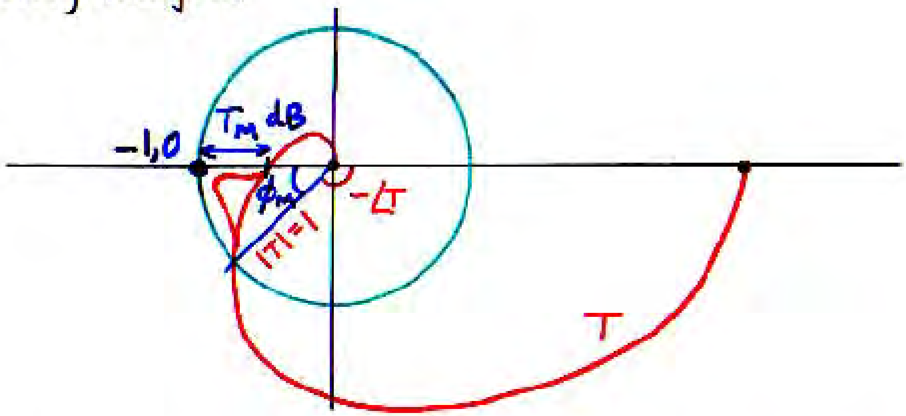


Phase margin $\phi_M \equiv 180° + \angle T$ when $|T| = 1$ (0dB)

negative (lag)

Gain margin $T_M|_{dB} \equiv -T|_{dB}$ when $\angle T = -180°$

Stability margins

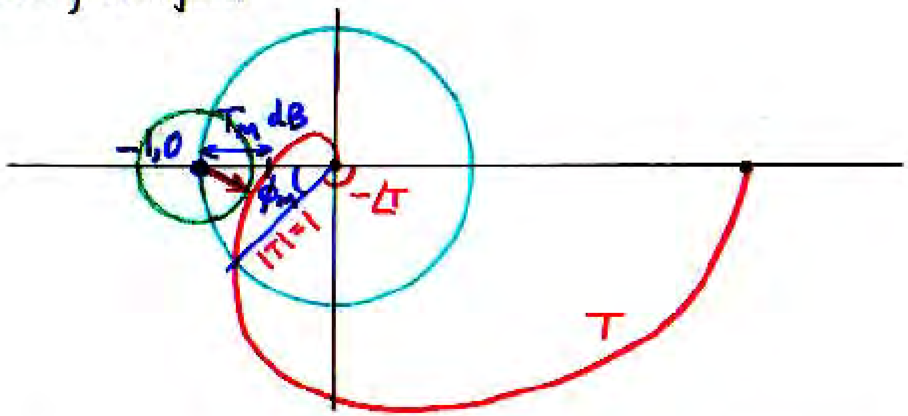


Phase margin $\phi_M \equiv 180° + \angle T$ when $|T| = 1$ (0dB)

negative (lag)

Gain margin $T_M|_{dB} \equiv -T|_{dB}$ when $\angle T = -180°$

Stability margins

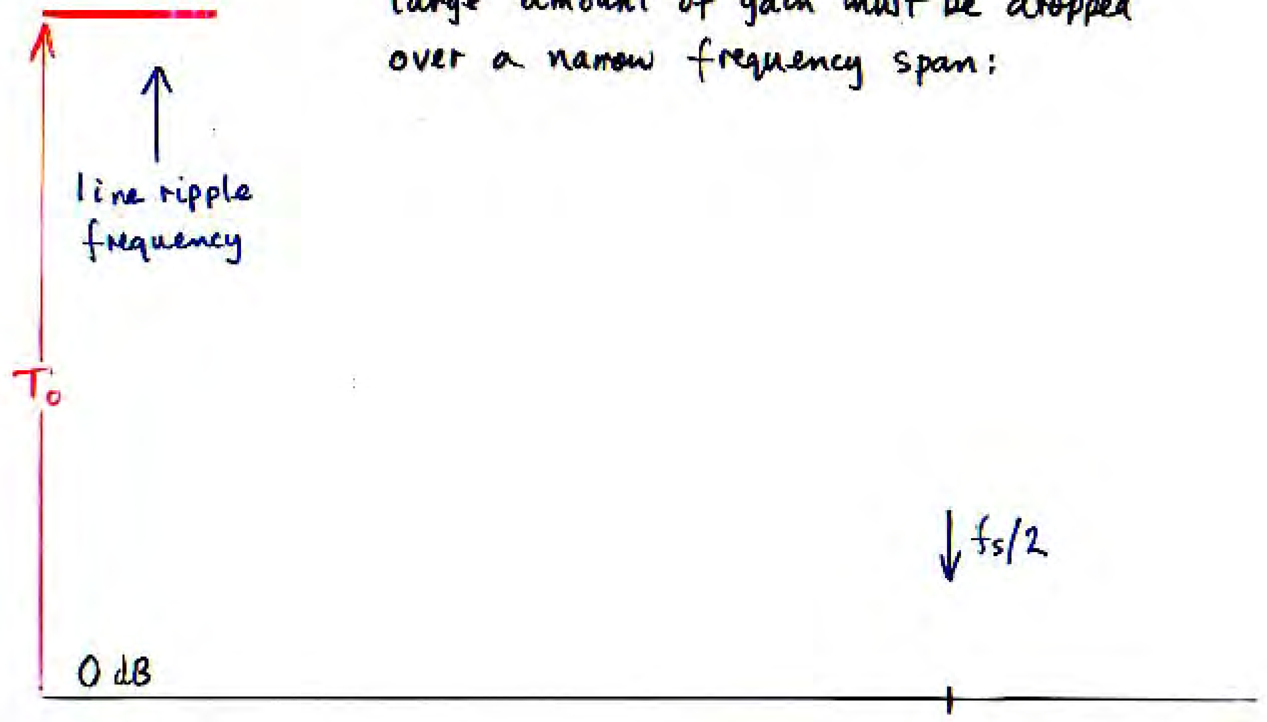


Phase margin $\phi_M \equiv 180° + \angle T$ when $|T| = 1$ (0dB)

negative (lag)

Gain margin $T_M|_{dB} \equiv -T|_{dB}$ when $\angle T = -180°$

A conditionally stable system may be necessary when a large amount of gain must be dropped over a narrow frequency span:



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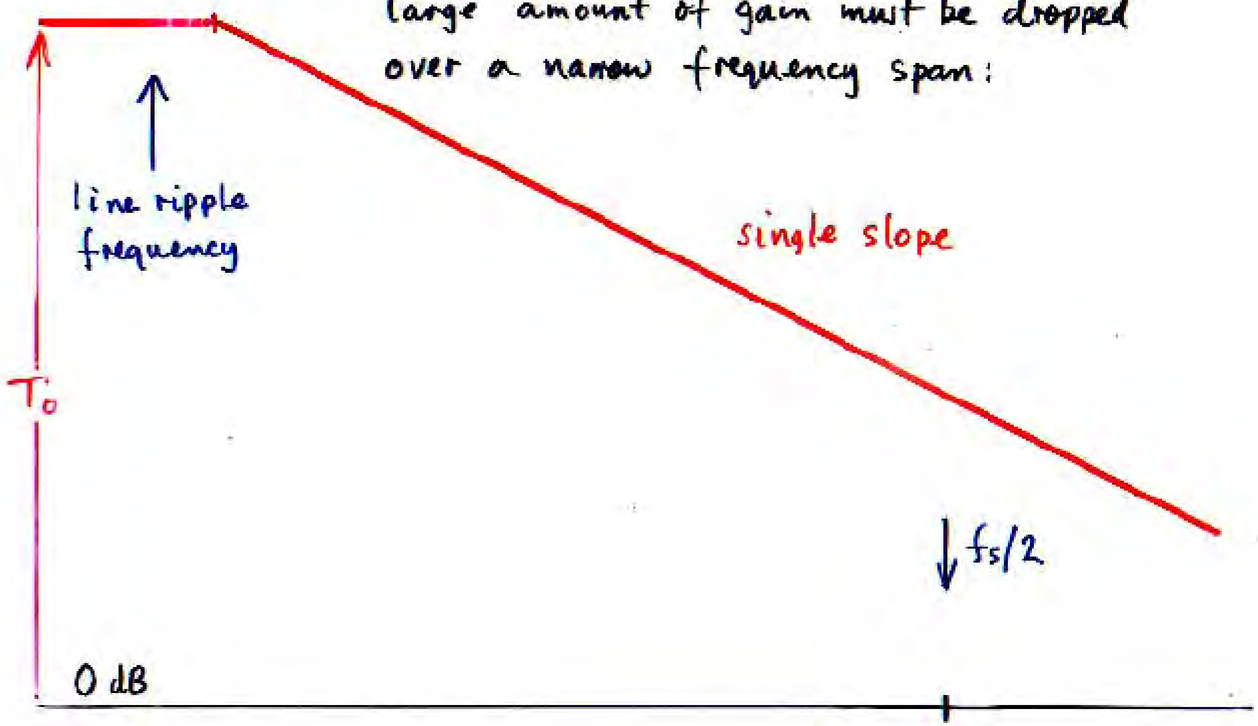

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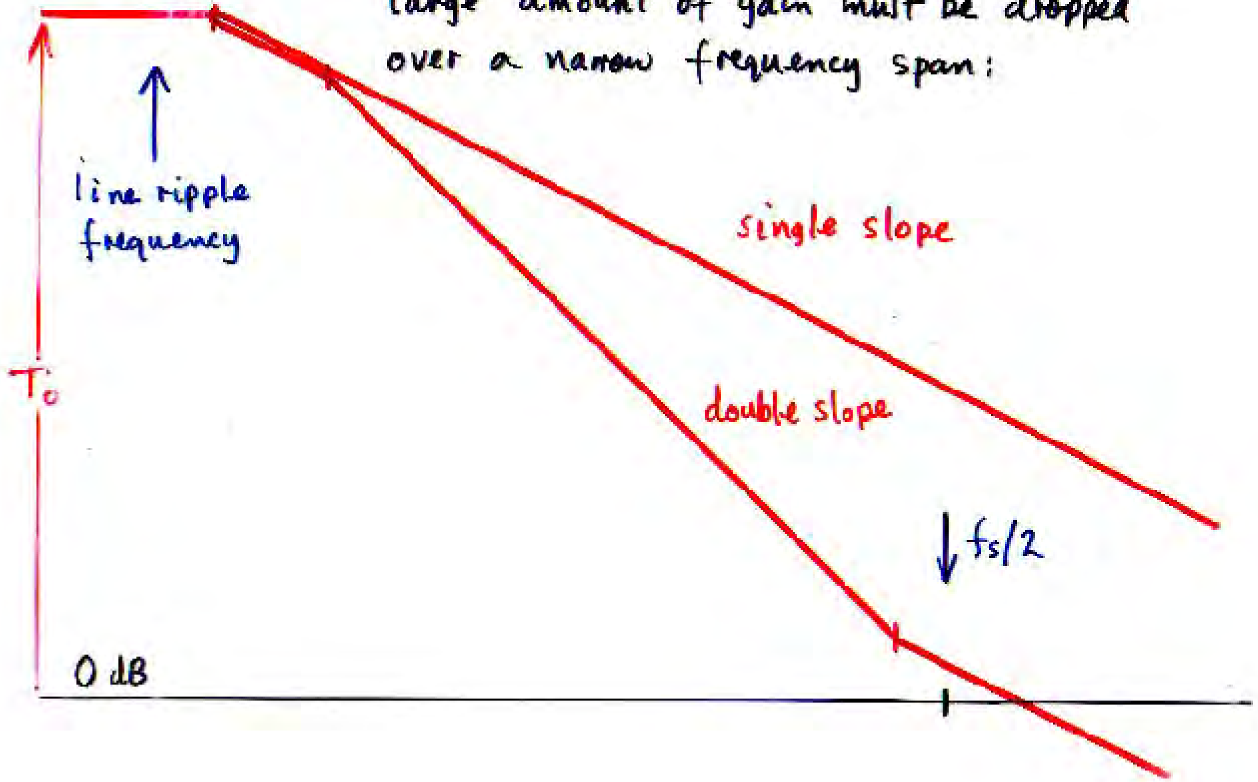

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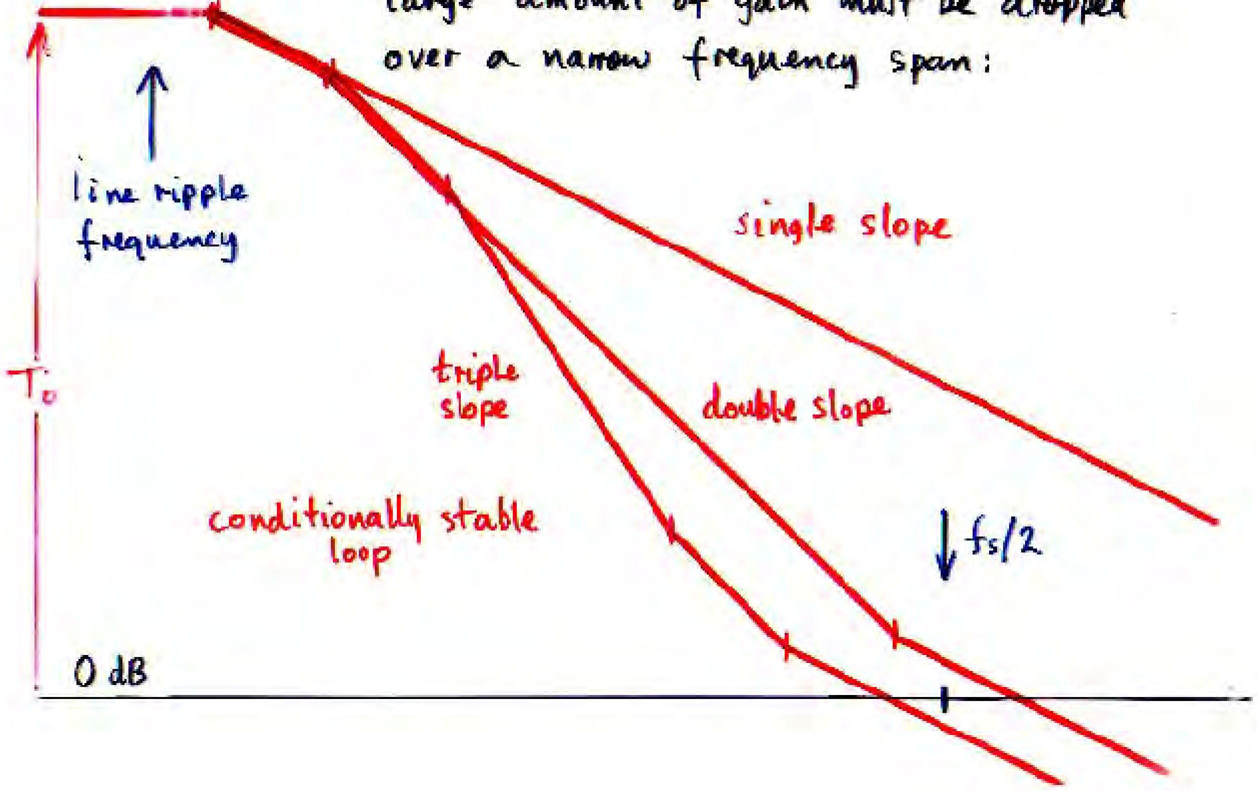
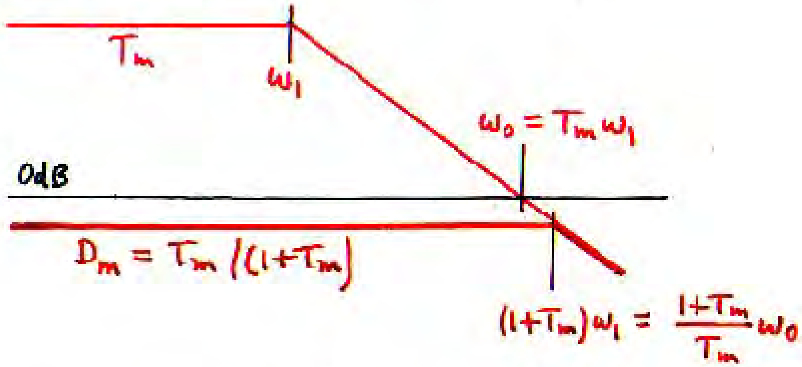


How much phase margin is necessary?

Determine the effect of the phase margin ϕ_M on the closed loop gain G. Since $G = G_\infty D$, the discrepancy factor $D = T/(1+T)$ is the parameter of interest.

Consider the relation between D and T.

1-pole response:

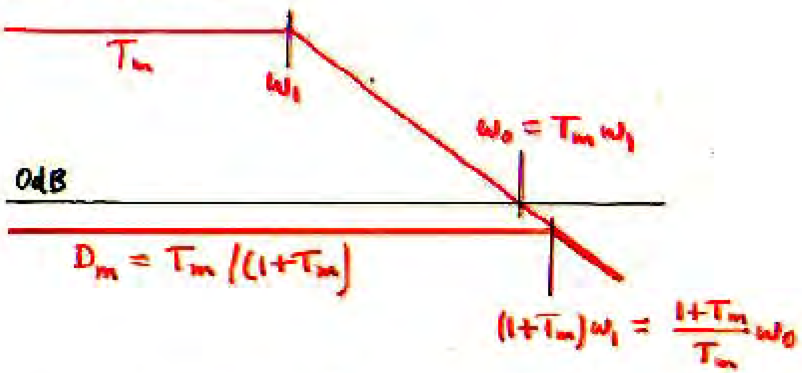


$$T = T_m \frac{1}{1+\frac{s}{\omega_1}} = \frac{1}{\frac{s}{\omega_0}\left(1+\frac{\omega_0}{T_m s}\right)}$$

$$D = \frac{T_m}{1+T_m}\frac{1}{1+\frac{s}{(1+T_m)\omega_1}} = \frac{T_m}{1+T_m}\frac{1}{1+\frac{T_m s}{(1+T_m)\omega_0}}$$

The phase margin is at least 90°, and the discrepancy factor also has one pole.

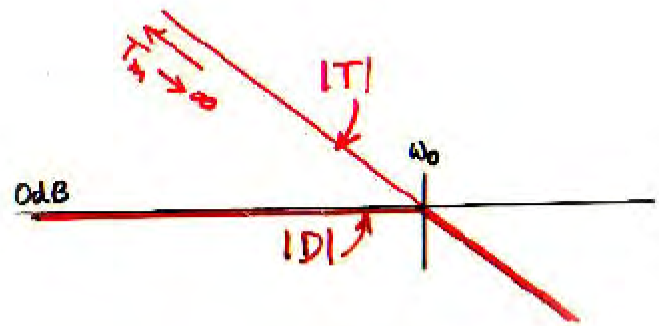
1-pole response:

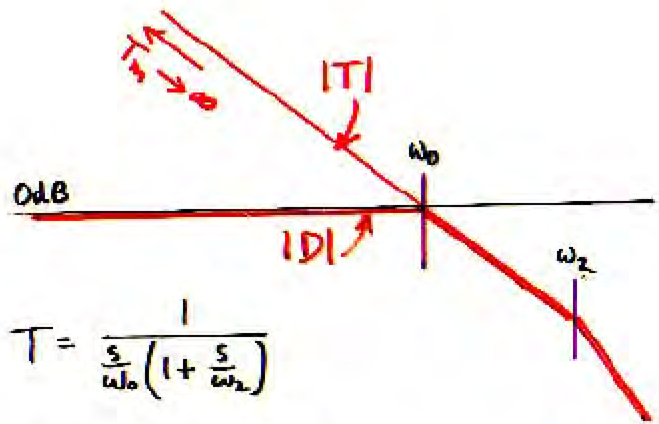


$$T = T_m \frac{1}{1+\frac{s}{\omega_1}} = \frac{1}{\frac{s}{\omega_0}\left(1+\frac{\omega_0}{T_m s}\right)} \xrightarrow{T_m \to \infty} \frac{1}{\frac{s}{\omega_0}}$$

$$D = \frac{T_m}{1+T_m} \frac{1}{1+\frac{s}{(1+T_m)\omega_1}} = \frac{T_m}{1+T_m} \frac{1}{1+\frac{T_m s}{(1+T_m)\omega_0}} \xrightarrow{T_m \to \infty} \frac{1}{1+\frac{s}{\omega_0}}$$

The phase margin is at least 90°, and the discrepancy factor also has one pole.
In the limiting case $T_m \to \infty$, the phase margin is 90°; and D still has one pole.


|T|
$T \to \infty$
ω_0
0dB
|D|



$$T = \frac{1}{\frac{s}{\omega_0}\left(1+\frac{s}{\omega_2}\right)}$$

If ω_2 is much higher than ω_0, obviously D has the same second pole.

Investigate what happens to D if ω_2 is close to, or even below, ω_0:

$$D = \frac{T}{1+T} = \frac{1}{1+\frac{s}{\omega_0}\left(1+\frac{s}{\omega_2}\right)} = \frac{1}{1+\frac{1}{Q}\left(\frac{s}{\omega_c}\right)+\left(\frac{s}{\omega_c}\right)^2}$$

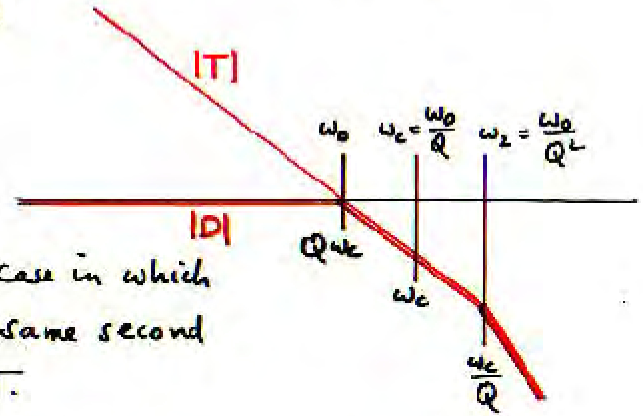
where

$$\omega_c \equiv \sqrt{\omega_0 \omega_2} \qquad Q \equiv \sqrt{\frac{\omega_0}{\omega_2}}$$

Consider ω_0 fixed, and ω_2 variable (Q variable)

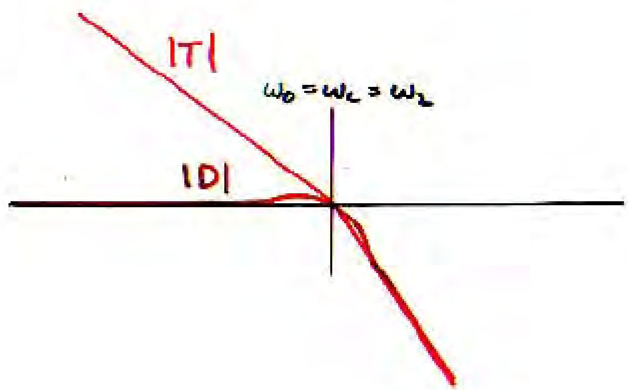
$$\omega_2 = \frac{\omega_0}{Q^2} \qquad \omega_c = \frac{\omega_0}{Q}$$

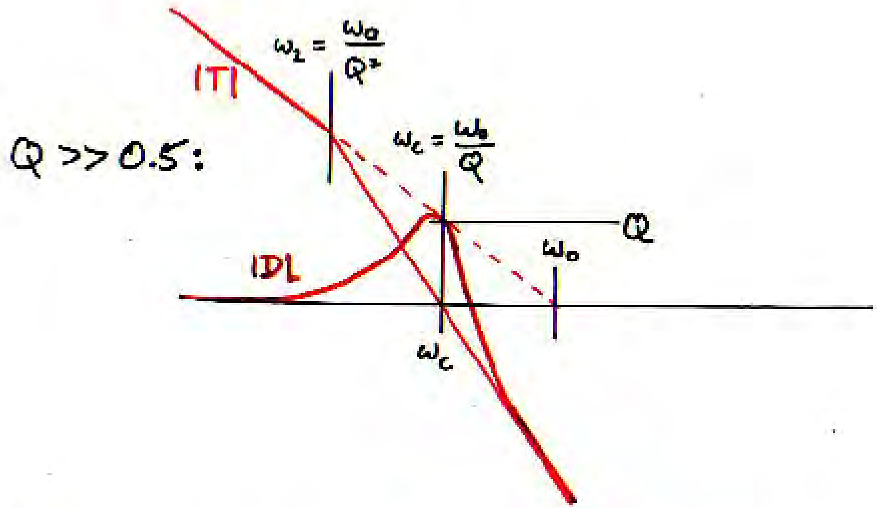
$Q \ll 0.5$:



This is the case in which D has the same second pole as T.

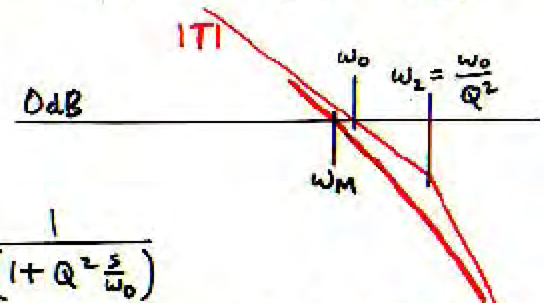
$Q = 1$:





Hence: discrepancy factor D peaks if second pole is not sufficiently far above ω_0, and D is characterized by the Q-factor of its quadratic, which is related to ω_2 (and ω_0) by $Q = \sqrt{\frac{\omega_0}{\omega_2}}$.

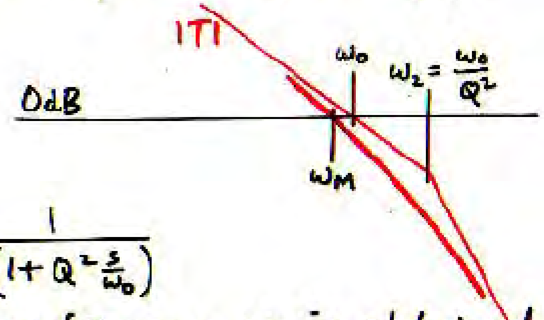
Now, investigate what happens to the phase margin ϕ_M if ω_2 is close to, or even below, ω_0: that is, investigate the relation between Q and ϕ_M.



$$T = \frac{1}{\frac{s}{\omega_0}\left(1 + Q^2 \frac{s}{\omega_0}\right)}$$

The crossover frequency ω_M is obtained by setting $|T| = 1$:

Now, investigate what happens to the phase margin ϕ_M if ω_2 is close to, or even below, ω_0: that is, investigate the relation between Q and ϕ_M.



$$T = \frac{1}{\frac{s}{\omega_0}\left(1 + Q^2 \frac{s}{\omega_0}\right)}$$

The crossover frequency ω_M is obtained by setting $|T| = 1$:

$$1 = \frac{1}{\left(\frac{\omega_M}{\omega_0}\right)^2 \left[1 + Q^4 \left(\frac{\omega_M}{\omega_0}\right)^2\right]}$$

$$Q^4 \left(\frac{\omega_M}{\omega_0}\right)^4 + \left(\frac{\omega_M}{\omega_0}\right)^2 - 1 = 0$$

$$\left(\frac{\omega_M}{\omega_0}\right)^2 = -\frac{c}{b}\frac{1}{F} = \frac{1}{\frac{1}{2} + \frac{1}{2}\sqrt{1 + 4Q^4}}$$

$$\frac{\omega_M}{\omega_0} = \sqrt{\frac{2}{1 + \sqrt{1 + 4Q^4}}}$$

The phase margin ϕ_M is found from $\angle T$ evaluated at the crossover frequency ω_M:

$$\phi_M \equiv 180° + \angle T\Big|_{\omega_M}$$

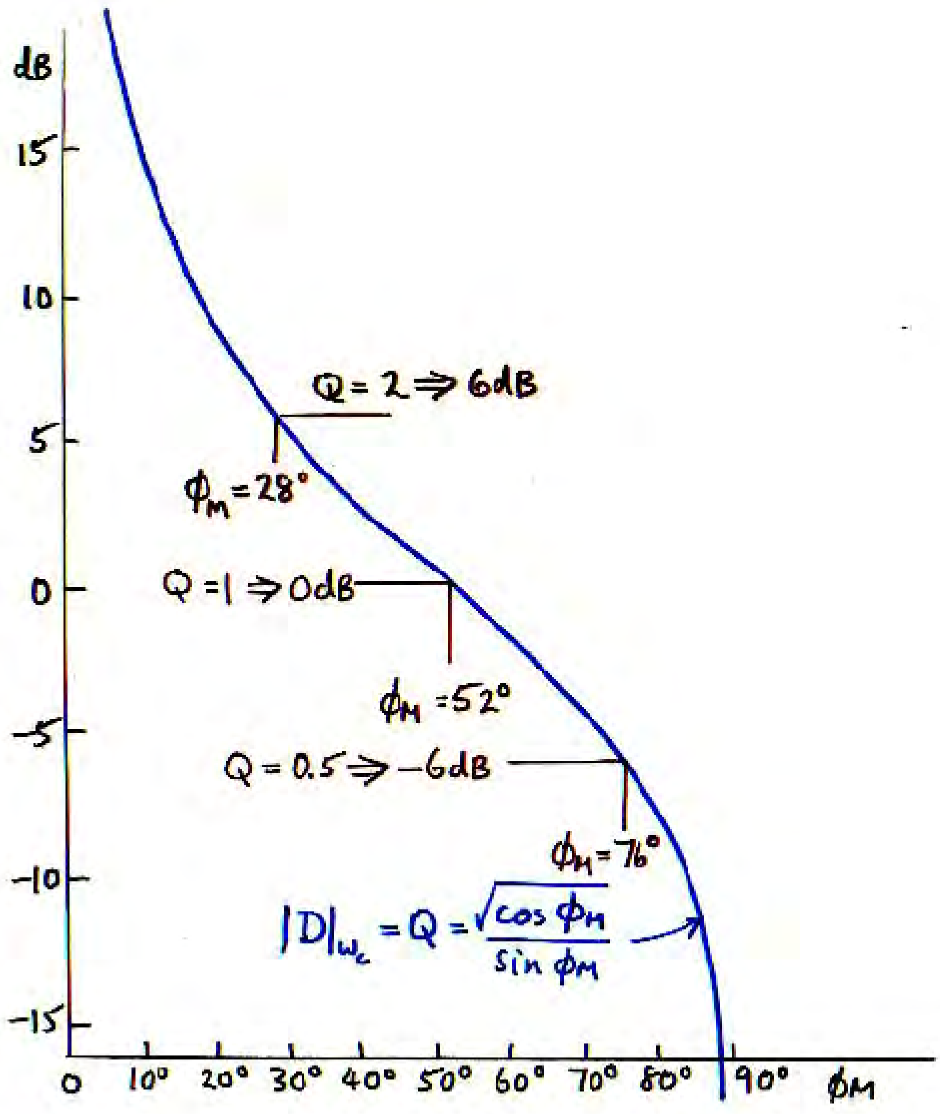
$$= 180 + \left(-90° - \tan^{-1}\frac{\omega_M}{\omega_0/Q^2}\right)$$

which leads to

$$\phi_M = \tan^{-1}\sqrt{\frac{1+\sqrt{1+4Q^4}}{2Q^4}}$$

By inversion:

$$Q = \frac{\sqrt{\cos\phi_M}}{\sin\phi_M}$$

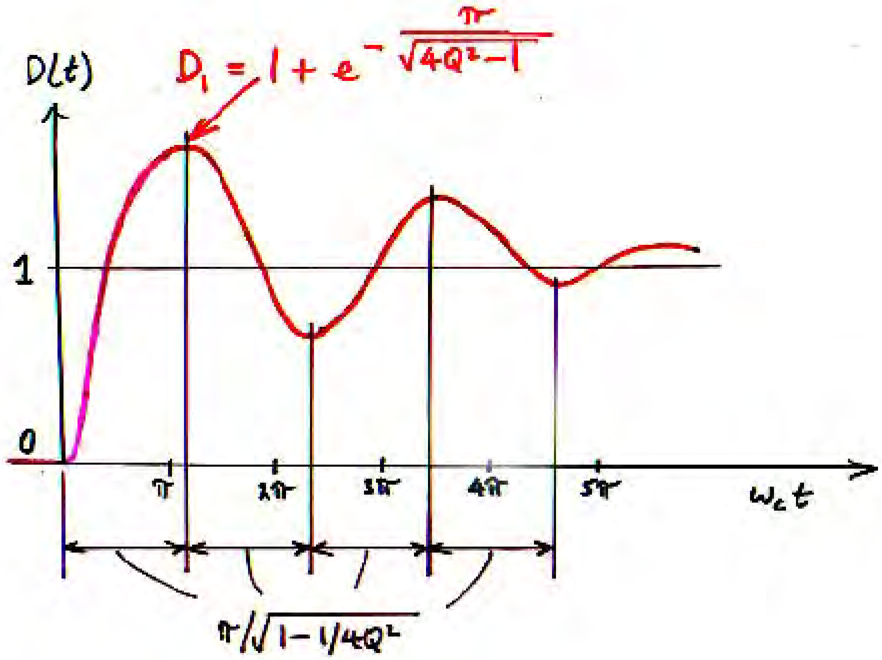


dB
15
10
5
0
-5
-10
-15
Q = 2 ⇒ 6dB
$\phi_M = 28°$
Q = 1 ⇒ 0dB
$\phi_M = 52°$
Q = 0.5 ⇒ -6dB
$\phi_M = 76°$
$|D|_{\omega_c} = Q = \frac{\sqrt{\cos\phi_M}}{\sin\phi_M}$
0 10° 20° 30° 40° 50° 60° 70° 80° 90° ϕ_M

A phase margin ϕ_M less than 76° causes complex roots in D, and therefore in the closed-loop gain G. In the frequency domain, this results in peaked high-frequency response; in the time domain, it results in transient overshoot. For constant G_∞, $G \propto D$ and the response to a step input is

$$D(t) = \mathcal{L}^{-1} \frac{1}{s} \frac{1}{1 + \frac{1}{Q}\left(\frac{s}{\omega_c}\right) + \left(\frac{s}{\omega_c}\right)^2}$$

$$\overset{Q>0.5}{=} 1 - \frac{1}{\sqrt{1-1/4Q^2}} e^{-\frac{\omega_c t}{2Q}} \sin\left[\sqrt{1-1/4Q^2}\,\omega_c t + \sin^{-1}\sqrt{1-1/4Q^2}\right]$$



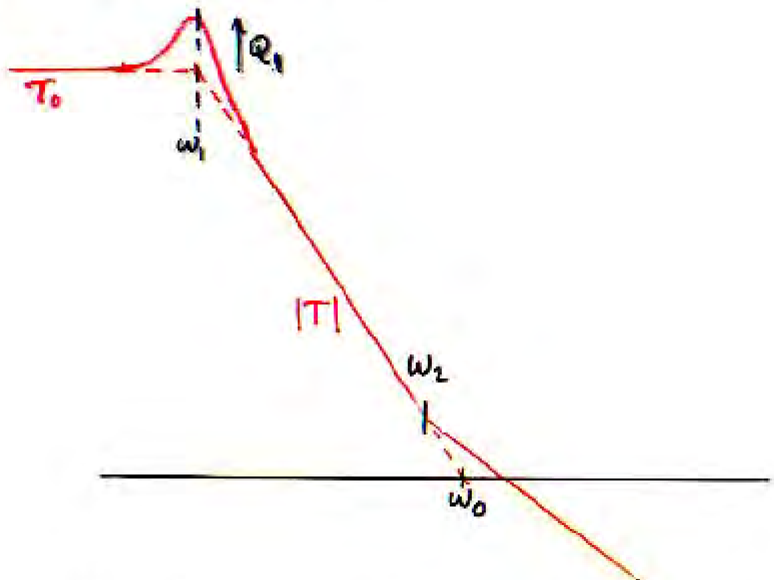
Q	ϕ_M	D_1
$1 \Rightarrow 0$dB	52°	1.16 : 16% overshoot
$2 \Rightarrow 6$dB	28°	1.44 : 44% overshoot

Exercise 10.1

Find the discrepancy factor D for a switching regulator loop gain T.

Exercise

Consider a loop gain that approaches extrapolated crossover frequency ω_0 at a double slope, -40dB/dec, and has a zero at ω_2:

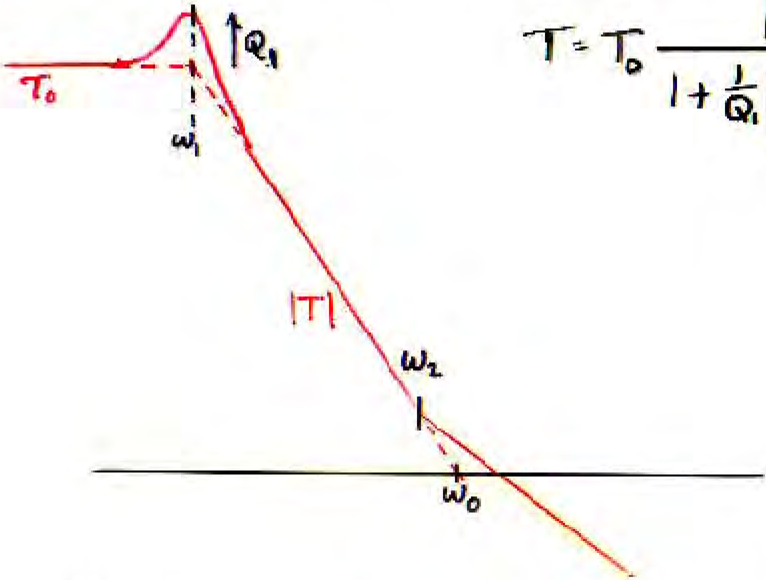


Express the discrepancy factor $D = T/(1+T)$ in normalized form, and identify a Q.

Exercise 10.1 - Solution

Exercise

Consider a loop gain that approaches extrapolated crossover frequency ω_0 at a double slope, -40dB/dec, and has a zero at ω_2:



$$T = T_0 \frac{1 + \frac{s}{\omega_2}}{1 + \frac{1}{Q_1}\left(\frac{s}{\omega_1}\right) + \left(\frac{s}{\omega_1}\right)^2}$$

Express the discrepancy factor $D = T/(1+T)$ in normalized form, and identify a Q.

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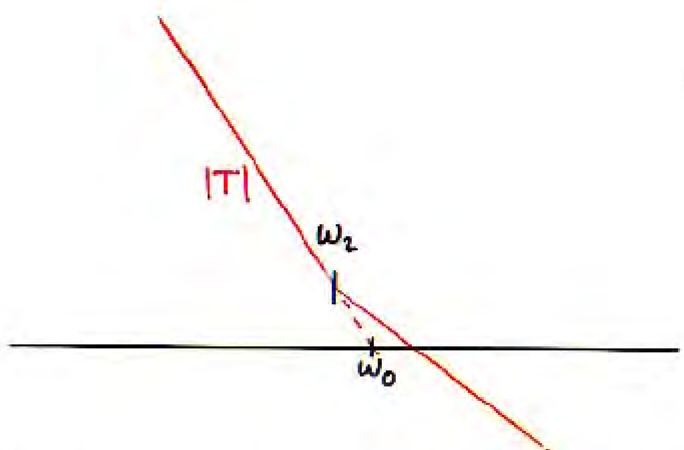

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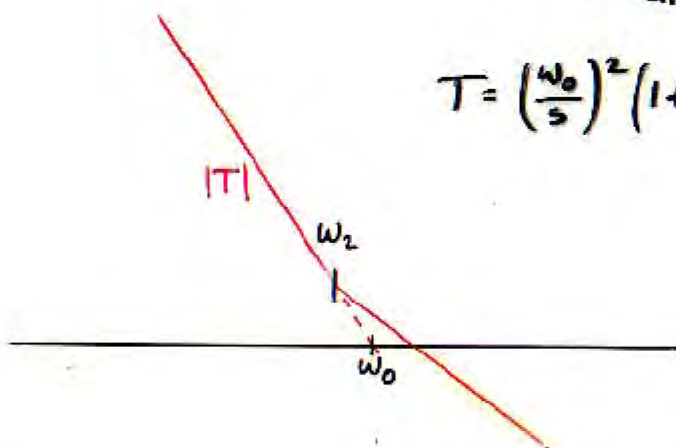
Consider a loop gain that approaches extrapolated crossover frequency ω_0 at a double slope, -40dB/dec, and has a zero at ω_2:

$$T = T_0 \frac{1 + \frac{s}{\omega_2}}{1 + \frac{1}{Q_1}\left(\frac{s}{\omega_1}\right) + \left(\frac{s}{\omega_1}\right)^2}$$

$$T = \left(\frac{\omega_0}{s}\right)^2 \left(1 + \frac{s}{\omega_2}\right)$$



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Exercise 10.1 - Solution

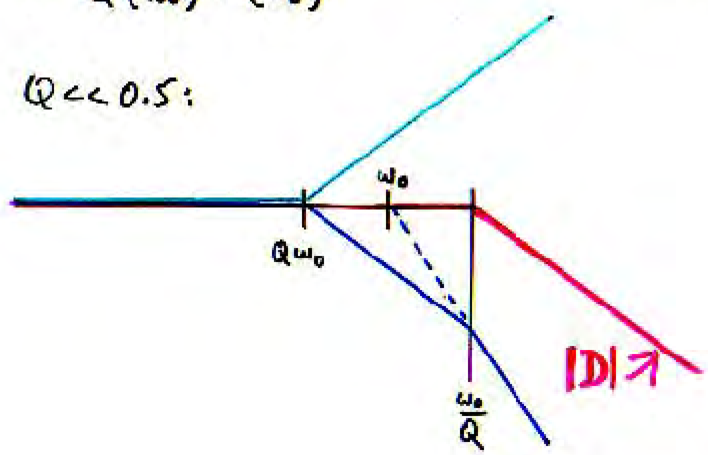
Exercise Solution

$$T = \left(\frac{\omega_0}{s}\right)^2\left(1+\frac{s}{\omega_2}\right)$$

$$D = \frac{T}{1+T} = \frac{1}{1+\frac{1}{T}} = \frac{1}{1+\left(\frac{s}{\omega_0}\right)^2\frac{1}{1+s/\omega_2}} = \frac{1+\frac{s}{\omega_2}}{1+\frac{s}{\omega_2}+\left(\frac{s}{\omega_0}\right)^2}$$

$$= \frac{1+\frac{1}{Q}\frac{s}{\omega_0}}{1+\frac{1}{Q}\left(\frac{s}{\omega_0}\right)+\left(\frac{s}{\omega_0}\right)^2} \quad \text{where } Q \equiv \frac{\omega_2}{\omega_0}$$

$Q << 0.5$:



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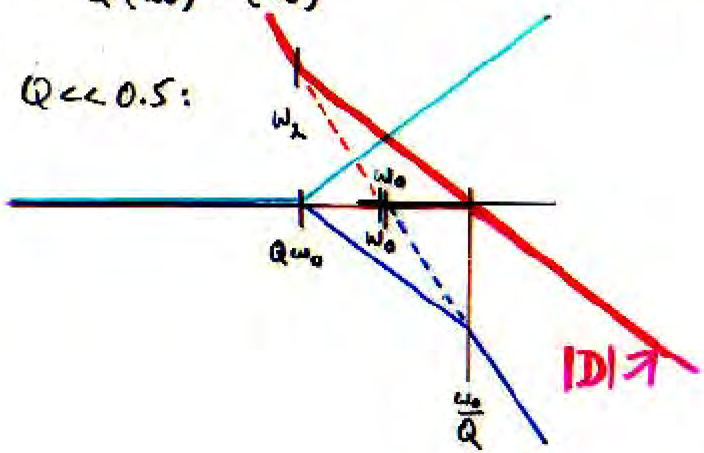
$Q \ll 0.5$:



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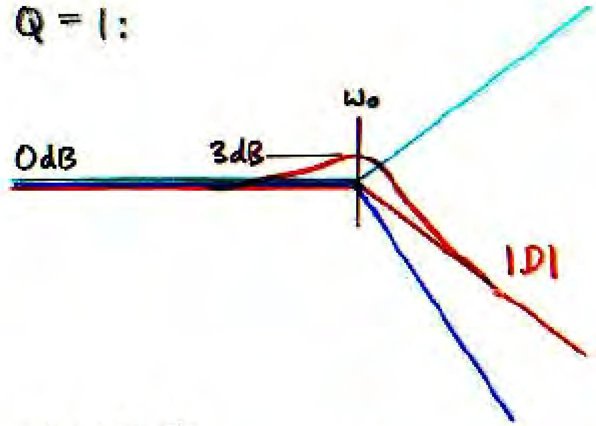
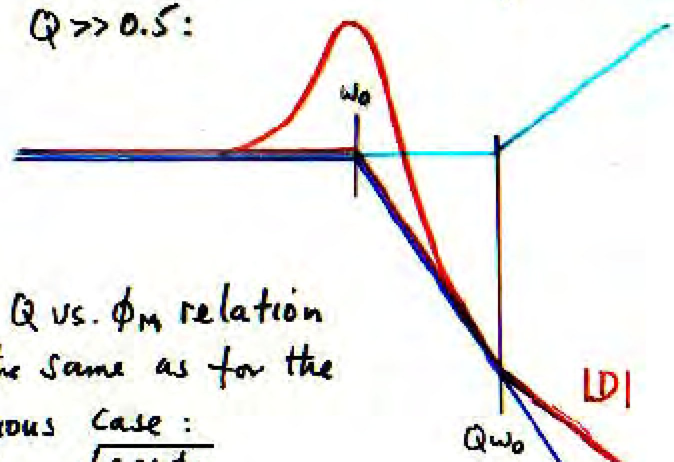


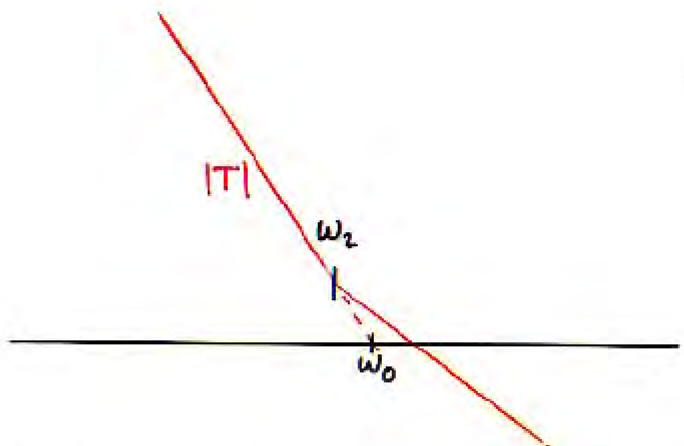


The Q vs. ϕ_M relation is the same as for the previous case:

$$Q = \frac{\sqrt{\cos\phi_M}}{\sin\phi_M}$$

Exercise

Consider a loop gain that approaches extrapolated crossover frequency ω_0 at a double slope, -40dB/dec, and has a zero at ω_2:



Express the discrepancy factor $D = T/(1+T)$ in normalized form, and identify a Q.

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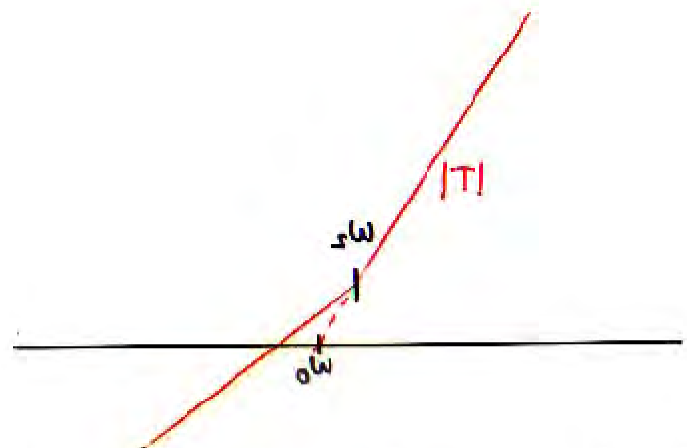

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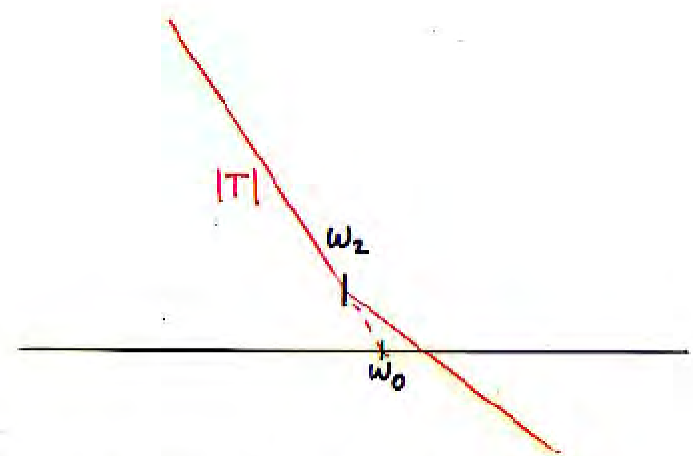


Express the discrepancy factor $D = T/(1+T)$ in normalized form, and identify a Q.

Generalization: How Much Phase Margin is Needed?

Depends on two considerations:

1. Effect of phase margin ϕ_M on closed-loop response via the Discrepancy Factor D. Too small a ϕ_M causes peaking in D.

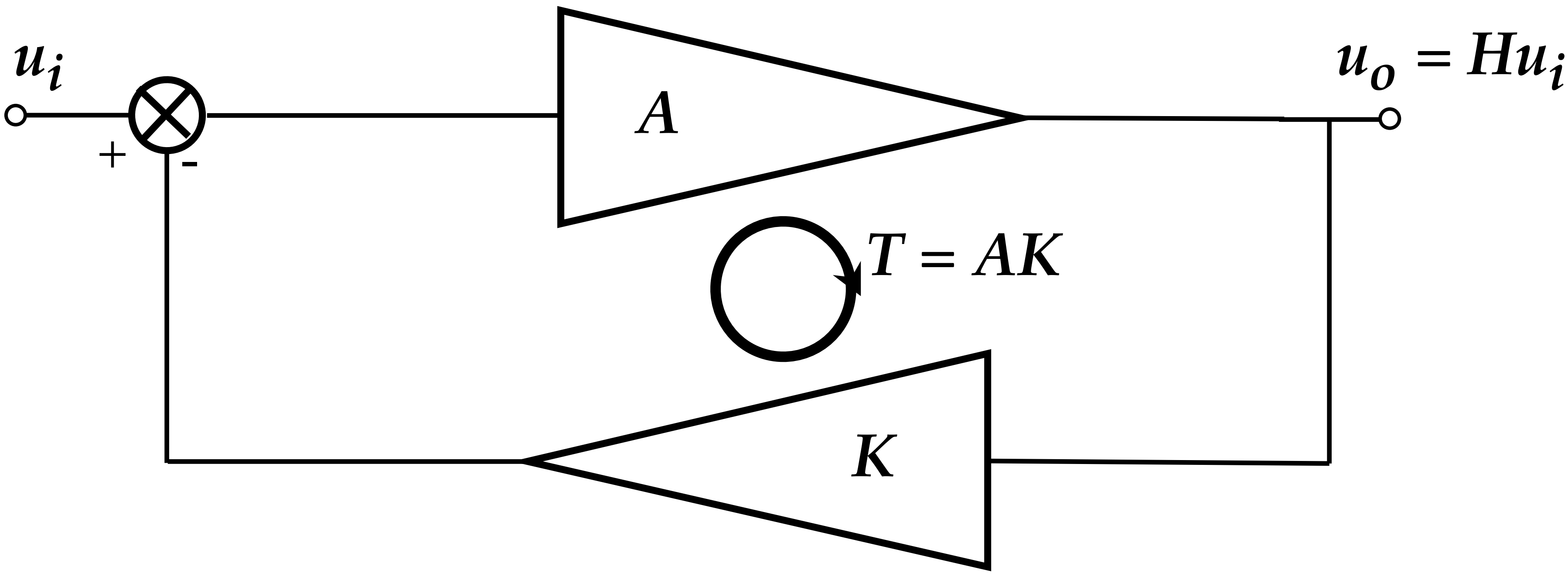
2. The sensitivity of ϕ_M to variations (worst-case). Avoid making ϕ_M strongly dependent on highly variable parameters.

11. NDI AND THE GFT:

Null Double Injection and the General Feedback Theorem

How to identify and include nonidealities in a third quantity, the Null Loop Gain T_n

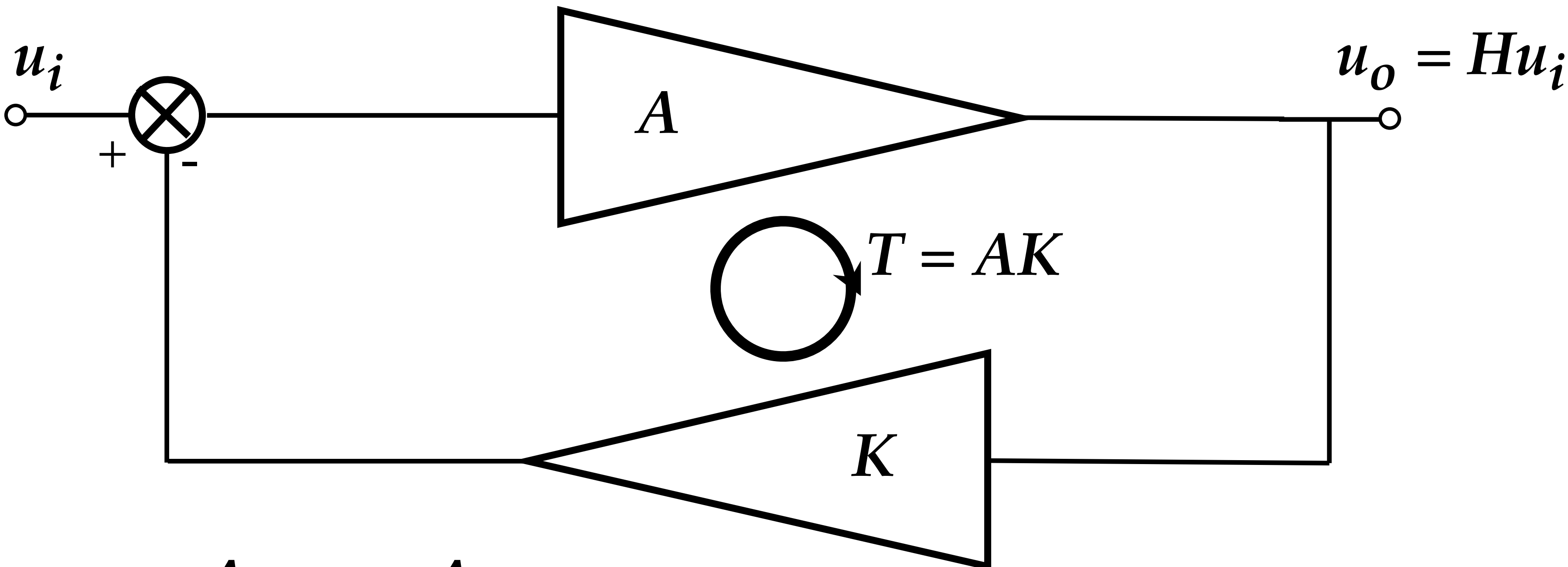
Conventional block diagram:



$$H = \frac{A}{1 + AK} = \frac{A}{1 + T}$$

where $T \equiv AK$ is the loop gain

Block diagram approach for closed-loop gain H:

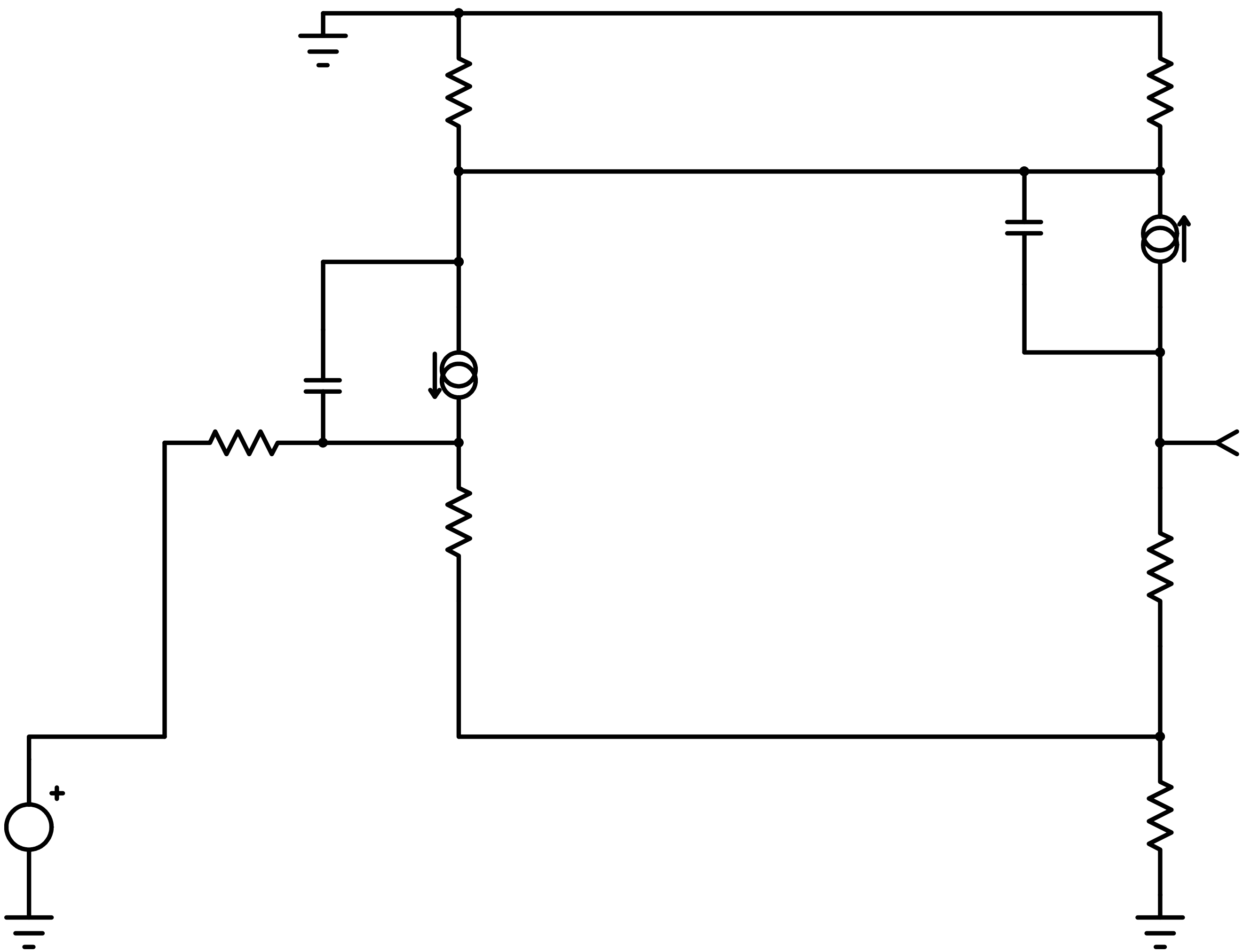


$$H = \frac{A}{1 + AK} = \frac{A}{1 + T}$$

where $T \equiv AK$ is the loop gain

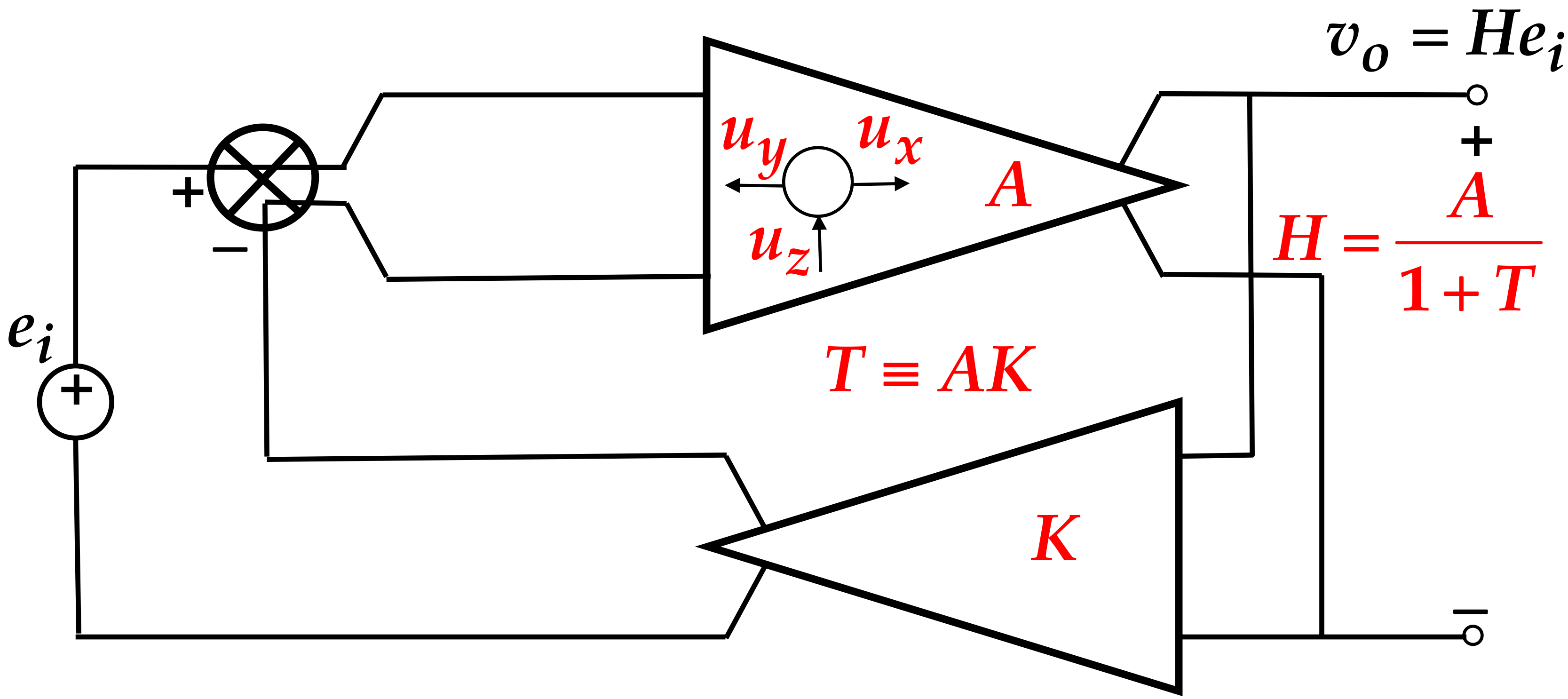
The block diagram says: if $A = 0$, then $H = 0$

But, this isn't true in an actual circuit model:



If the second device fails open, the input signal can still reach the output by going the "wrong way" through the feedback path.

The formula is wrong because it is based on an incomplete model:



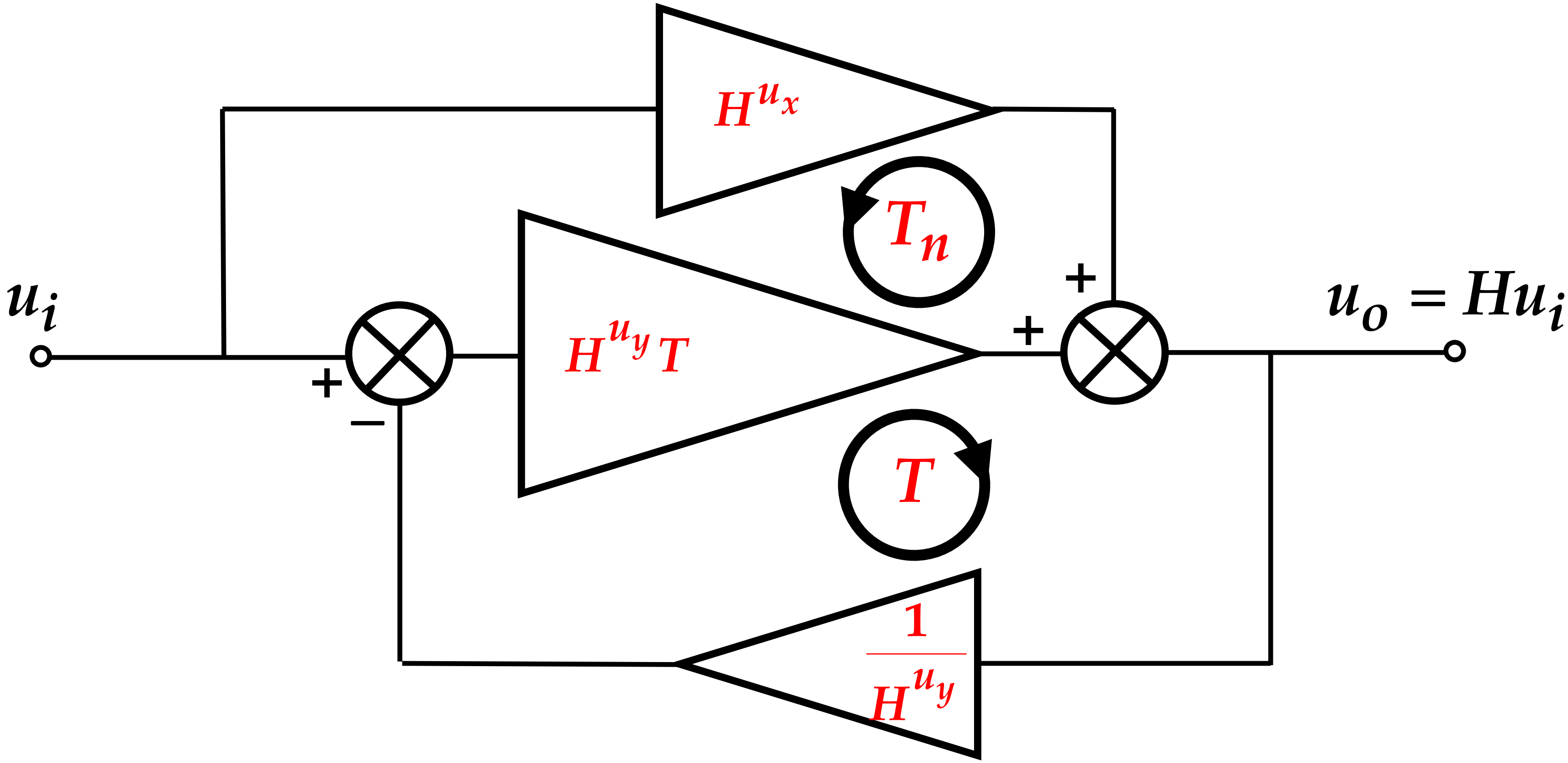
Two deficiencies:

1. **Requires an ideal injection point**

2. **Ignores nonidealities**

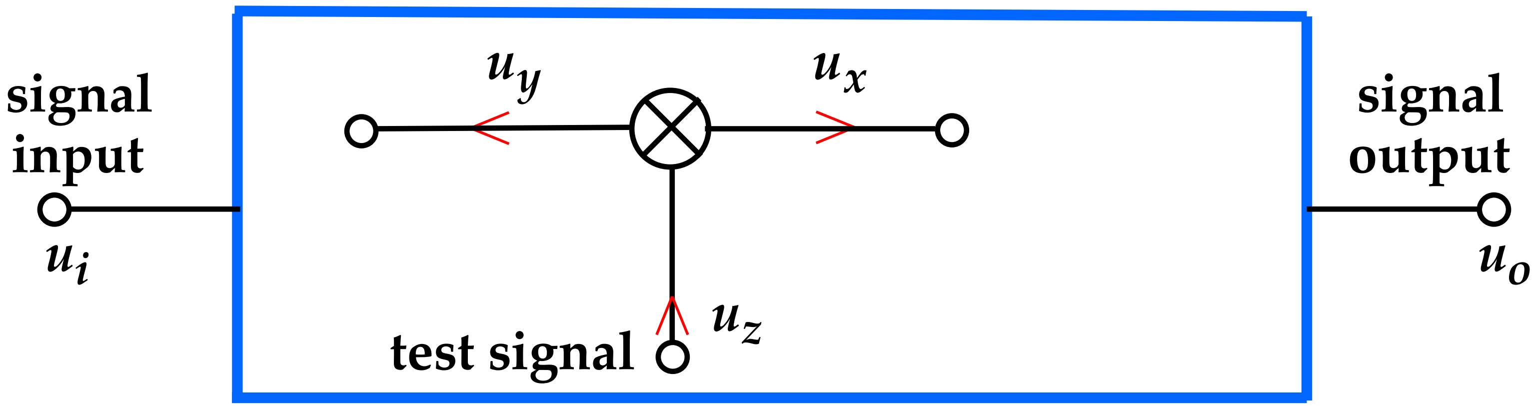
In contrast, the Dissection Theorem, which is a formula in similar format, is not based on a model.

On the contrary, the block diagram is a *result* of the formula, not its *origin*, and contains no assumptions or approximations.



How can the DT be made to represent a feedback system?

Dissection Theorem (DT)



ndi ndi ndi

$$H \equiv \left.\frac{u_o}{u_i}\right|_{u_z=0} \qquad H = H^{u_y}\frac{1+\frac{1}{T_n}}{1+\frac{1}{T}} = H^{u_y}\frac{T}{1+T} + H^{u_x}\frac{1}{1+T}$$

si

first level TF

second level TFs

$$H^{u_y} \equiv \left.\frac{u_o}{u_i}\right|_{u_y=0} \qquad T_n \equiv \left.\frac{u_y}{u_x}\right|_{u_o=0}$$

Redundancy Relation:

$$\frac{H^{u_y}}{H^{u_x}} = \frac{T_n}{T}$$

$$H^{u_x} \equiv \left.\frac{u_o}{u_i}\right|_{u_x=0} \qquad T \equiv \left.\frac{u_y}{u_x}\right|_{u_i=0}$$

There are many reasons why the Dissection Theorem is useful.

The *minimum* benefit of the DT is that it embodies the "Divide and Conquer" approach, because one complicated calculation is replaced by three calculations, two of which are ndi calculations and are therefore *simpler* and *easier* than si calculations.

Not only does the DT implement the Design & Conquer objective, but the DT is itself a Low Entropy Expression, and *much greater* benefits accrue if the second level TFs have useful physical interpretations.

Thus, the second level TFs themselves contain the useful design-oriented information and you may never need to actually substitute them into the theorem.

For example, if $T, T_n >> 1$, $H \approx H^{u_y}$

How to determine the physical interpretations of the second level TFs?

What kind of signal (voltage or current) is injected, and where it is injected, defines an "injection configuration."

Therefore, ***the key decision in applying the DT is choosing a test signal injection point so that at least one of the second level TFs has the physical interpretation you want it to have.***

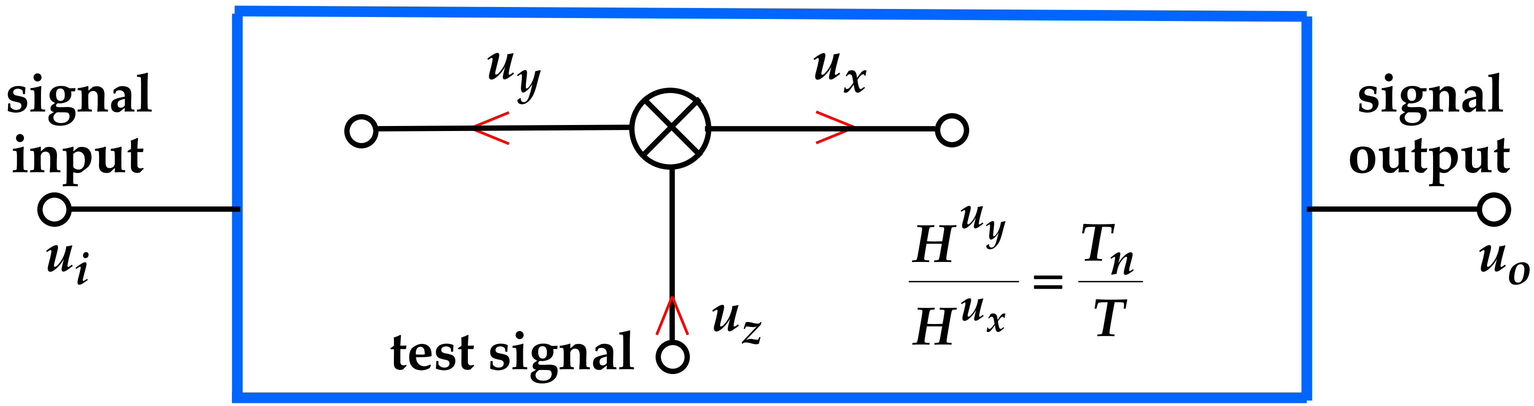
Specific injection configurations for the DT lead to the:

Extra Element Theorem (EET)

Chain Theorem (CT)

General Feedback Theorem (GFT)

Dissection Theorem (DT)



ndi ndi ndi

$$H = H^{u_y}\frac{1+\frac{1}{T_n}}{1+\frac{1}{T}} = H^{u_y}\frac{T}{1+T} + H^{u_x}\frac{1}{1+T}$$

si

first level TF second level TFs

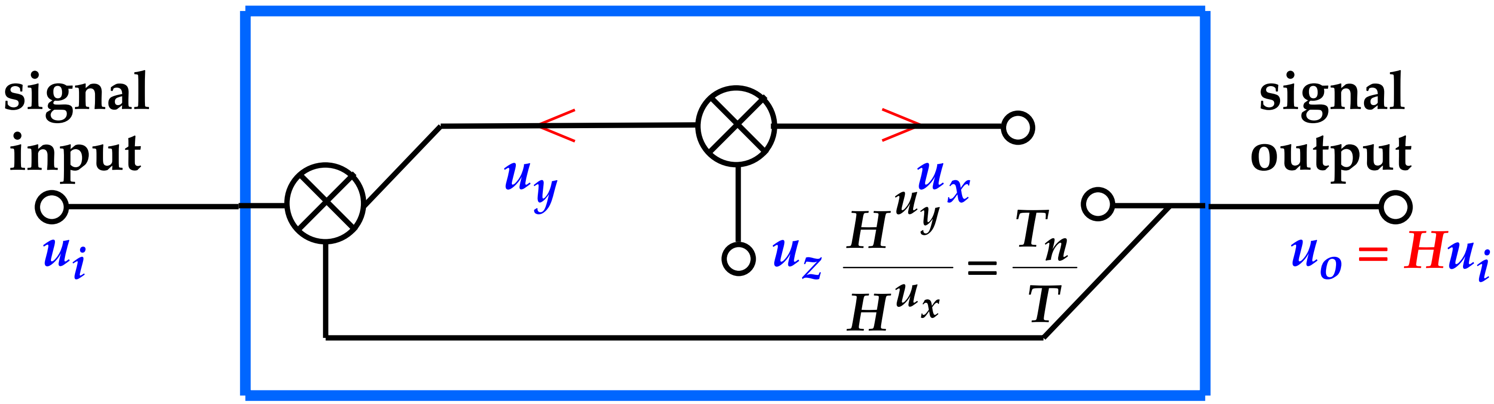
$$H \equiv \left.\frac{u_o}{u_i}\right|_{u_z=0} \qquad H^{u_y} \equiv \left.\frac{u_o}{u_i}\right|_{u_y=0} \qquad H^{u_x} \equiv \left.\frac{u_o}{u_i}\right|_{u_x=0} \qquad T \equiv \left.\frac{u_y}{u_x}\right|_{u_i=0} \qquad T_n \equiv \left.\frac{u_y}{u_x}\right|_{u_o=0}$$

gain

What test signal injection configuration makes the DT represent a feedback system?

We want H^{u_y} to represent the ideal closed-loop gain, and so the test signal u_z must be injected at the error signal summing point.
At the same time, u_x is the signal going forward around the feedback loop, so T represents the loop gain:

Test signal injection at the error signal summing point:



ndi ndi ndi

$$H = H^{u_y}\frac{1+\frac{1}{T_n}}{1+\frac{1}{T}} = \underbrace{H^{u_y}\frac{T}{1+T} + H^{u_x}\frac{1}{1+T}}_{\text{second level TFs}}$$

si

first level TF

$$H \equiv \left.\frac{u_o}{u_i}\right|_{u_z=0} \qquad H^{u_y} \equiv \left.\frac{u_o}{u_i}\right|_{u_y=0} \qquad H^{u_x} \equiv \left.\frac{u_o}{u_i}\right|_{u_x=0} \qquad T \equiv \left.\frac{u_y}{u_x}\right|_{u_i=0} \qquad T_n \equiv \left.\frac{u_y}{u_x}\right|_{u_o=0}$$

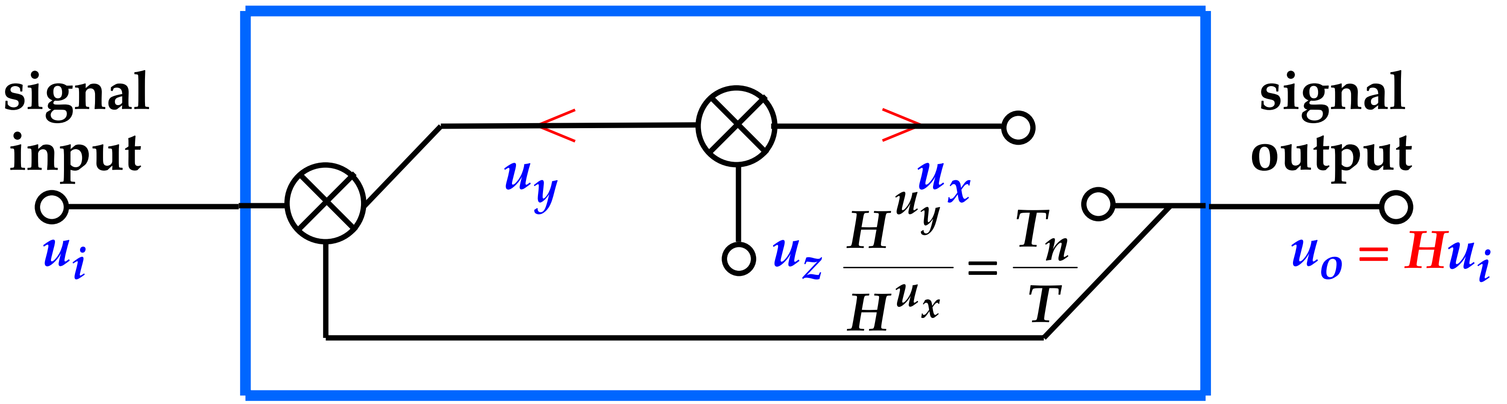
closed-loop gain

ideal closed-loop gain

loop gain

null loop gain

Test signal injection at the error signal summing point:



$$H = H^{u_y} \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}} = H^{u_y} \frac{T}{1+T} + H^{u_x} \frac{1}{1+T}$$

It is seen that the closed-loop gain H is the weighted sum of two components:

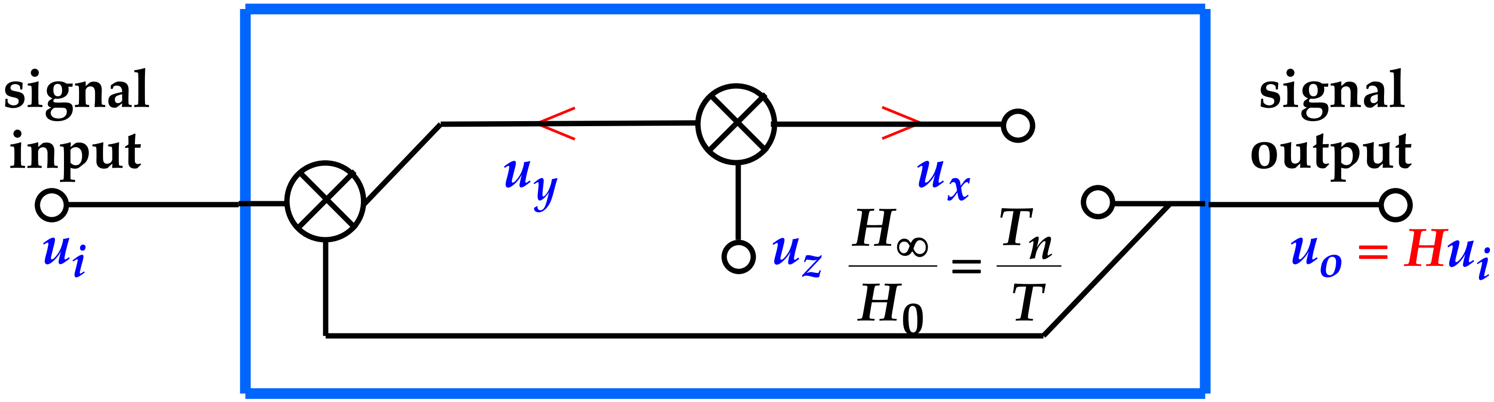
closed-loop gain when $T = \infty$: $H_\infty \equiv H^{u_y}$

(the "ideal closed-loop gain")

closed-loop gain when $T = 0$: $H_0 \equiv H^{u_x}$

(the "direct forward transmission")

With these new definitions, the DT morphs into the
General Feedback Theorem (GFT):



ndi ndi ndi

$$H = H_\infty \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}} = H_\infty \frac{T}{1+T} + H_0 \frac{1}{1+T}$$

si

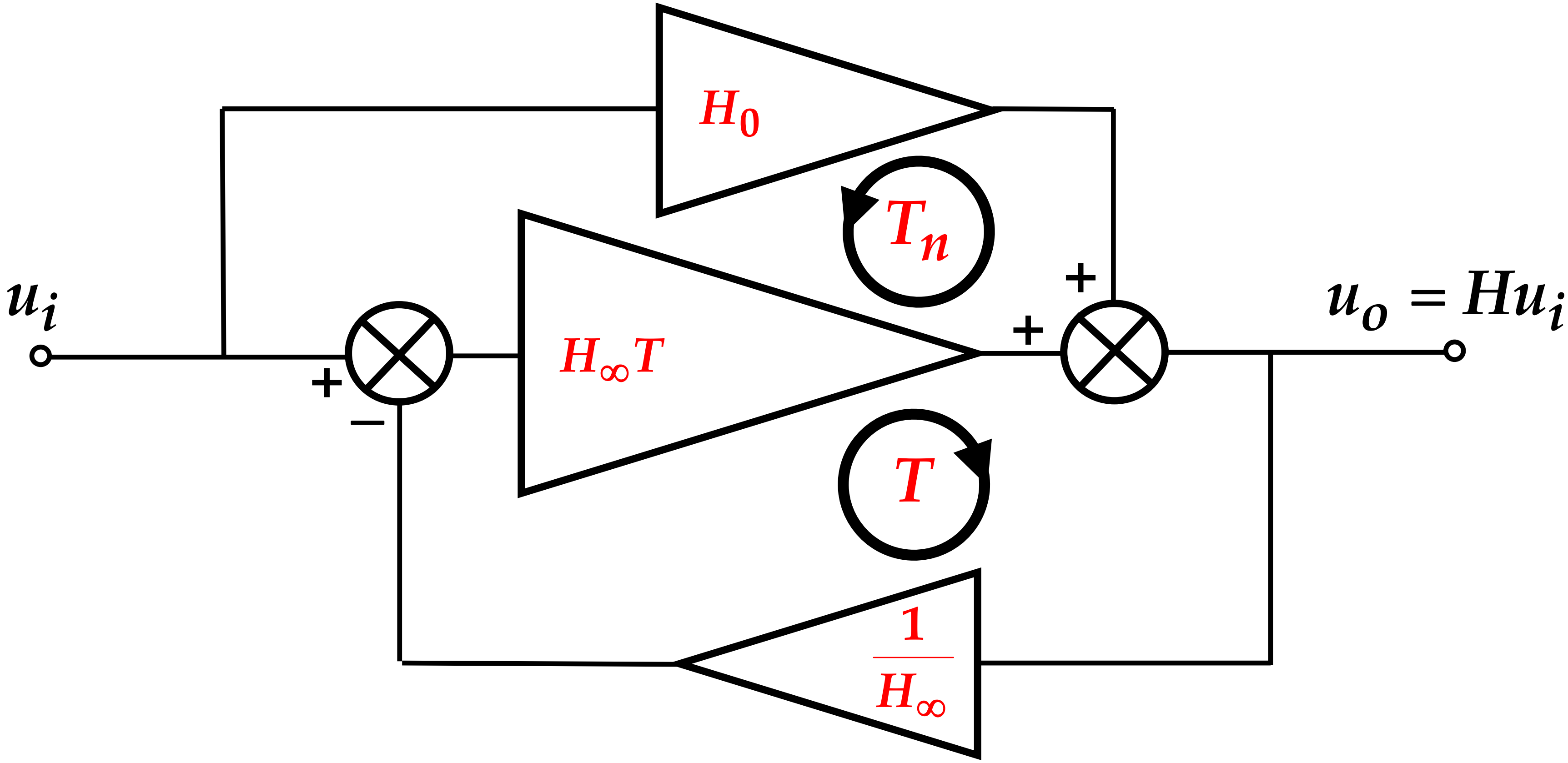
first level TF second level TFs

$$H \equiv \left.\frac{u_o}{u_i}\right|_{u_z=0} \qquad H_\infty \equiv \left.\frac{u_o}{u_i}\right|_{u_y=0} \qquad H_0 \equiv \left.\frac{u_o}{u_i}\right|_{u_x=0} \qquad T \equiv \left.\frac{u_y}{u_x}\right|_{u_i=0} \qquad T_n \equiv \left.\frac{u_y}{u_x}\right|_{u_o=0}$$

closed-loop gain | ideal closed-loop gain | direct forward transmission | loop gain | null loop gain

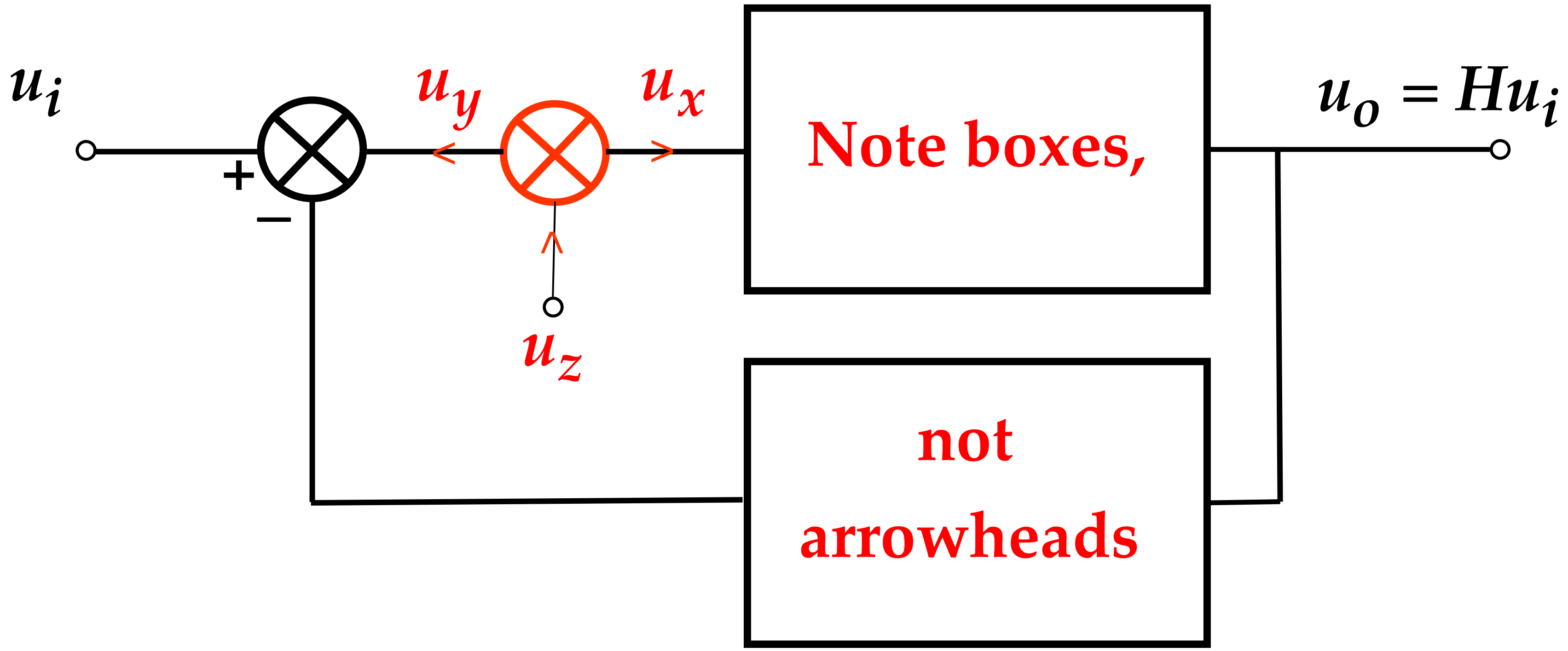
The augmented feedback block diagram:

This is the same block diagram that represents the conventional model, plus the H_0 block, and the corresponding null loop gain T_n, which represent *nonidealities* not accounted for in the conventional model.



One of these nonidealities is the direct forward transmission through the feedback path "in the wrong direction."

The GFT Approach:



Inject a test signal u_z at error summing point.

The GFT gives all the second-level TFs directly in terms of the circuit elements.

As already seen, the GFT can be expressed as the weighted sum of two components:

$$H = H_\infty \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}} = H_\infty \frac{T}{1+T} + H_0 \frac{1}{1+T}$$

It is useful to define the weighting factors as "discrepancy factors" D and D_0 :

$$D \equiv \frac{T}{1+T} \qquad\qquad D_0 \equiv \frac{1}{1+T}$$

Also define a "null discrepancy factor" $\quad D_n \equiv 1+\frac{1}{T_n}$

$$H = H_\infty D D_n = H_\infty D + H_0 D_0$$

When $T >> 1$, $D \to 1$ and $D_0 \to 1/T$ When $T_n >> 1$, $D_n \to 1$

When $T << 1$, $D \to T$ and $D_0 \to 1$ When $T_n << 1$, $D_n \to 1/T_n$

Different versions of the GFT:

$$H = H_\infty \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}} = H_\infty \frac{T}{1+T} + H_0 \frac{1}{1+T}$$

$$H = H_\infty D D_n = H_\infty D + H_0 D_0$$

Thus, the direct forward transmisson nonideality H_0 appears either:

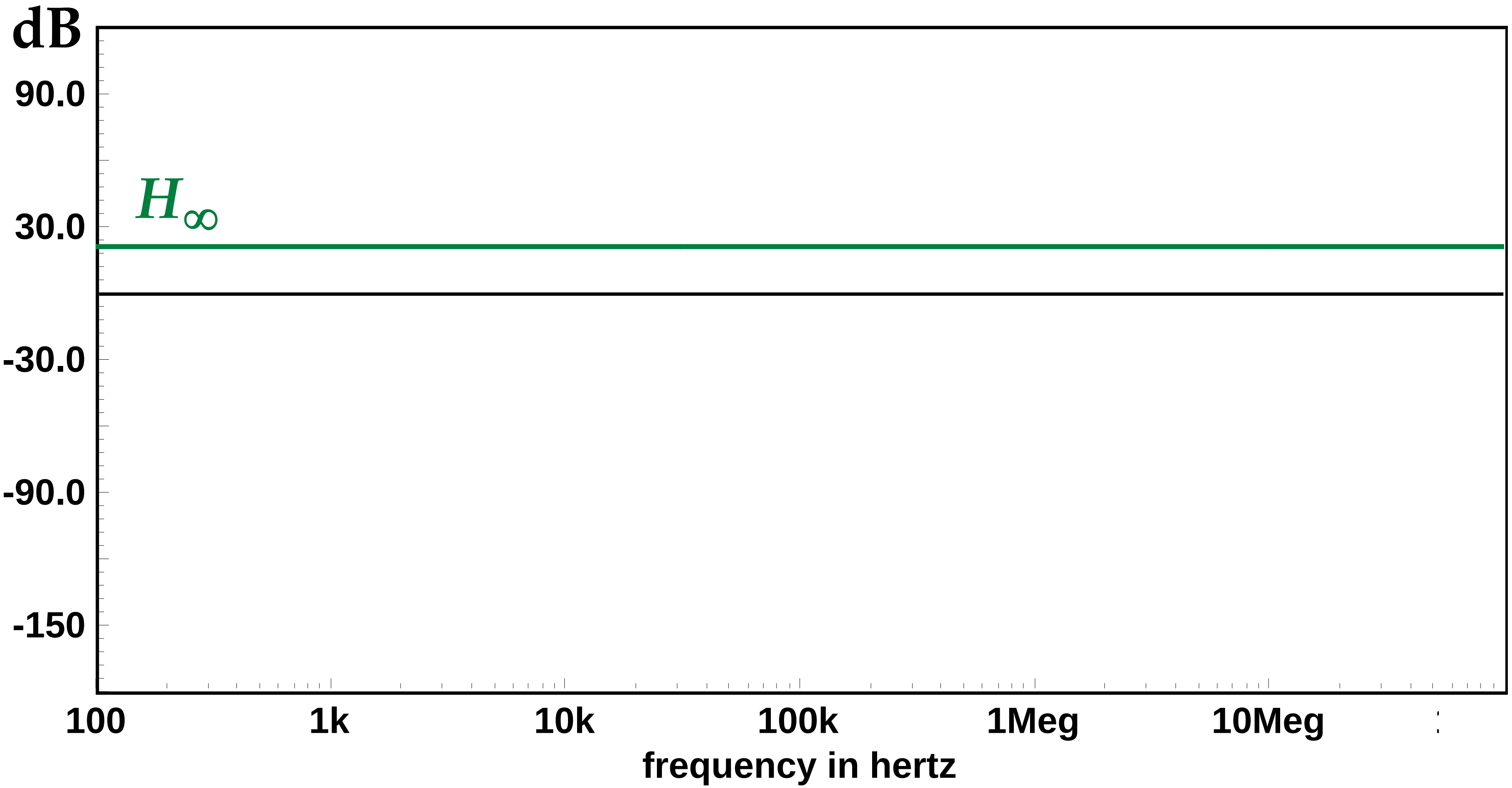
as a multiplier term, indirectly via T_n or D_n, or

as an additive term, directly

The job of a *designer*, as distinct from that of an *analyst*, is to construct hardware that meets specifications within certain tolerances.

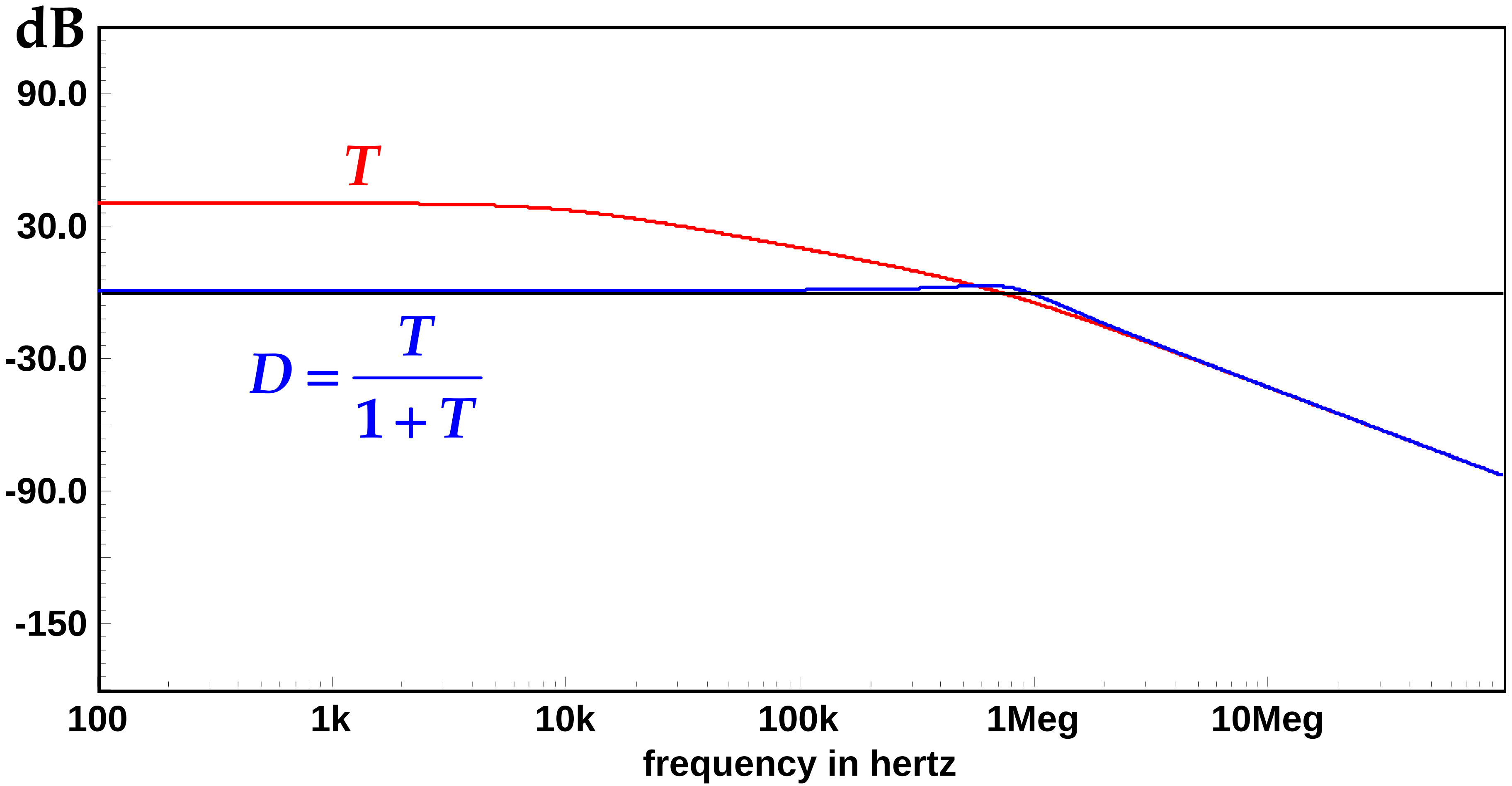
If you are designing a feedback amplifier, you effectively proceed through four steps:

Design Step #1



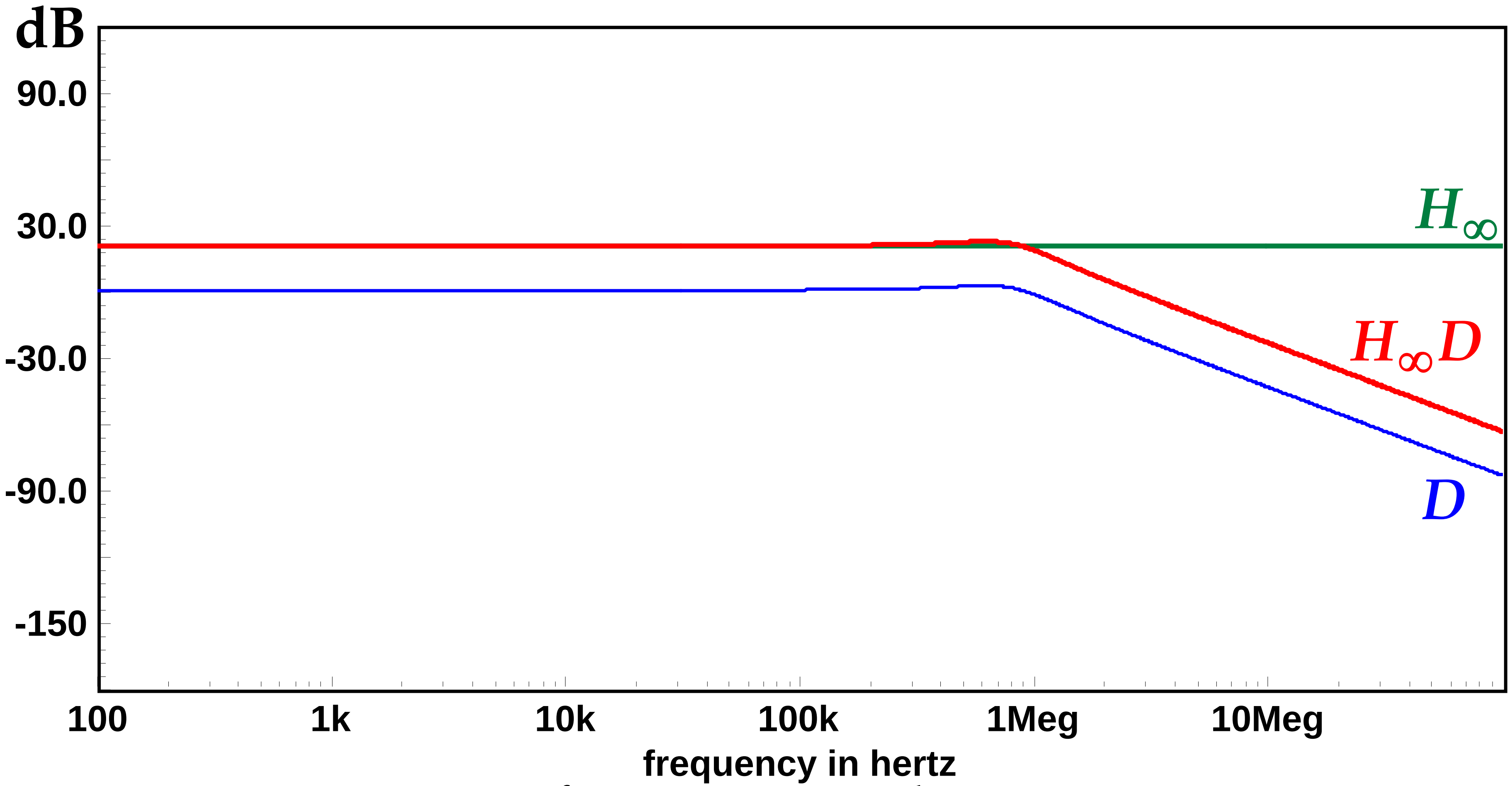
Design the feedback network $K = 1/H_\infty$ so that H_∞ meets the specification

Design Step #2



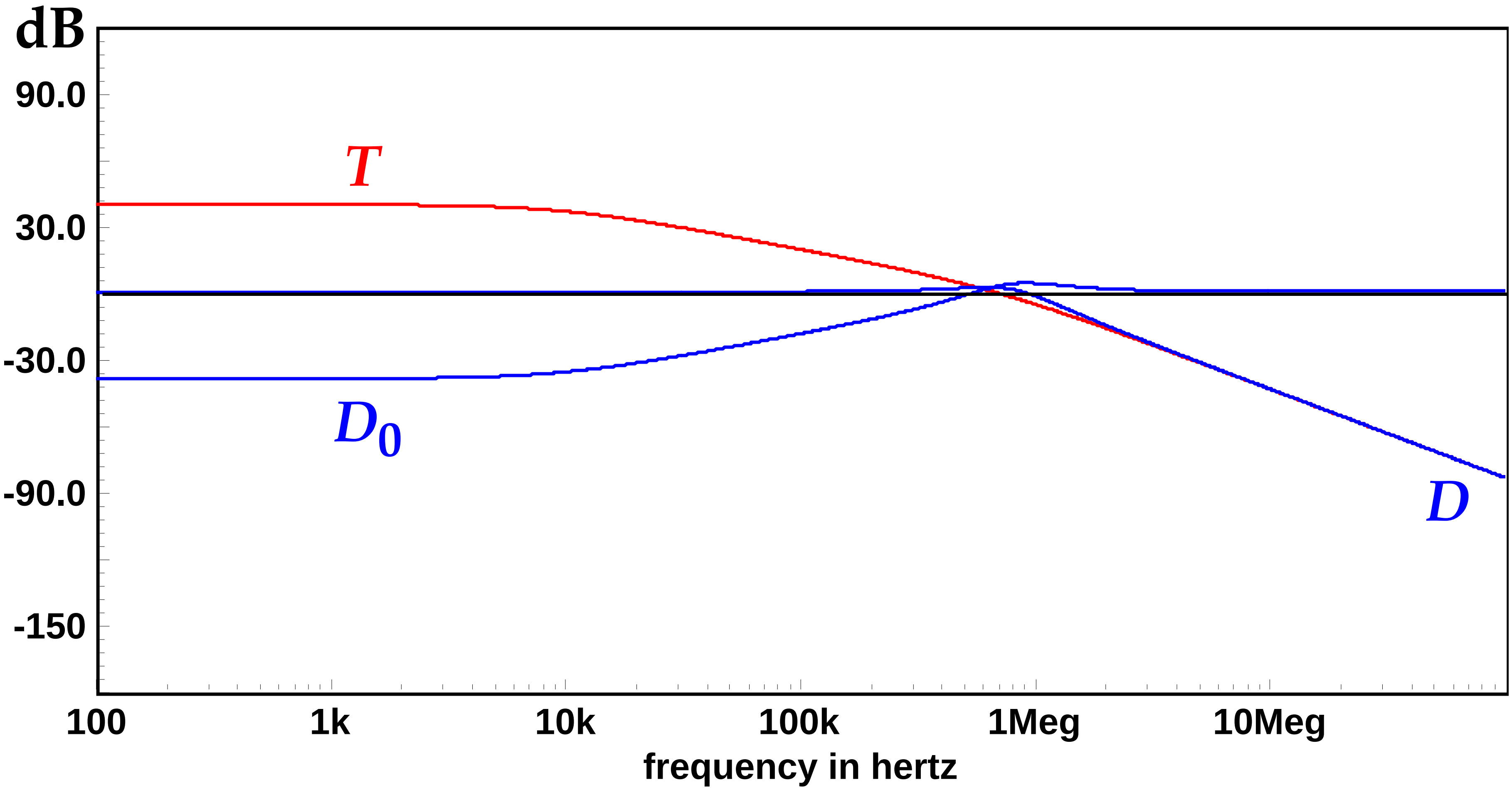
Design the loop gain so that T is large enough that the actual H meets the specification within the allowed tolerances

Design Step #2 cont.



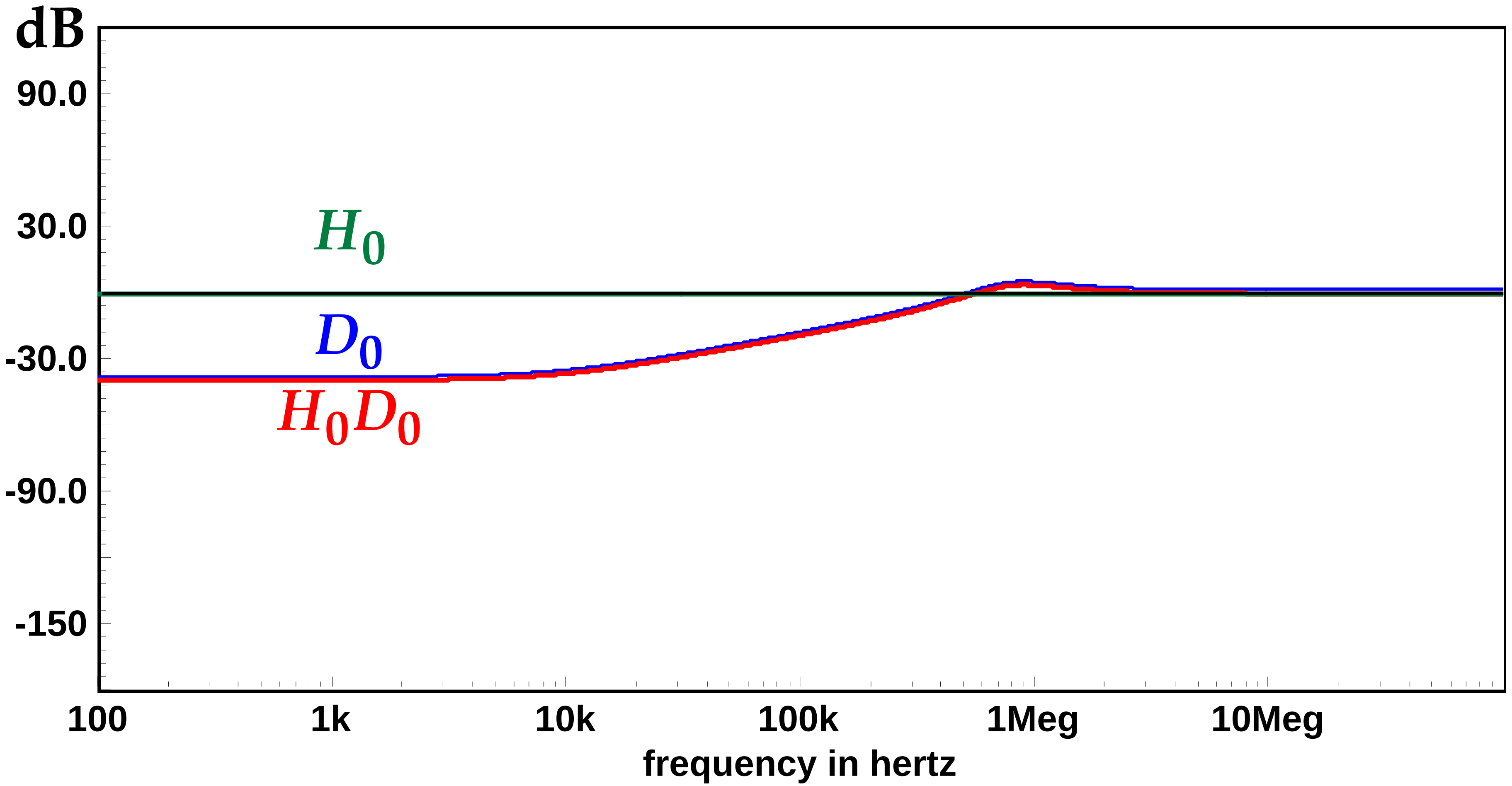
Construct $H_\infty D$ **from** H_∞ **and** D

Design Step #3



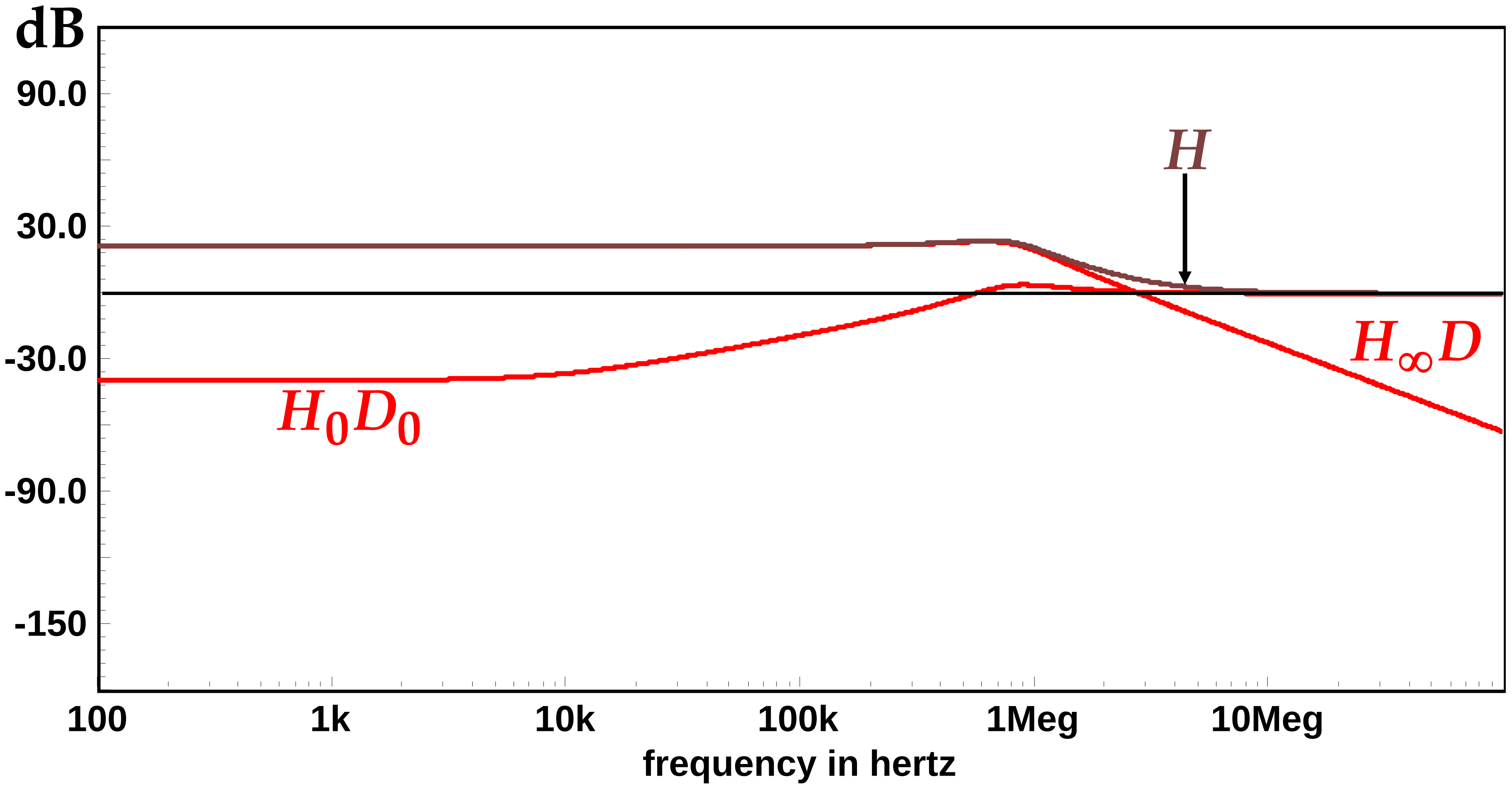
Design *T* to be large enough up to a sufficiently high frequency. Usually the most difficult step; includes the stability requirement

Design Step #4

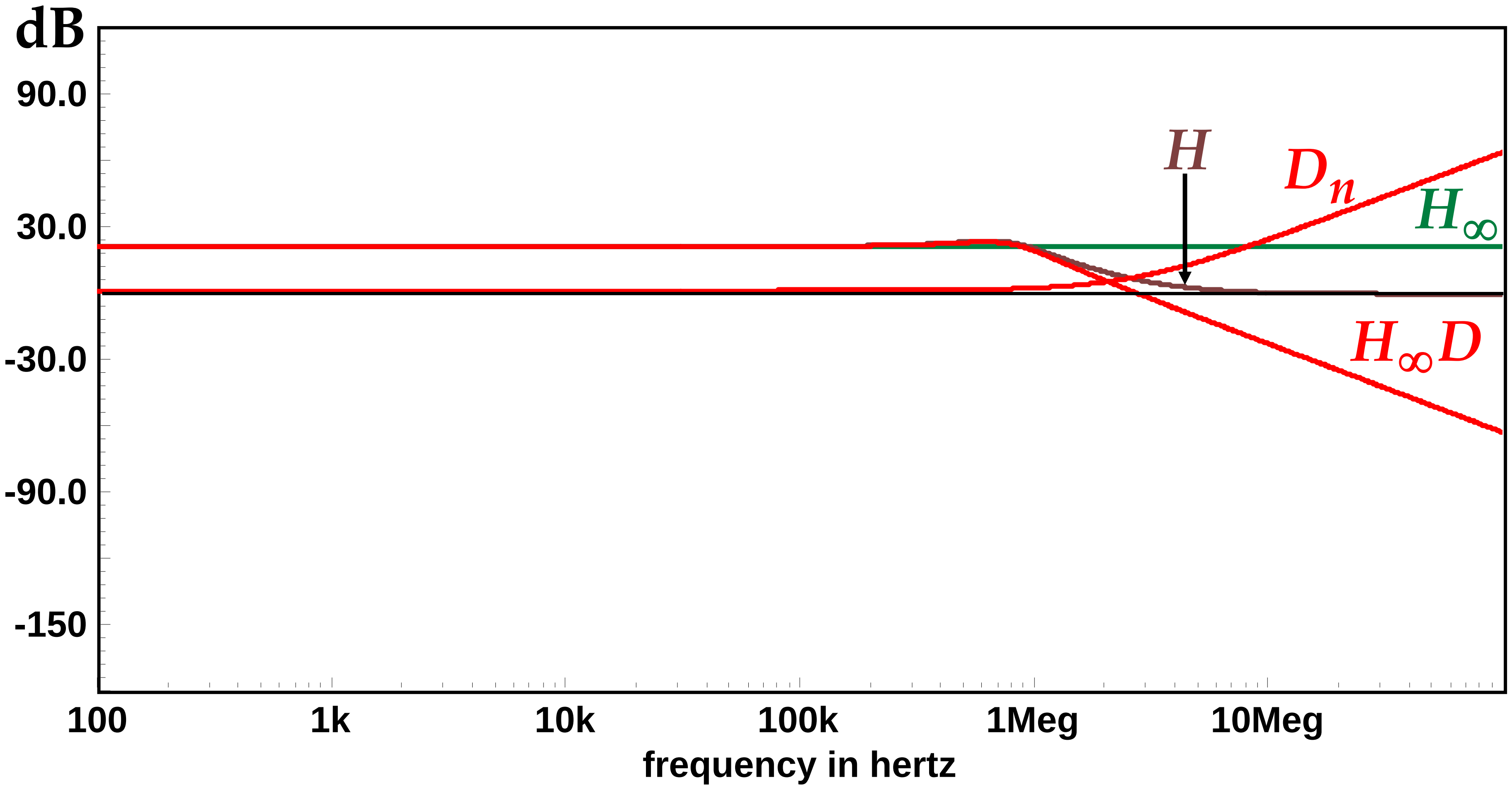


Design the direct forward transmission so that H_0 is small enough that the actual H meets the specification within the allowed tolerances

Design Step #4 cont.



Construction of H with $H_0 D_0$ as an additive term to $H_\infty D$



Construction of H with D_n as a multiplier term to $H_\infty D$

Calculation of the second level TFs H_∞, T, H_0, T_n by injection of a test signal u_z.

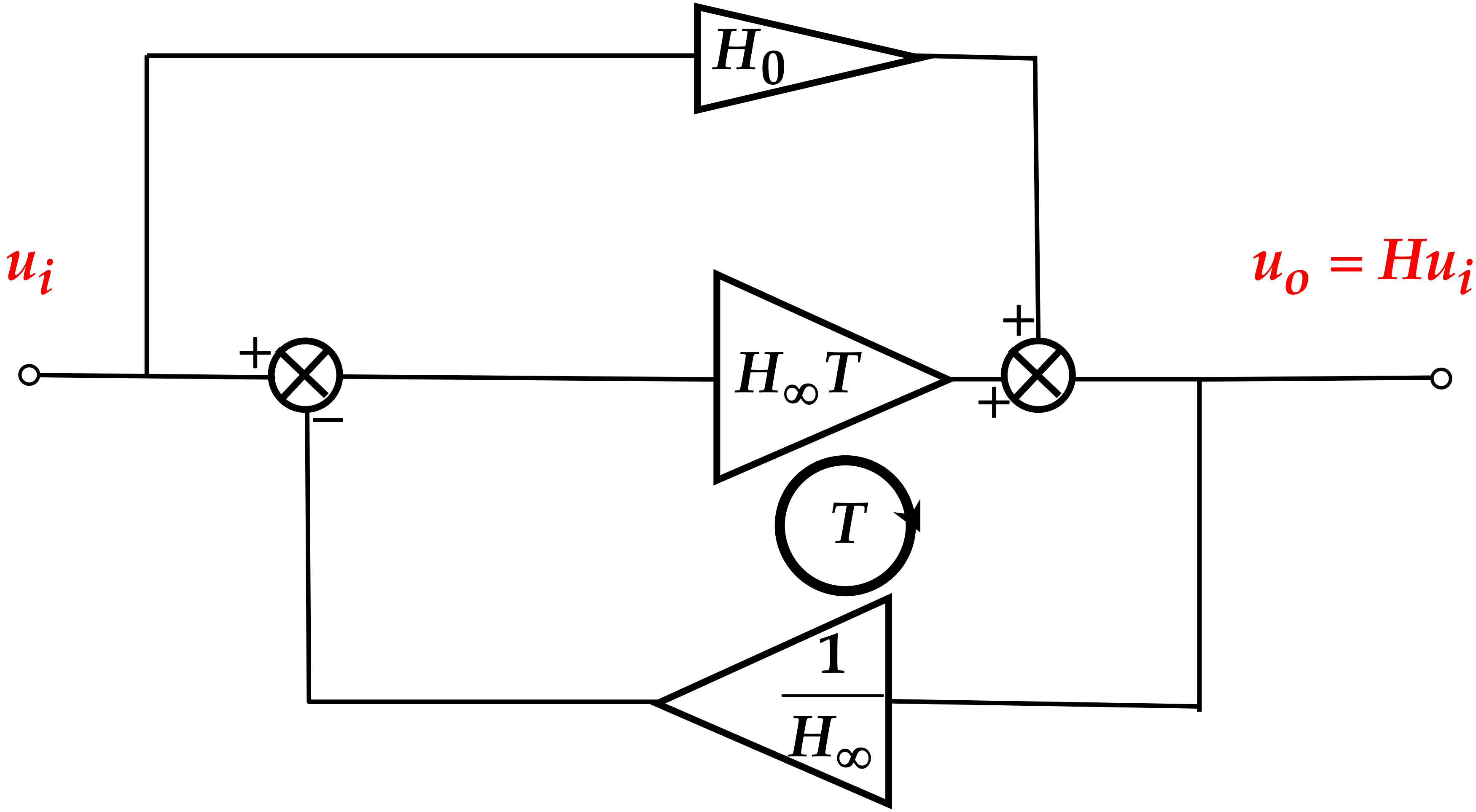
Ultimately, we want to calculate the second-level TFs directly from the circuit model, but first we'll do it from the block diagram.

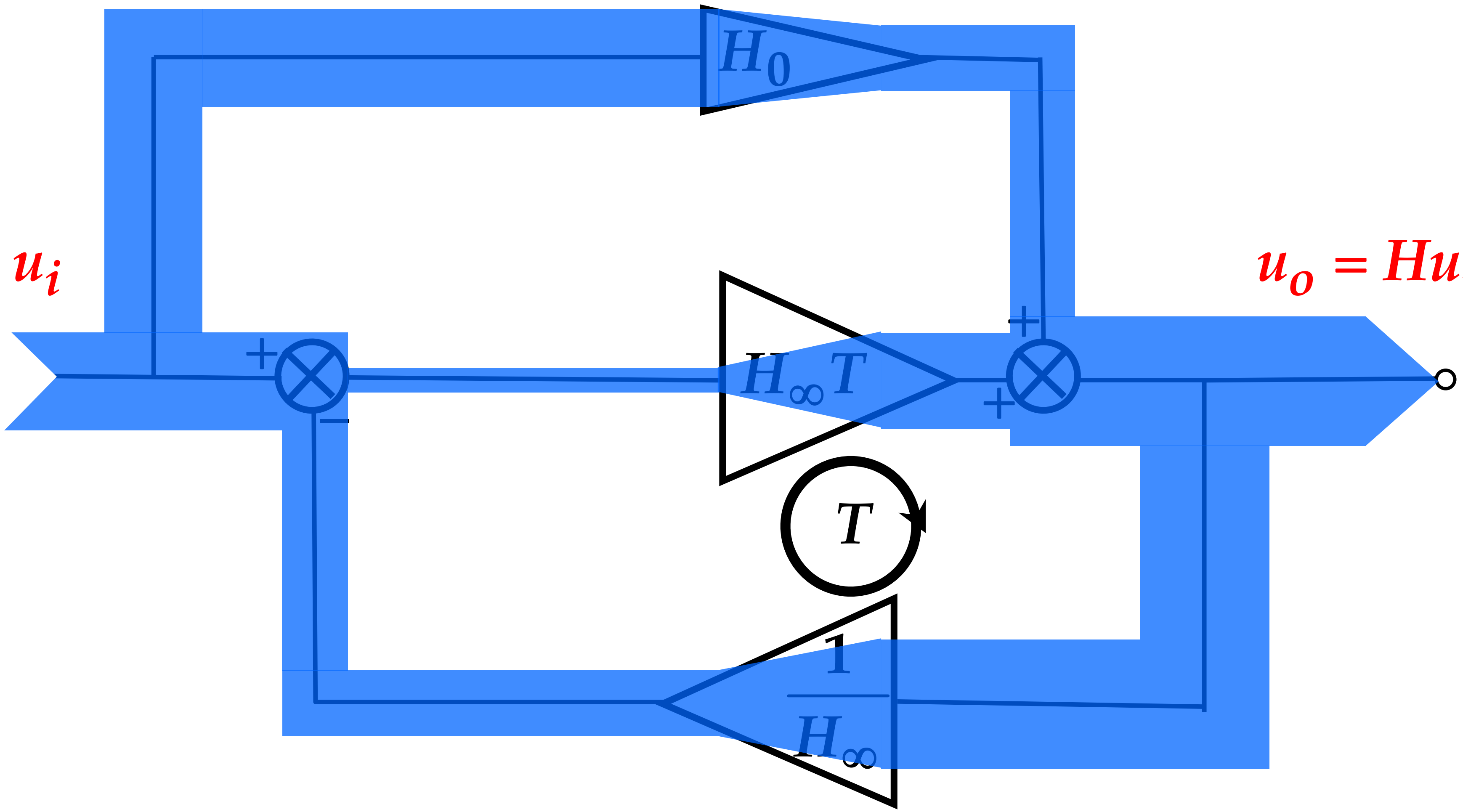
We want to do this by *signal injection*, without disturbing the circuit configuration and therefore without disturbing the circuit determinant.

Normally, we inject a single signal and calculate a TF whose input is that signal. This is single injection (si).

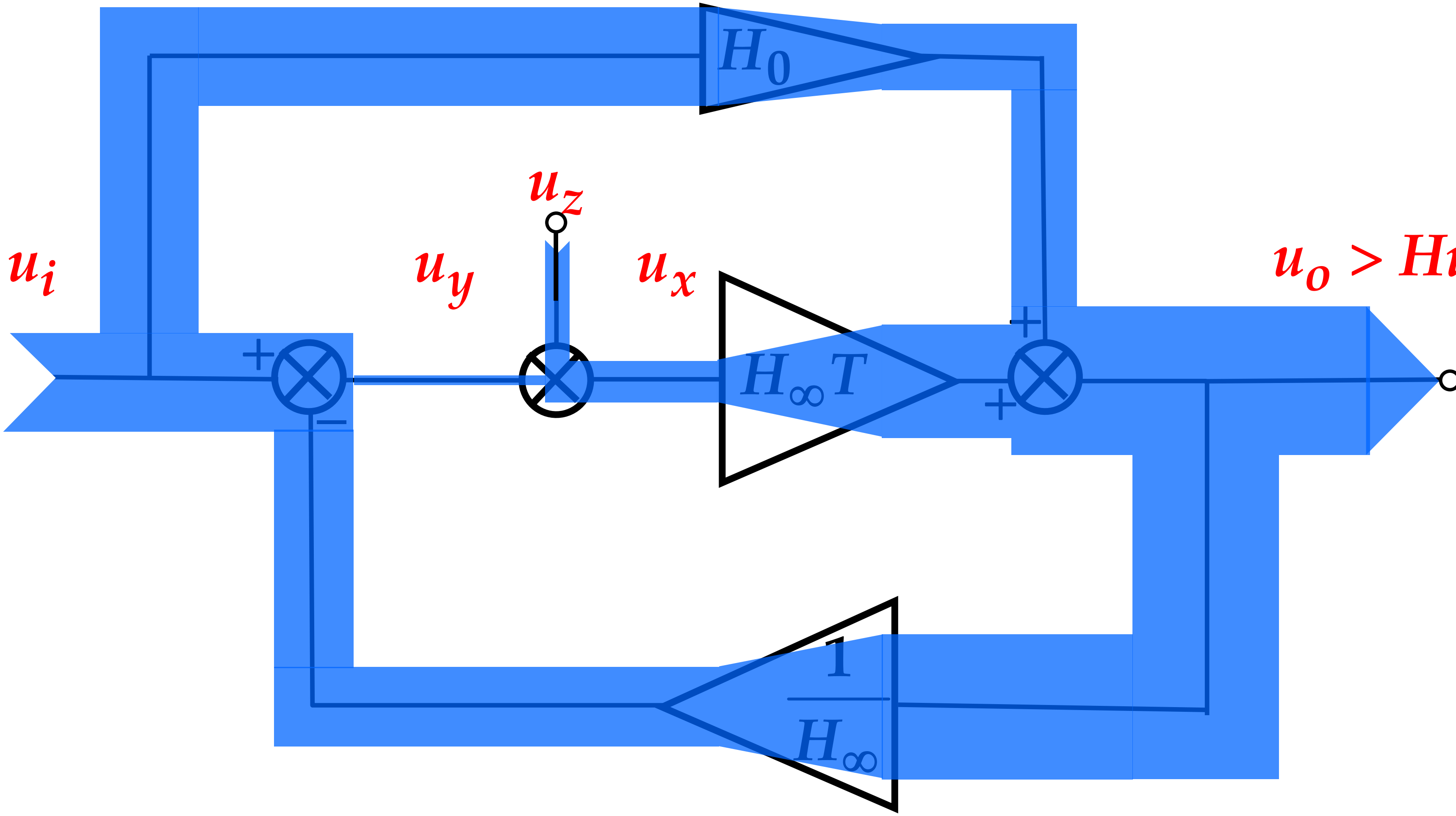
However, a powerful analytic technique is to inject two signals, mutually adjust them to null some dependent signal, and calculate a TF whose input is one or the other of the two injected signals. This is *null double injection* (ndi).

Consider the second-level TF H_∞, the ideal closed-loop gain. The actual closed-loop gain H falls short of H_∞ because the error signal, which is the difference between the input signal u_i and the fedback signal Ku_o, is not zero when the loop gain is not infinite.

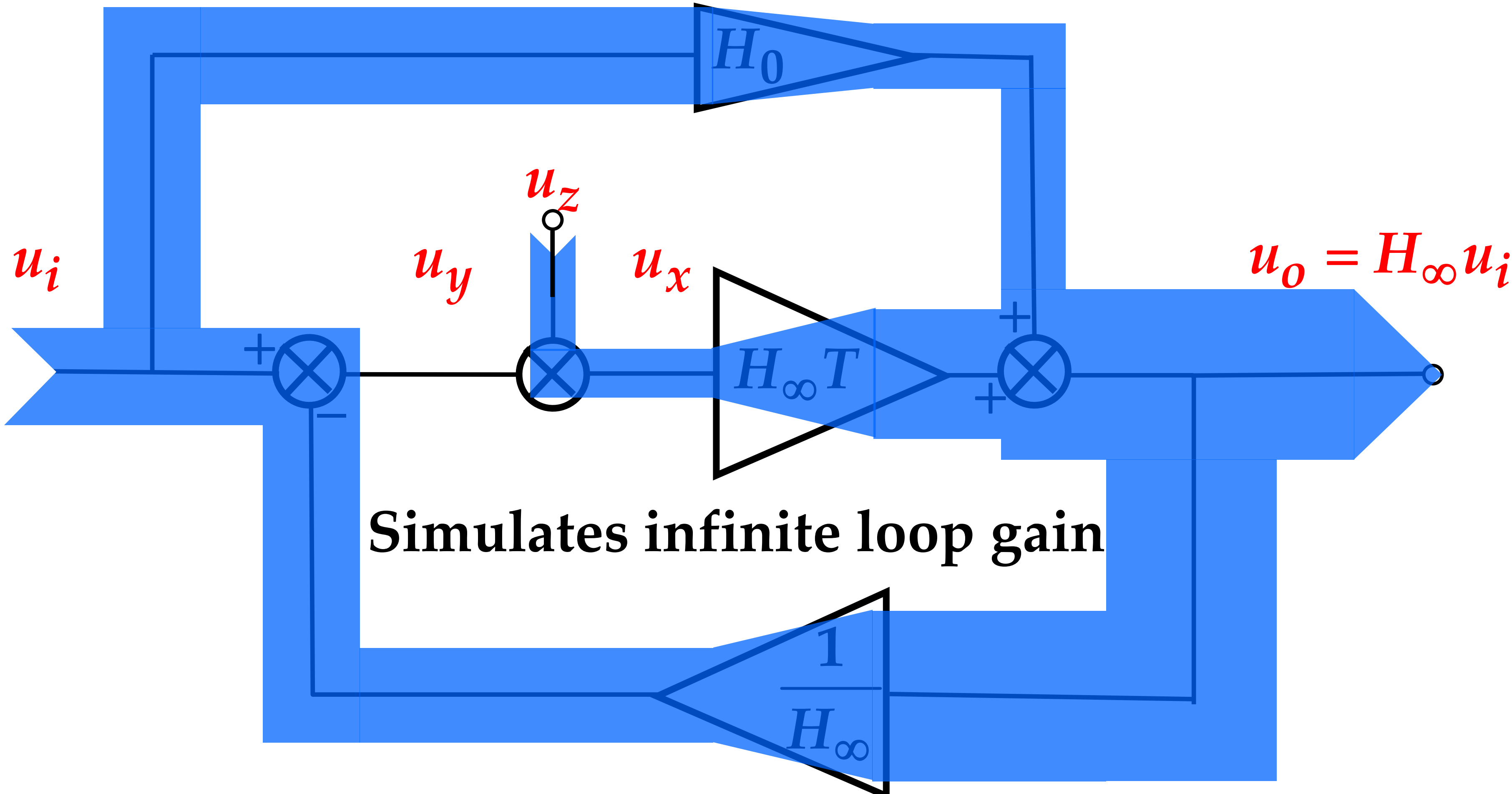

H_0
u_i
$u_o = Hu_i$
$H_\infty T$
T
$\frac{1}{H_\infty}$
+
−
+
+



Normal closed-loop operation



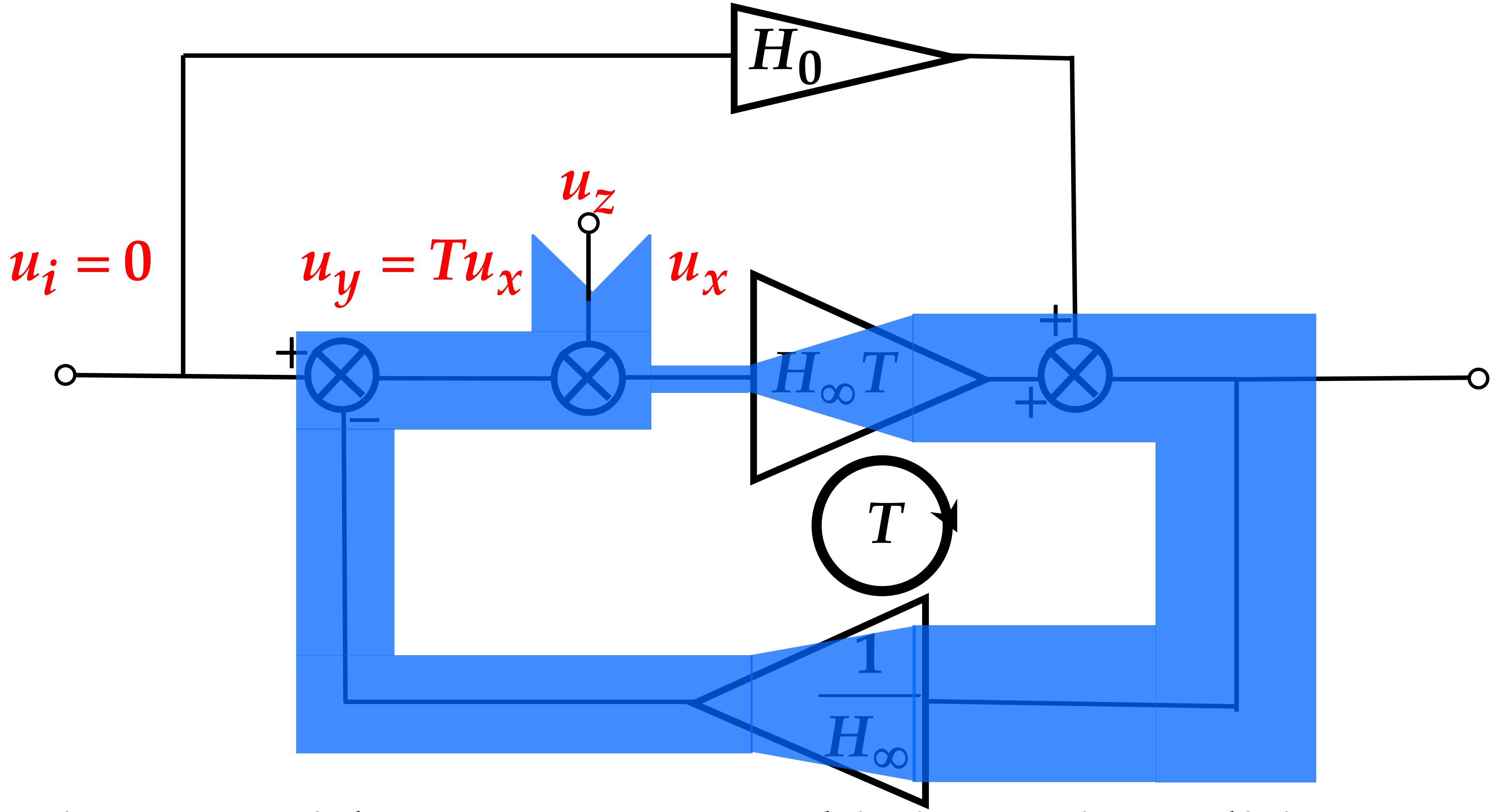
Inject a test signal u_z that adds to the error signal: the output u_o increases and the error signal u_y decreases



Simulates infinite loop gain

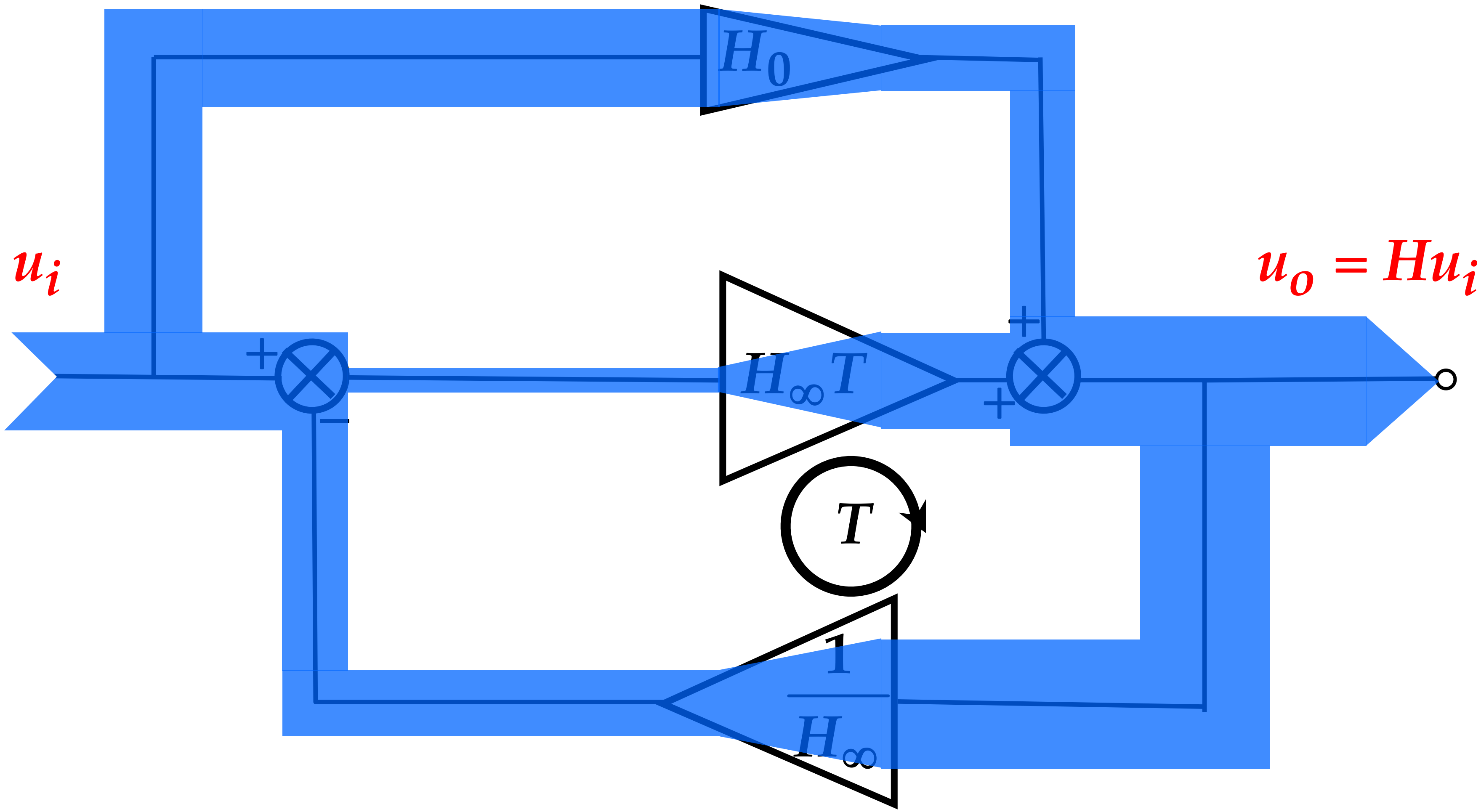
Increase u_z **until** u_y **falls to zero: this is an ndi** ***condition*****.**

The ndi ***calculation*** **is** $H_\infty = \left.\frac{u_o}{u_i}\right|_{u_y=0}$

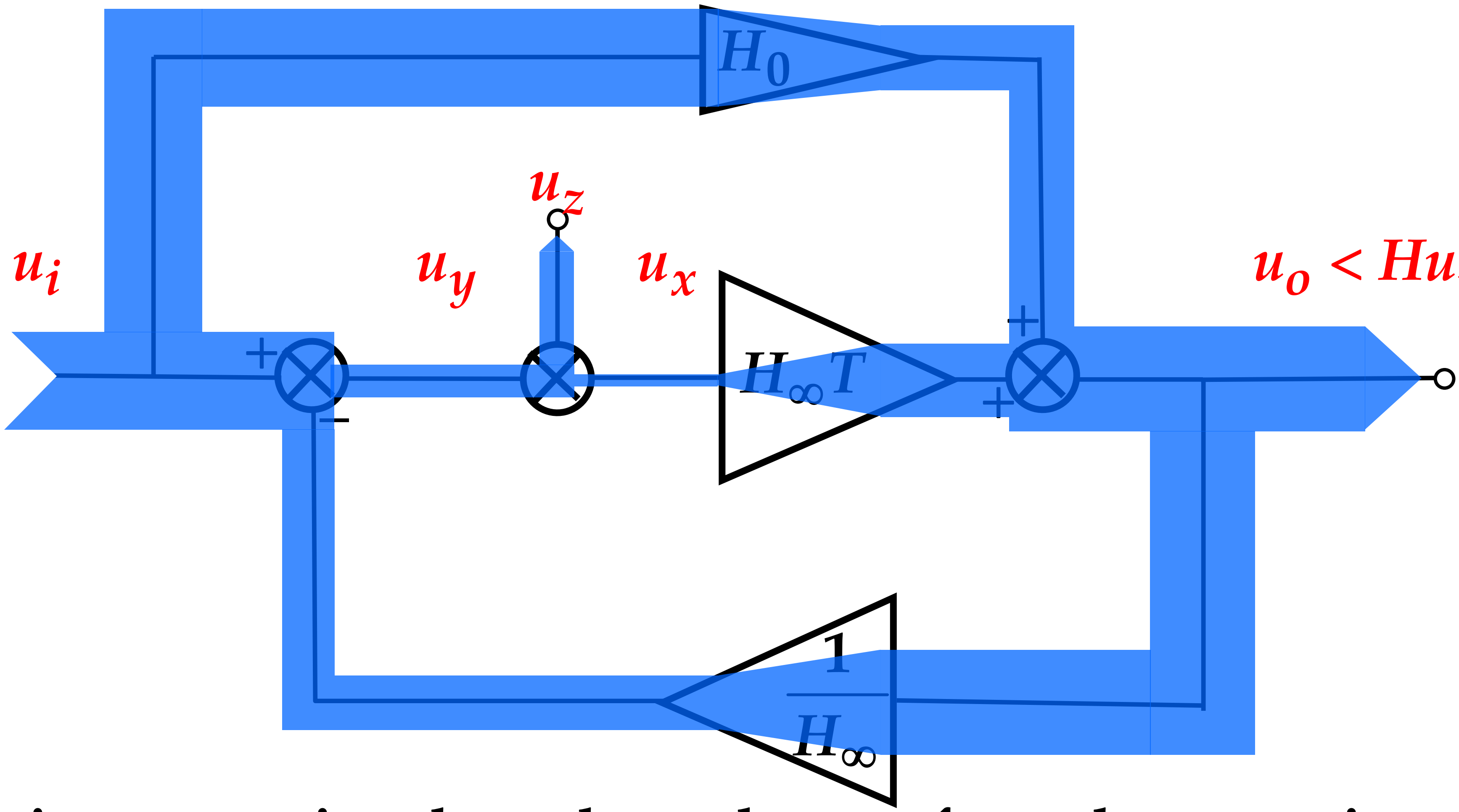


Inject u_z with u_i set to zero: this is an si condition.

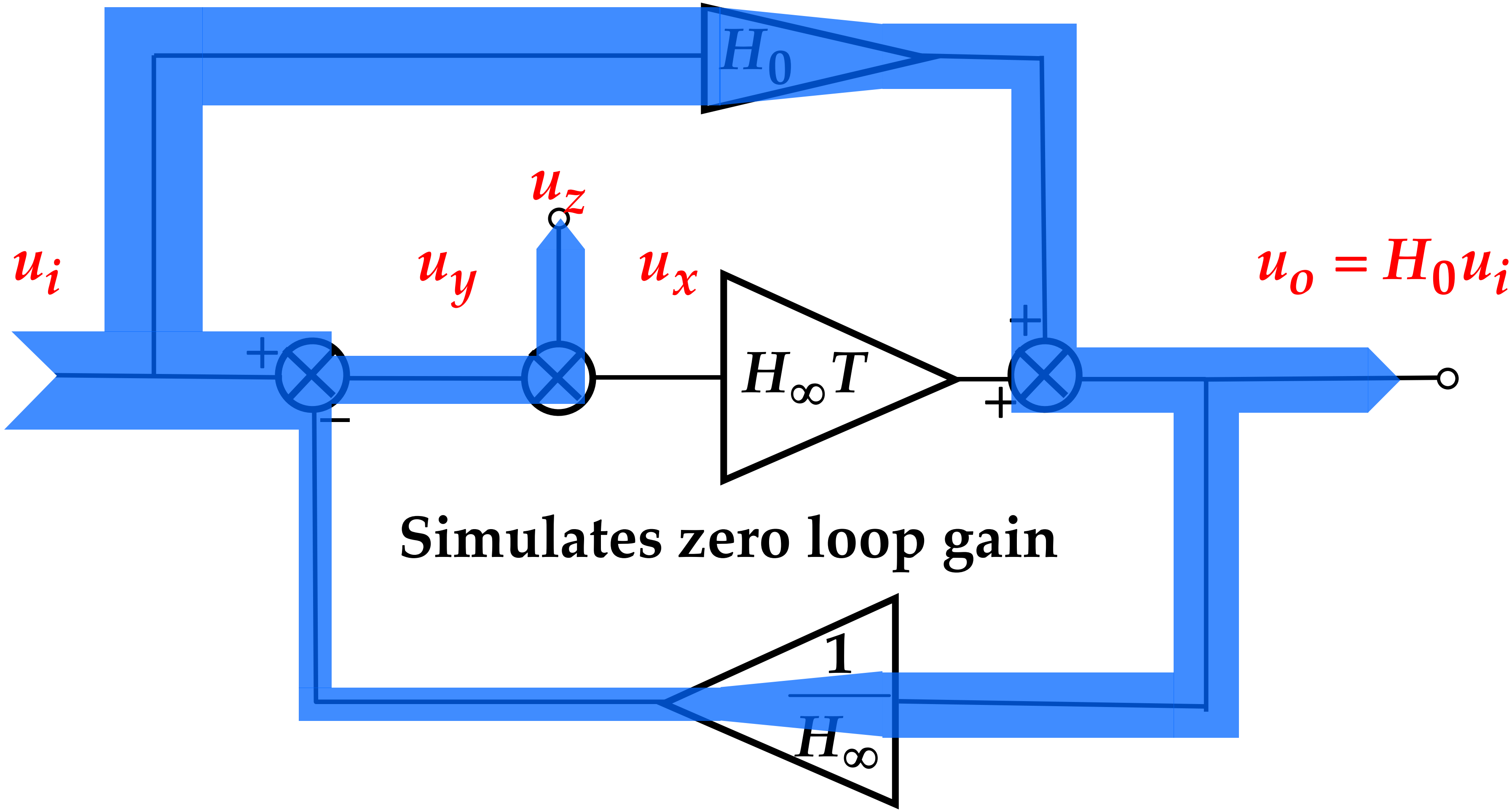
The si *calculation* is $T = \left.\frac{u_y}{u_x}\right|_{u_i=0}$



Back to normal closed-loop operation

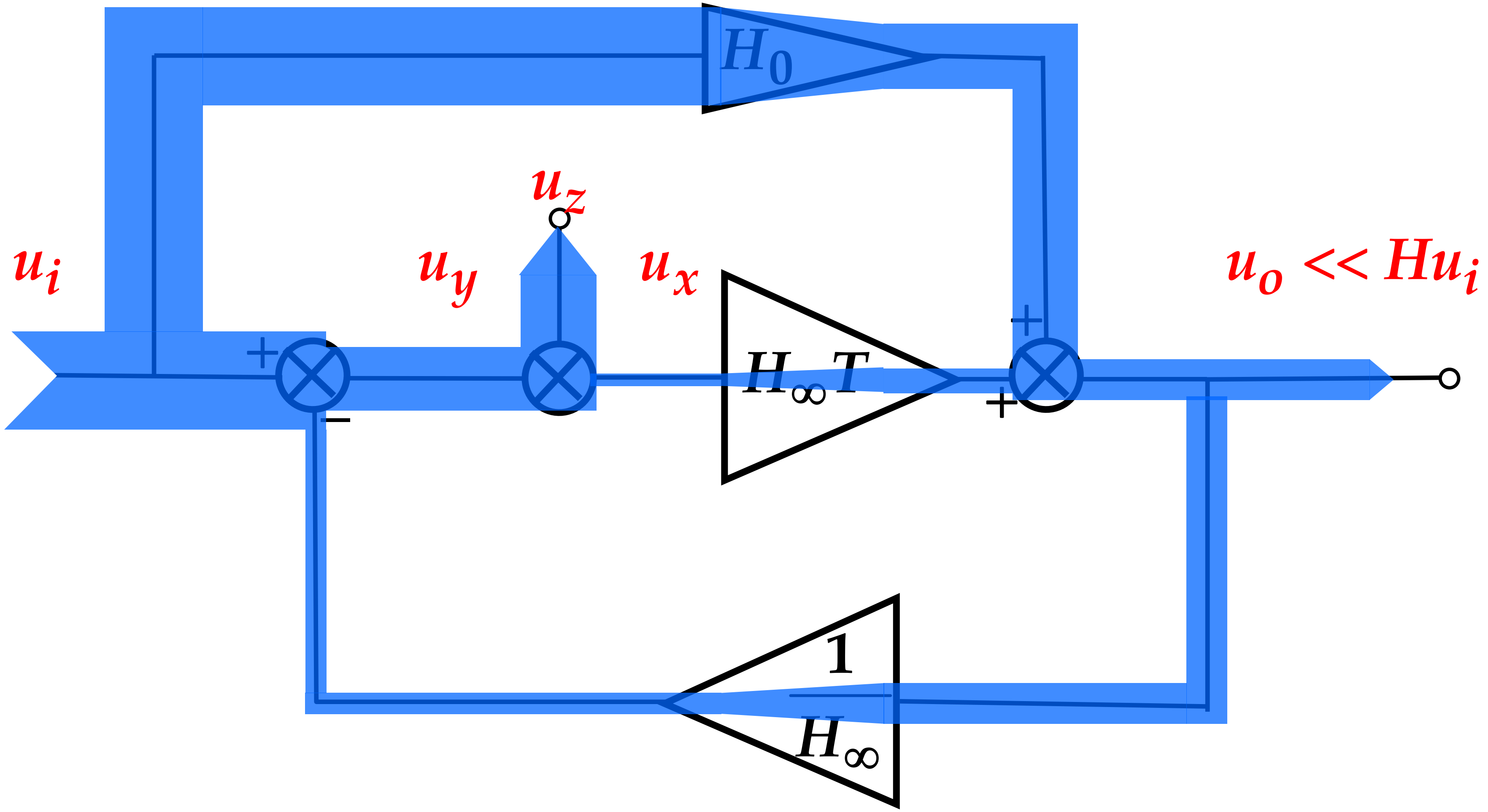


Inject a test signal u_z that subtracts from the error signal: the output u_o decreases and u_x decreases

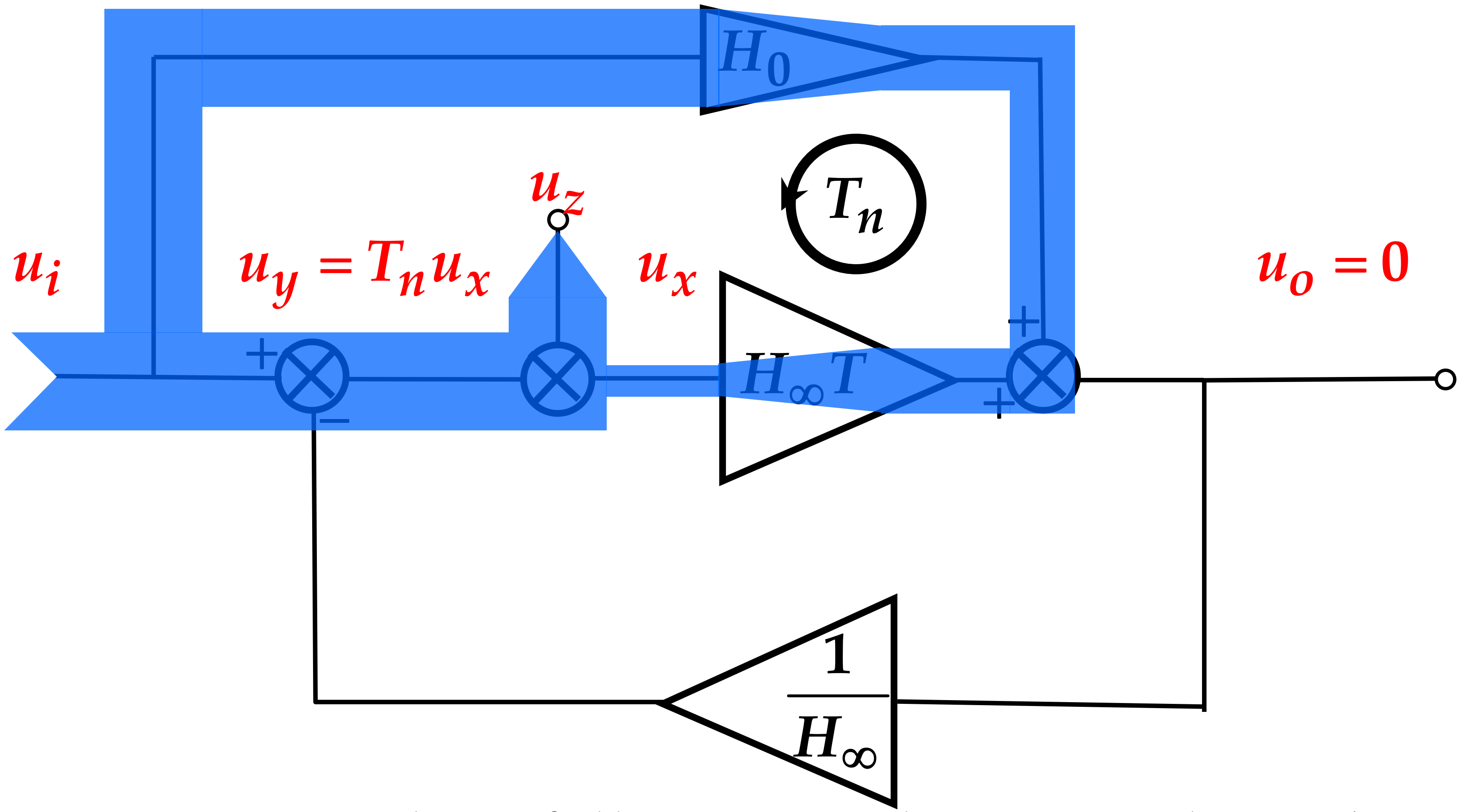


Increase u_z until u_x falls to zero: this is an ndi *condition*.

The ndi *calculation* is $H_0 = \left.\dfrac{u_o}{u_i}\right|_{u_x=0}$

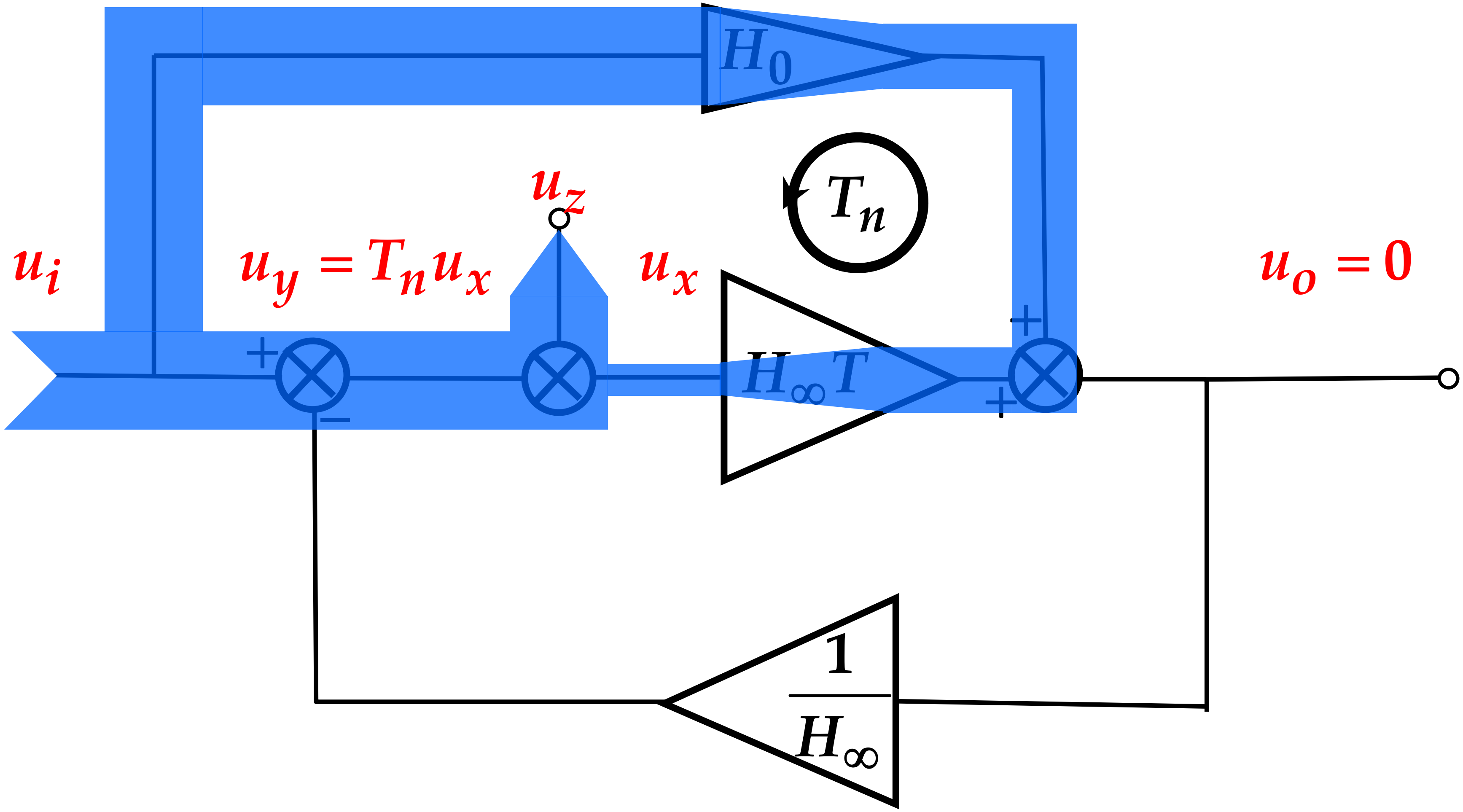


Further increase of u_z causes u_x to reverse, and a further drop in u_o



Increase u_z until u_o falls to zero: this is an ndi *condition*.

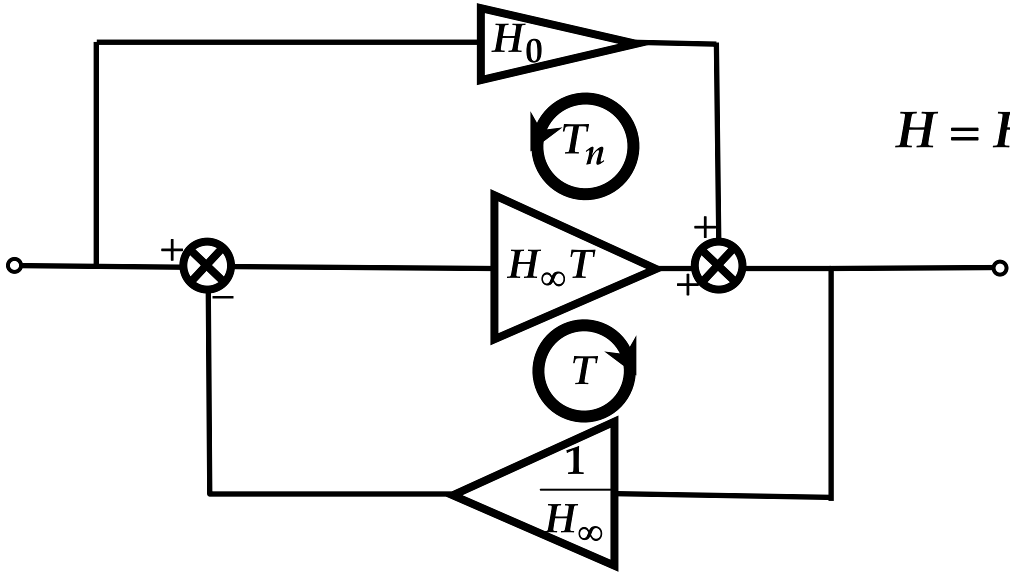
The ndi *calculation* is $T_n = \left.\frac{u_y}{u_x}\right|_{u_o=0}$



Under this ndi condition of nulled u_o,

$H_\infty T u_x = H_0 u_i$. But $u_y = u_i$, so $u_y / u_x = H_\infty T / H_0$,

which is how T_n was originally defined.



$$H = H_\infty \frac{T}{1+T} + H_0 \frac{1}{1+T} = H_\infty \frac{1 + \frac{H_0}{H_\infty T}}{1 + \frac{1}{T}}$$

Redundancy relation:

$$\frac{T_n}{T} = \frac{H_\infty}{H_0}$$

ideal closed-loop gain: $H_\infty = \left.\frac{u_o}{u_i}\right|_{u_y=0}$ **ndi calculation**

principal loop gain: $T = \left.\frac{u_y}{u_i}\right|_{u_i=0}$ **si calculation**

direct fwd transmission: $H_0 = \left.\frac{u_o}{u_i}\right|_{u_x=0}$ **ndi calculation**

null loop gain: $T_n = \left.\frac{u_y}{u_x}\right|_{u_o=0}$ **ndi calculation**

Test Signal Injection Configuration

In order for the *definitions* of the four second-level TFs H_{∞}, T, H_0, T_n to have the *interpretations* shown, it is necessary that the Test Signal Injection Configuration satisfy the conditions adopted in the preceding derivation:

Test Signal Injection Configuration

In order for the *definitions* of the four second-level TFs H_∞, T, H_0, T_n to have the *interpretations* shown, it is necessary that the Test Signal Injection Configuration satisfy the conditions adopted in the preceding derivation:

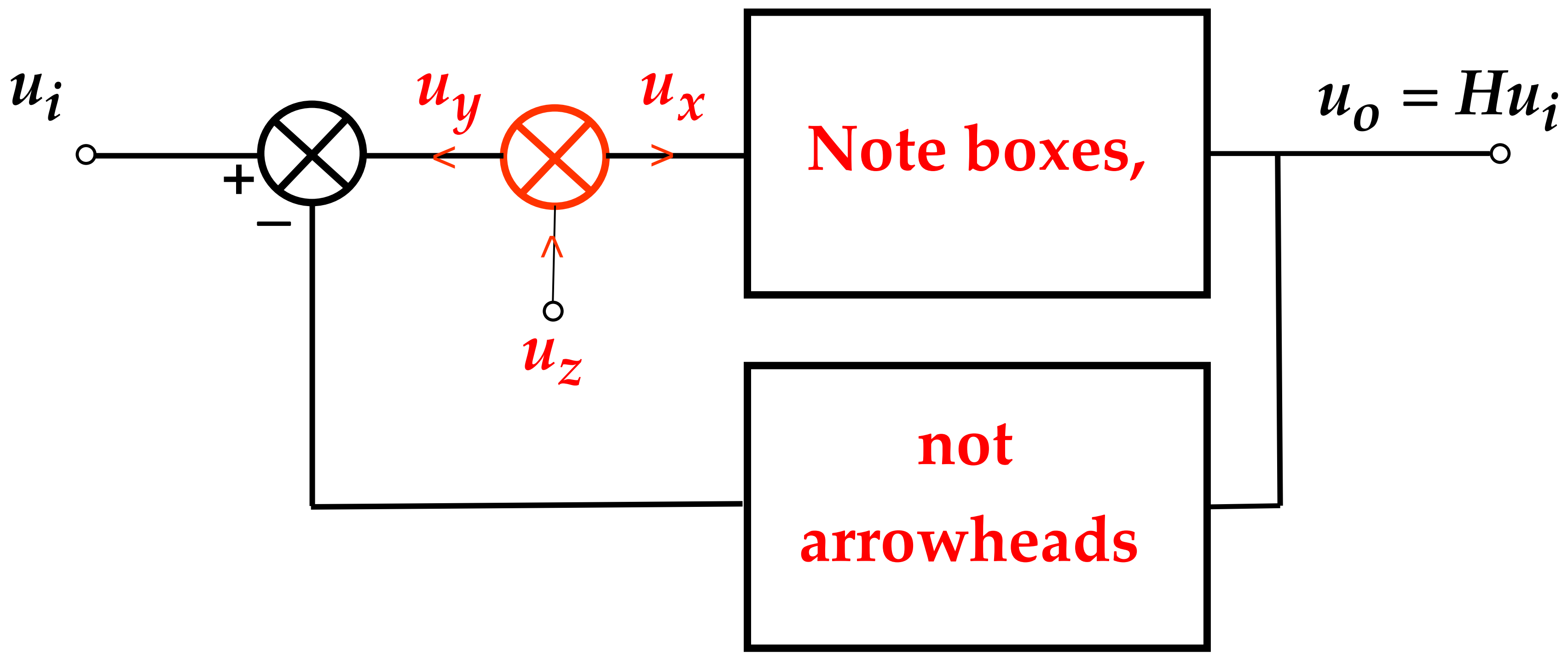
1. The test signal must be injected so that u_y is the error signal. This makes H_∞ equal to the Ideal Closed Loop Gain $1/K$, the reciprocal of the feedback ratio.

Test Signal Injection Configuration

In order for the *definitions* of the four second-level TFs H_∞, T, H_0, T_n to have the *interpretations* shown, it is necessary that the Test Signal Injection Configuration satisfy the conditions adopted in the preceding derivation:

2. The test signal must be injected inside the major loop, but outside any minor loops. This makes T represent the Principal Loop Gain.

The GFT Approach:



Inject a test signal u_z at error summing point.
The GFT gives all the second-level TFs directly in terms of the circuit elements.

Nonidealities:

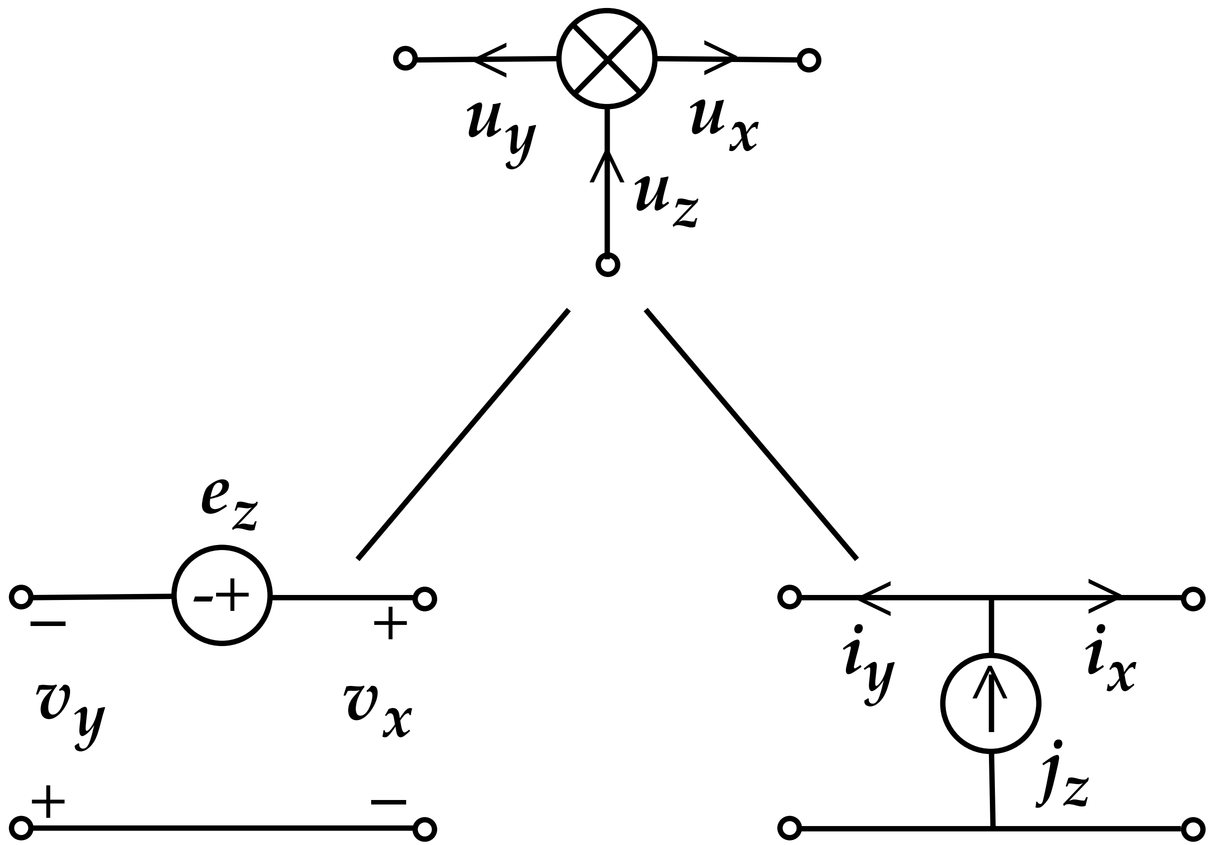
1. Reverse transmission through the feedback path

2. Reverse transmission through the forward path

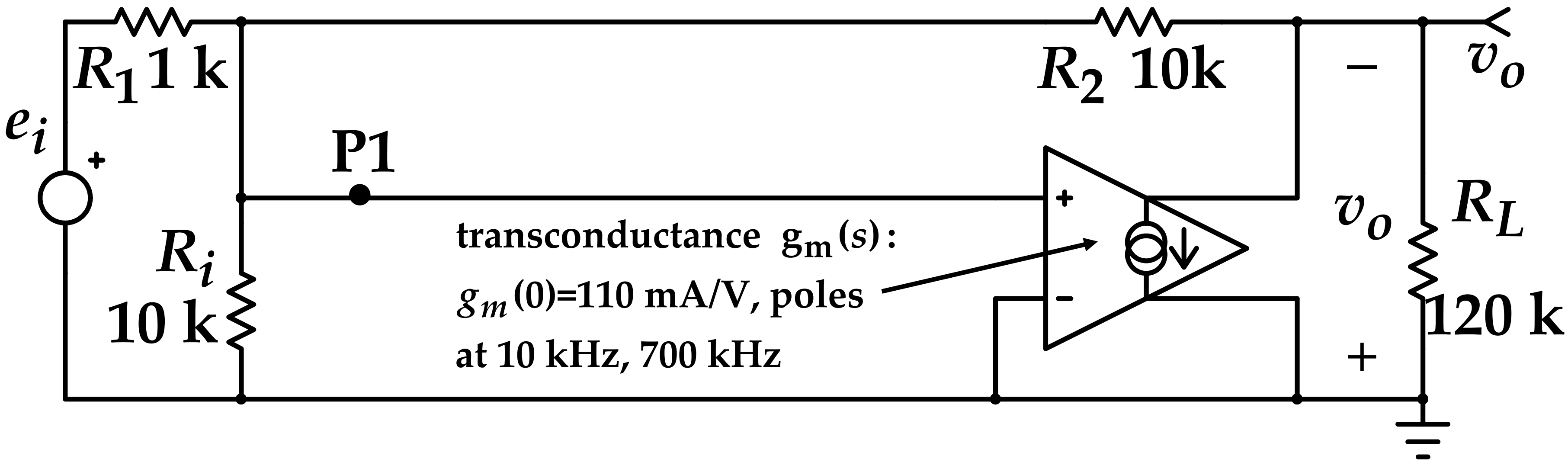
3. If both paths have reverse transmission, there is a nonzero reverse loop gain

All nonidealities are automatically accounted for by the GFT

Are u_x, u_y, u_z voltages or currents?

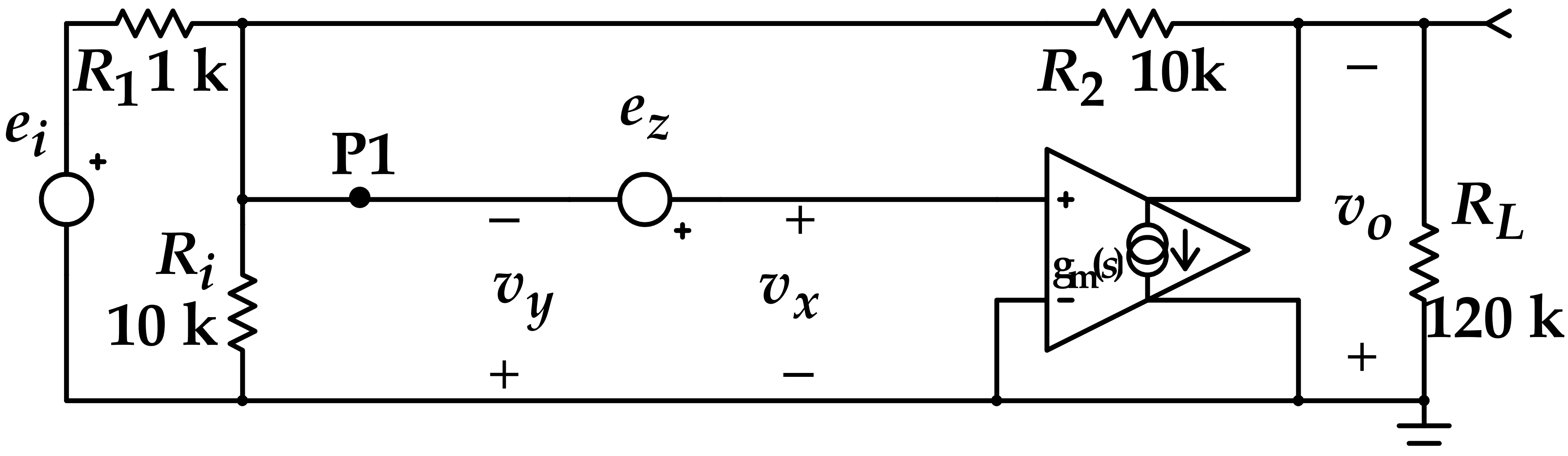


Inverting Opamp



We want to get $H_\infty = \dfrac{R_2}{R_1}$, **so inject** e_z **to add to the error voltage at P1:**

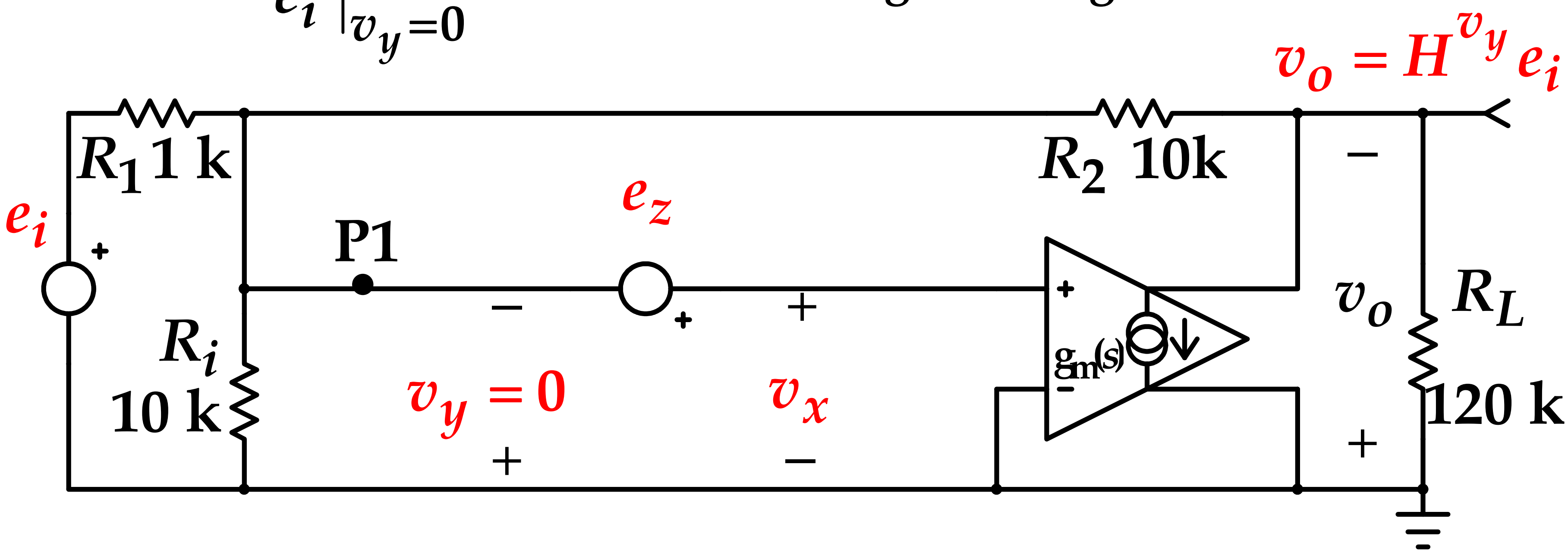
Ideal voltage injection point



An ideal voltage injection point is where v_y comes from an ideal (zero impedance) voltage generator,
or **where v_x looks into an infinite impedance.**

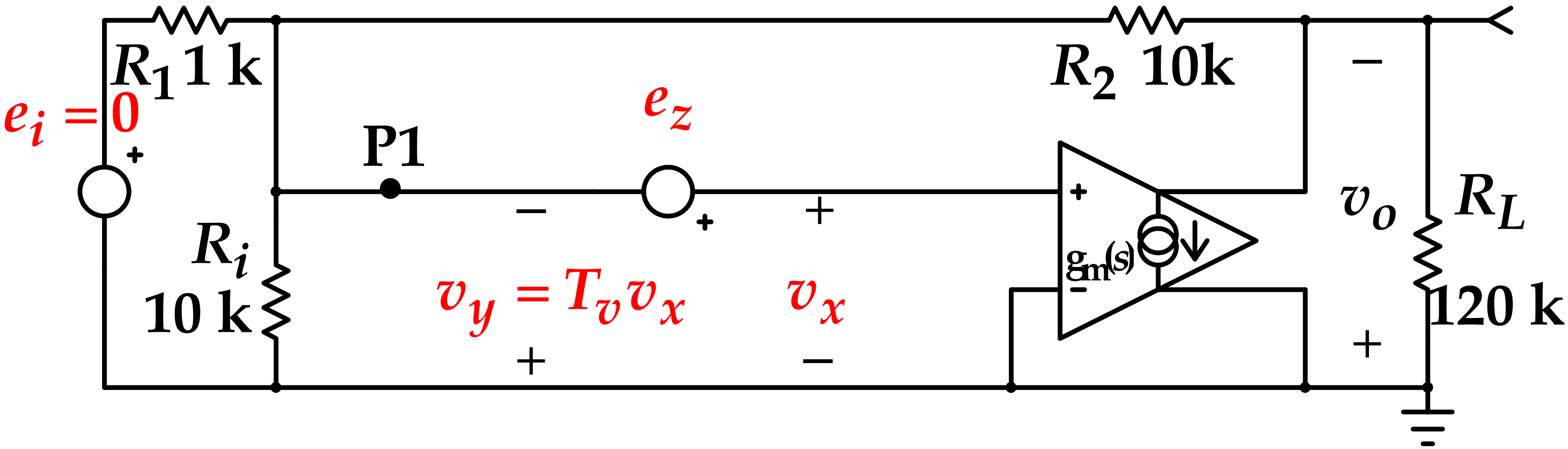
$$H_\infty = \left.\frac{v_o}{e_i}\right|_{v_y=0} \equiv H^{v_y}$$

A superscript indicates the signal being nulled



$$H^{v_y} = \frac{R_2}{R_1} = 10 \Rightarrow 20 \text{ dB}$$

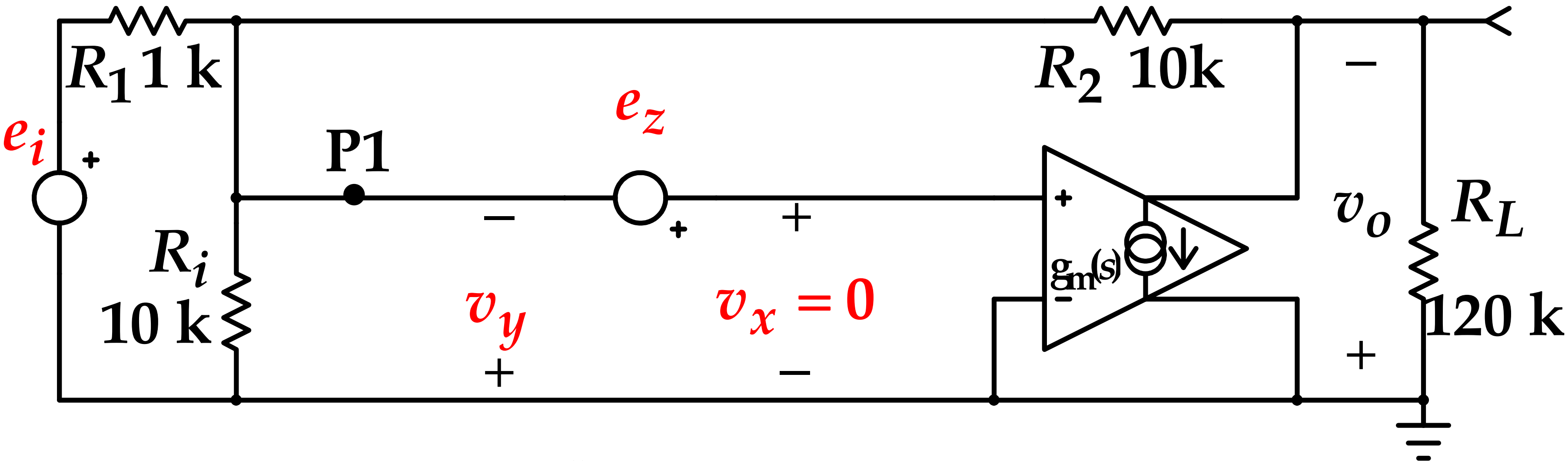
$$T = \left.\frac{v_y}{v_x}\right|_{e_i=0} \equiv T_v$$



$$T_v(0) = g_m(0)[R_L \| (R_1 \| R_i + R_2)] \frac{R_1 \| R_i}{R_1 \| R_i + R_2} = 92 \Rightarrow 39.2 \text{ dB}$$

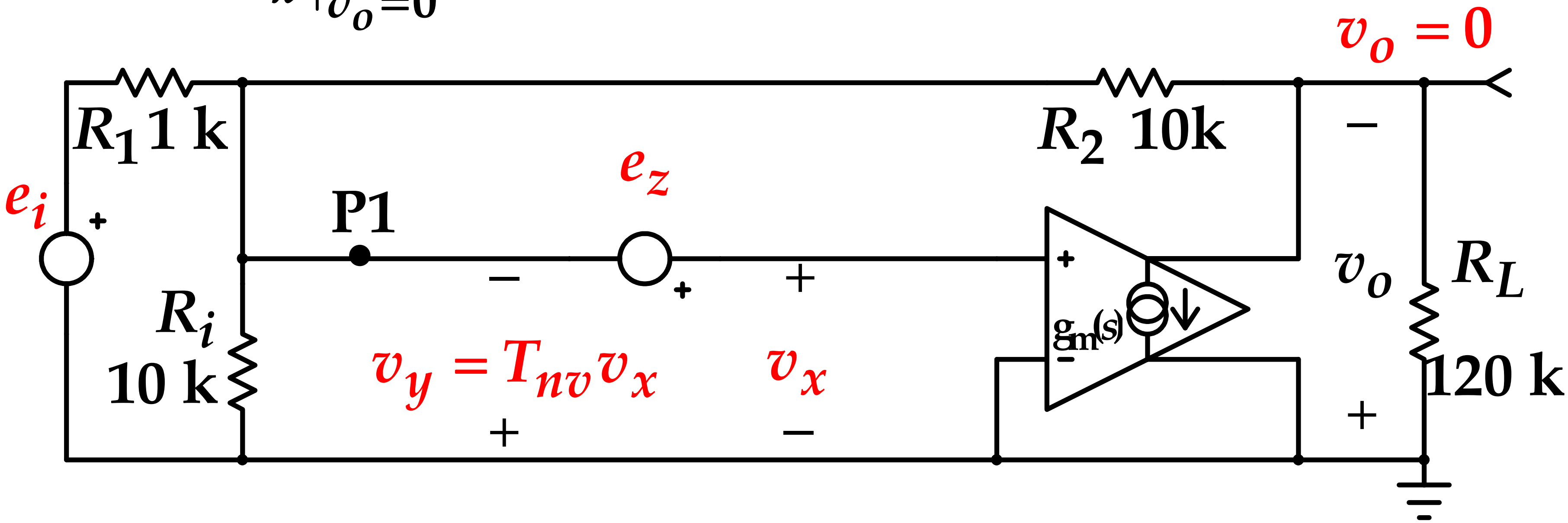
$$H_0 = \left.\frac{v_o}{e_i}\right|_{v_x=0} \equiv H^{v_x}$$

$v_o = H^{v_x} e_i$



$$H^{v_x} = \frac{R_L}{R_2 + R_L} \frac{R_i \| (R_2 + R_L)}{R_1 + R_i \| (R_2 + R_L)} = 0.83 \Rightarrow -1.6 \text{ dB}$$

$$T_n = \left.\frac{v_y}{v_x}\right|_{v_o=0} \equiv T_{nv}$$



$$T_{nv}(0) = g_m(0) R_2 = 1100 \Rightarrow 60.8 \text{ dB}$$

The redundancy relation $\dfrac{T_{nv}(0)}{T_v(0)} = \dfrac{H^{v_y}}{H^{v_x}} = 12.0$

is verified.

Note that T_n is a simpler result from a shorter calculation than is H_0, which is usually the case.

Therefore, it may be preferable to use the second of the two versions of H:

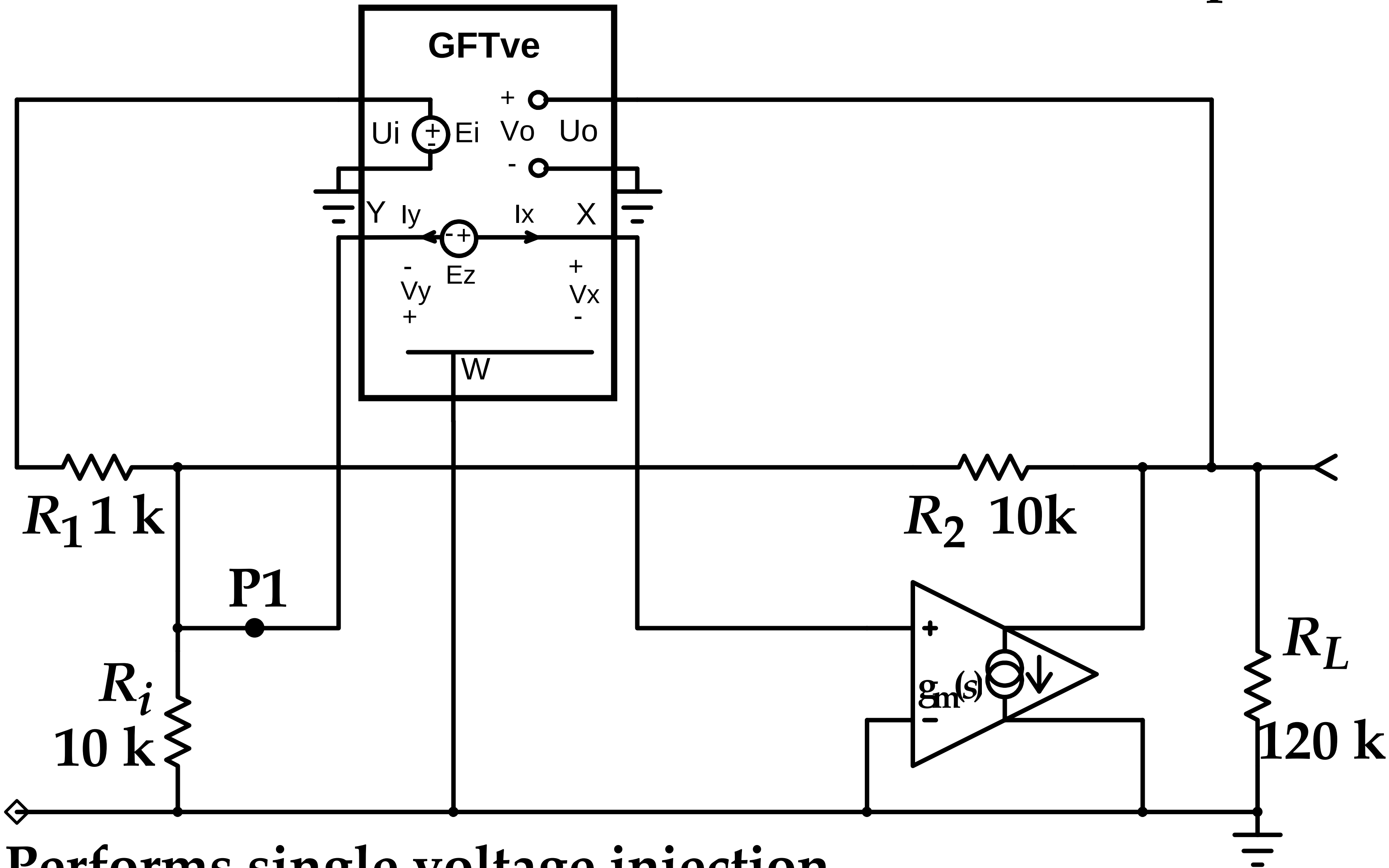
First version:

$$H = H_\infty \frac{T}{1+T} + H_0 \frac{1}{1+T} H = H_\infty D + H_0 D_0$$

Second version:

$$H = H_\infty \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}} = H_\infty D D_n$$

Intusoft ICAP/4 Circuit Simulator with GFT Template



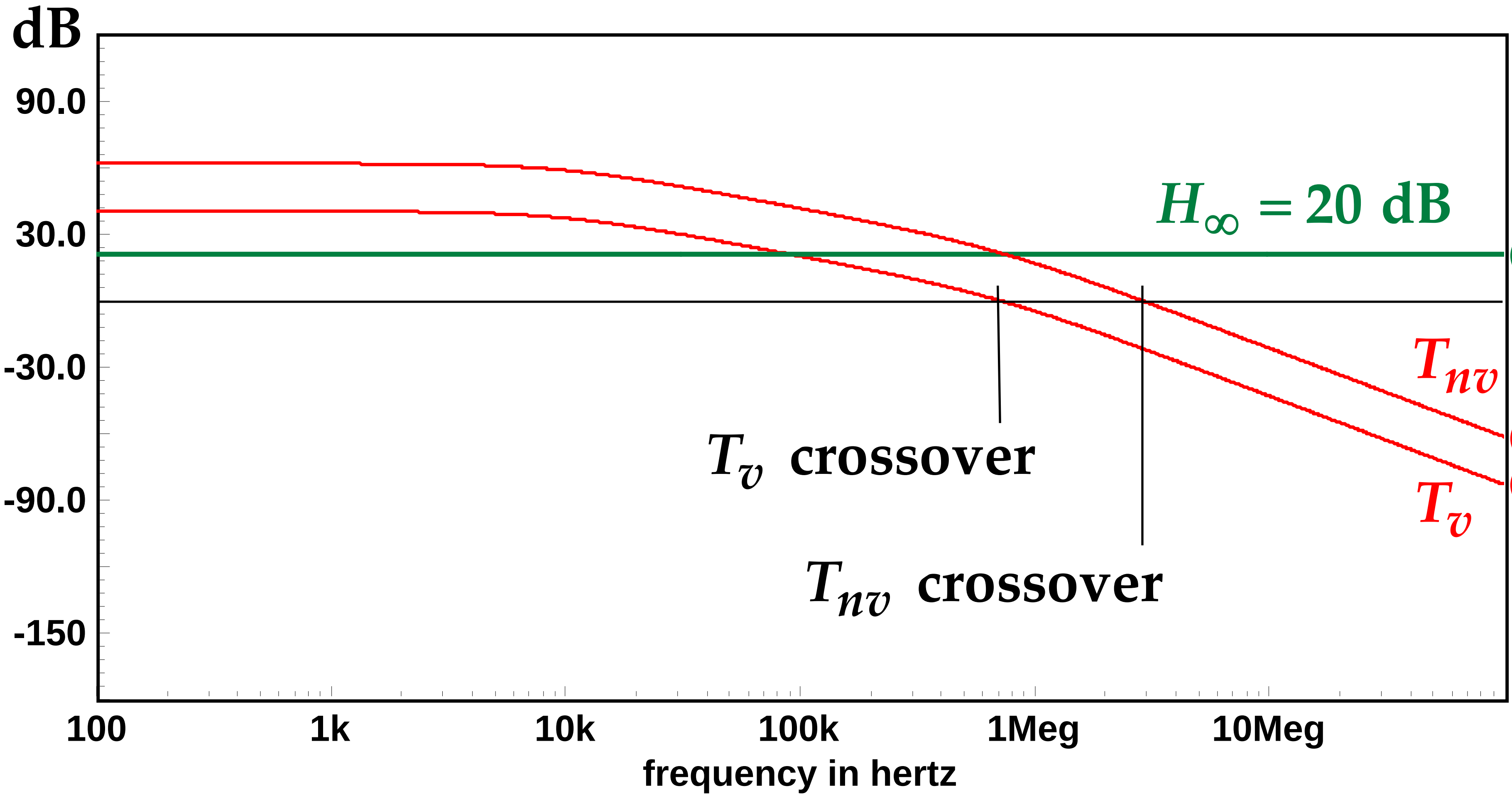
Performs single voltage injection at the ideal point P1

The GFT Template calculates all the second-level TFs H_∞, T, H_0, T_n and inserts them into any version of the first-level TF H, for comparison with the directly calculated H (the "normal" closed-loop gain with no injected test signal).

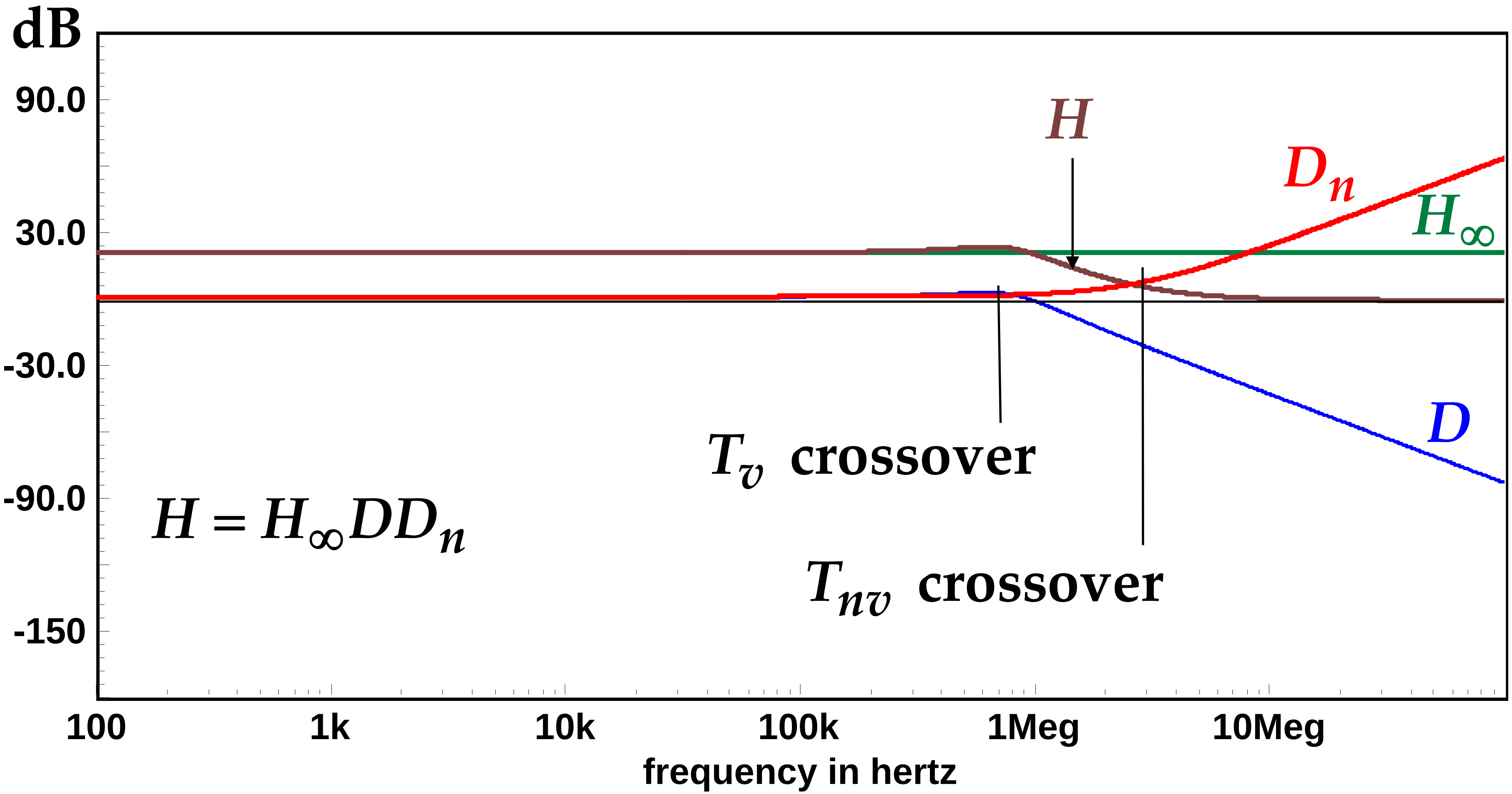
All TFs can be displayed as magnitude and phase Bode plots.

The results for the above circuit were those used previously to illustrate the four Feedback Amplifier Design Steps.

Check that H is what was expected



Check that H[calculated] is same as H[direct simulation]



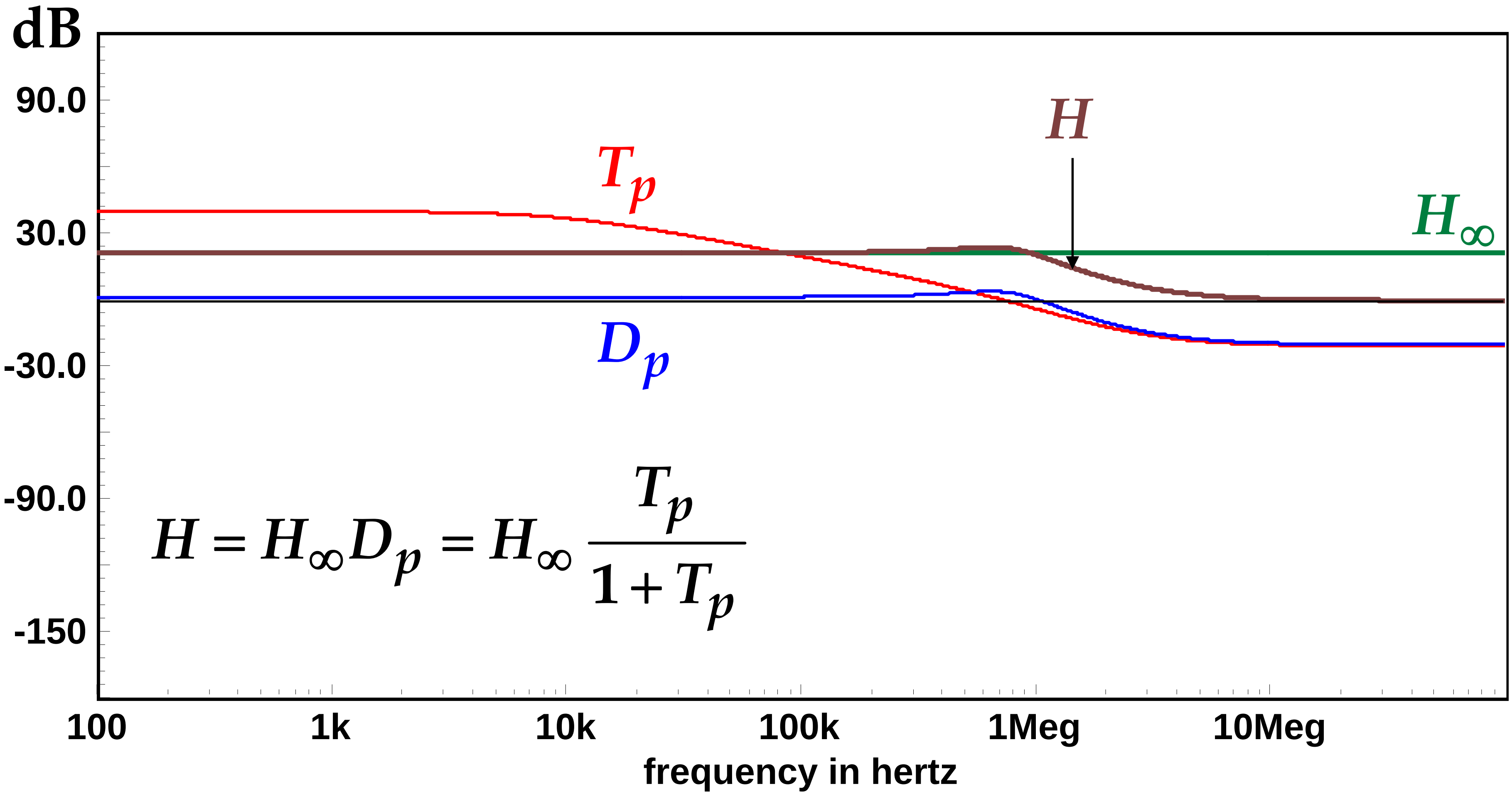
A third version of H can be found by forcing the result to be of the form $H = H_\infty \frac{1}{1+1/T_p}$**:**

$$H = H_\infty D D_n = H_\infty \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}} = H_\infty \frac{1+\frac{1}{T_n}}{1+\frac{1}{T_n}+\frac{1}{T}-\frac{1}{T_n}}$$

$$= H_\infty \frac{1}{1+\frac{T_n}{1+T_n}\left(\frac{1}{T}-\frac{1}{T_n}\right)} = H_\infty \frac{1}{1+\frac{1}{T_p}} = H_\infty D_p$$

where $T_p \equiv \frac{D_n}{\frac{1}{T}-\frac{1}{T_n}}$ **is a "pseudo loop gain"**

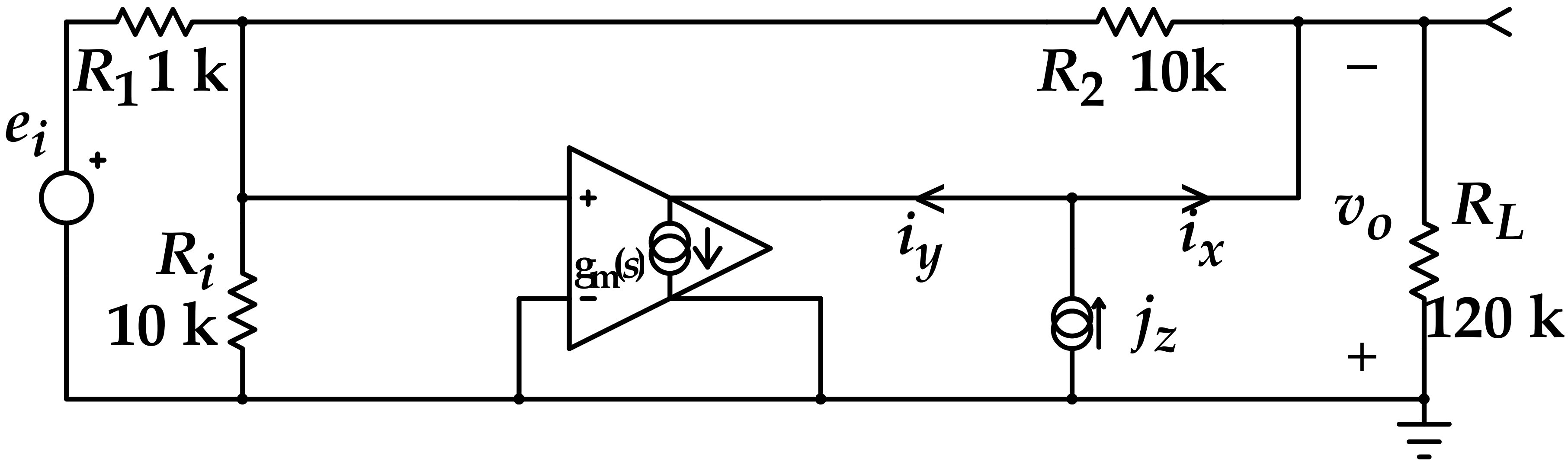
and $D_p \equiv \frac{T_p}{1+T_p}$ **is a "pseudo discrepancy factor"**



$$H = H_\infty D_p = H_\infty \frac{T_p}{1+T_p}$$

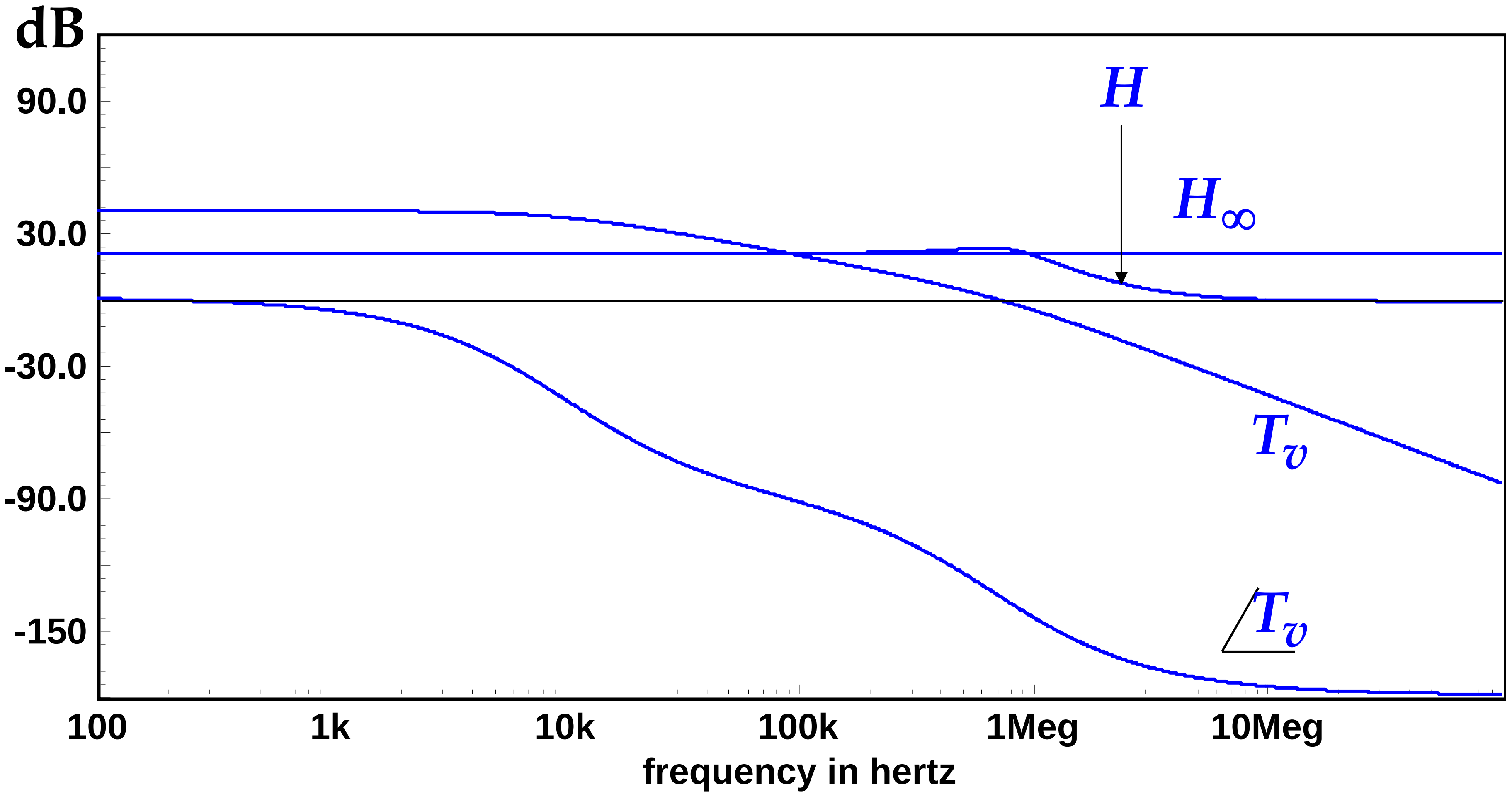
The pseudo loop gain T_p may be hard to relate to the circuit model, because it contains both T and T_n

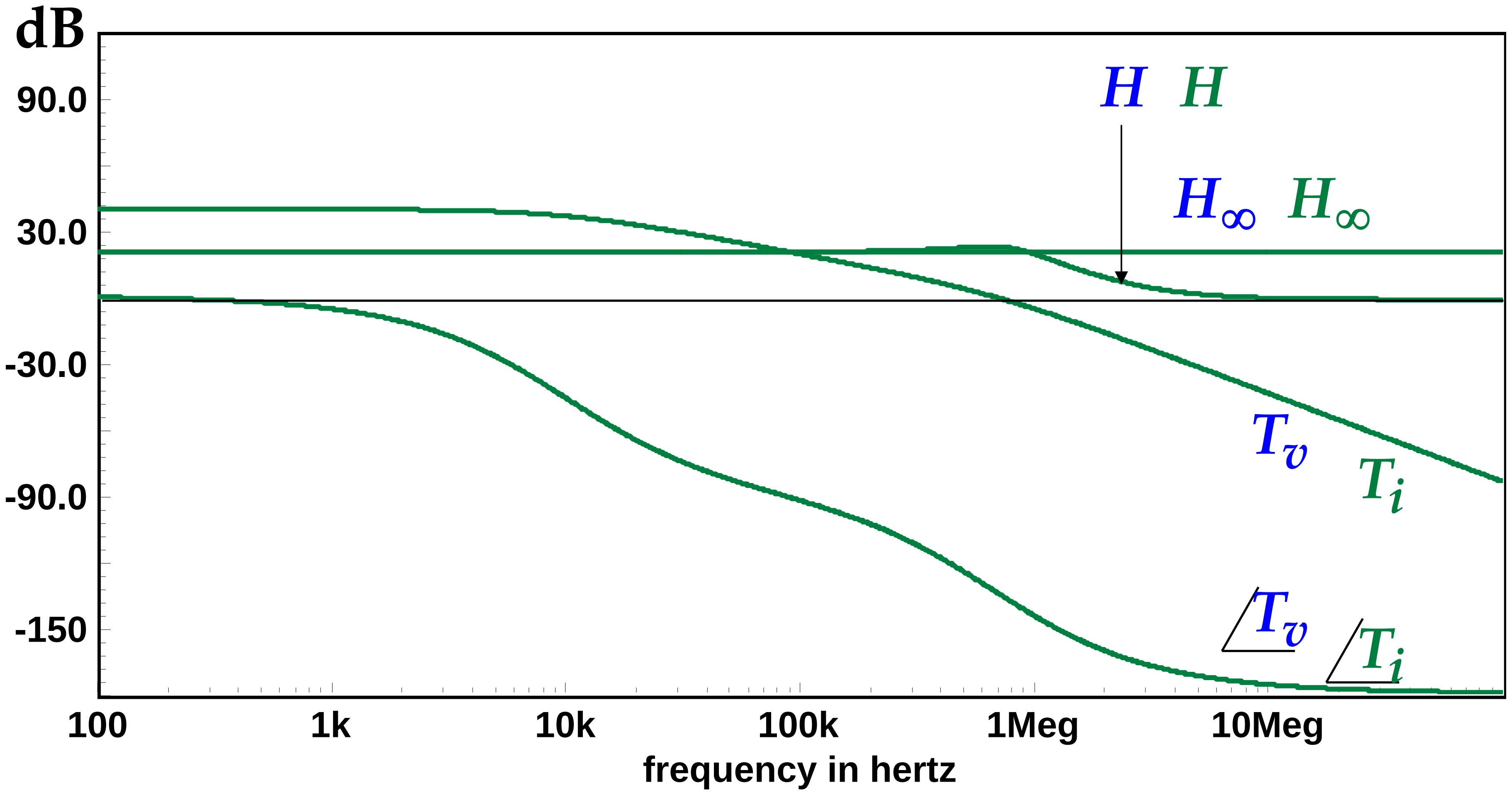
Ideal current injection point



An ideal current injection point is where i_y comes from an ideal (infinite impedance) current generator,
***or* where i_x looks into a zero impedance.**

Results are exactly the same as for the ideal voltage injection point.


dB
90.0
30.0
-30.0
-90.0
-150
H
H∞
Tv
Tv
100
1k
10k
100k
1Meg
10Meg
frequency in hertz


dB
90.0
30.0
-30.0
-90.0
-150
H H
H∞ H∞
Tv
Ti
Tv
Ti
100
1k
10k
100k
1Meg
10Meg
frequency in hertz

12. DNTI AND THE 2EET:

Double Null Triple Injection and the Two Extra Element Theorem

A short and easy way to find the poles and zeros of a circuit containing two reactances

Benefits of the EET:

1. It is very easy to use.
2. It saves a lot of work.
3. The result is automatically in Low-Entropy form.

Bottom Line:

Two contexts in which the EET is particularly useful:

1. A transfer function has already been analyzed, and later an extra element is to be added to the model: the EET avoids the analysis having to be restarted from scratch, since only the two dpi's have to be calculated (on the original model) in order to evaluate the required correction factor upon the already known transfer function.

2. A transfer function is to be analyzed for the first time: if one element is designated as "extra," the analysis can be performed on the simpler model in the absence of the designated element, and the result modified by the EET correction factor upon restoration of the "extra" element.

The Extra Element Theorem (EET) can be used successively to add one element after another.

For example, a first extra element Z_1 gives (for reference gain with all extra elements infinite):

$$A\Big|_{Z_1} = A_{ref} \frac{1 + \frac{Z_{n1}}{Z_1}}{1 + \frac{Z_{d1}}{Z_1}}$$

Then, a second extra element Z_2 can be added:

$$A\Big|_{Z_1, Z_2} = A_{ref} \frac{1 + \frac{Z_{n1}}{Z_1}}{1 + \frac{Z_{d1}}{Z_1}} \frac{1 + \frac{Z_{n2}}{Z_2}}{1 + \frac{Z_{d2}}{Z_2}}$$

Of course, the driving-point impedances (dpi's) Z_{n2} and Z_{d2} must be calculated in the presence of Z_1.

It would be useful to have a theorem that would give $A|_{Z_1,Z_2}$ in terms of A_{ref} when both extra elements Z_1 and Z_2 are added simultaneously, in which the dpi's for each extra element would be calculated in the absence of the other extra element.

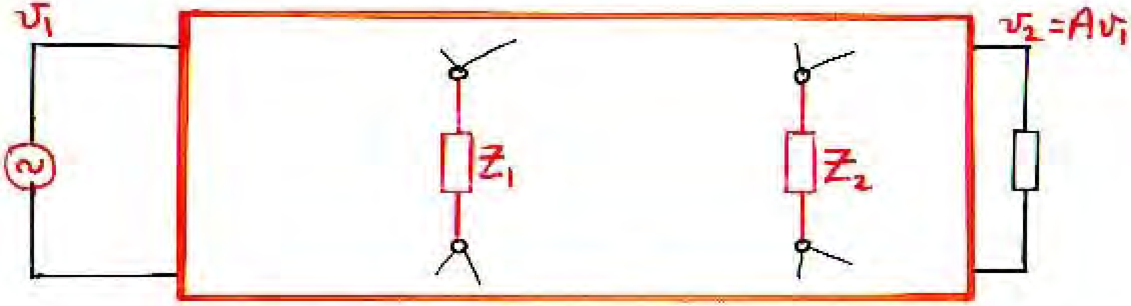
Obviously, in the special case that each dpi is independent of the other extra element (no interaction between the extra elements), the result should be the same as obtained by adding each element independently of the other. (Example: addition of the two coupling capacitances C_1 and C_3 to the common-emitter—emitter-follower amplifier pair.)

In the case in which there is interaction between the two extra elements, a generalized Two Extra Element Theorem (2EET) is needed. (Example: addition of the coupling capacitance C_1 and the emitter bypass capacitance C_2 to the common-emitter stage.)

The Two Extra Element Theorem can be obtained by successive application of the Extra Element Theorem. The derivation, not given here, leads to the following results.

The gain A of a system in the presence of two elements Z_1 and Z_2 can be calculated as a correction factor upon the gain when the two elements have certain "reference" values.

If the reference values are $Z_1 = \infty$ and $Z_2 = \infty$, the result is:

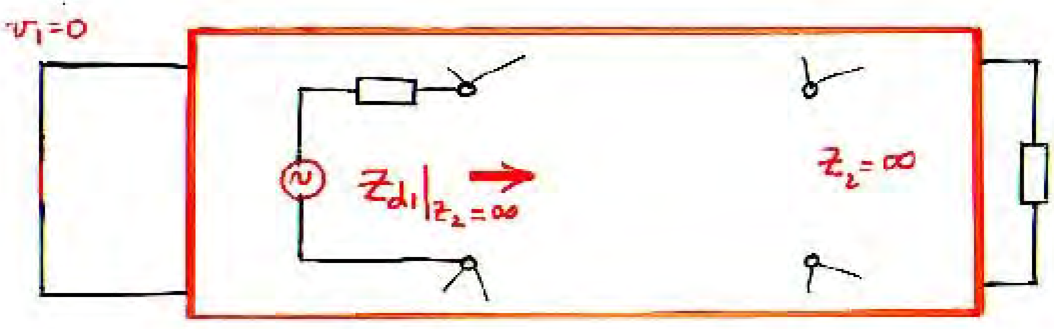


$$A = A\Big|_{\substack{Z_1=\infty \\ Z_2=\infty}} \frac{1 + \dfrac{Z_{n1}|_{Z_2=\infty}}{Z_1} + \dfrac{Z_{n2}|_{Z_1=\infty}}{Z_2} + K_n \dfrac{Z_{n1}|_{Z_2=\infty}}{Z_1}\dfrac{Z_{n2}|_{Z_1=\infty}}{Z_2}}{1 + \dfrac{Z_{d1}|_{Z_2=\infty}}{Z_1} + \dfrac{Z_{d2}|_{Z_1=\infty}}{Z_2} + K_d \dfrac{Z_{d1}|_{Z_2=\infty}}{Z_1}\dfrac{Z_{d2}|_{Z_1=\infty}}{Z_2}}$$

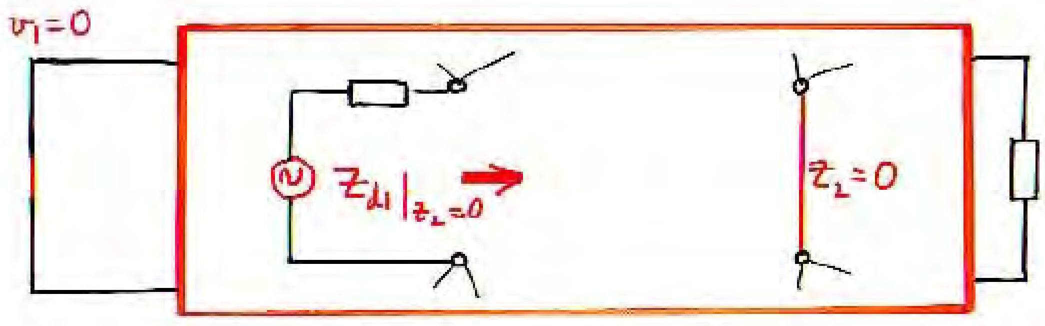
where

$$K_n \equiv \frac{Z_{n1}|_{Z_2=0}}{Z_{n1}|_{Z_2=\infty}} = \frac{Z_{n2}|_{Z_1=0}}{Z_{n2}|_{Z_1=\infty}} \qquad K_d \equiv \frac{Z_{d1}|_{Z_2=0}}{Z_{d1}|_{Z_2=\infty}} = \frac{Z_{d2}|_{Z_1=0}}{Z_{d2}|_{Z_1=\infty}}$$

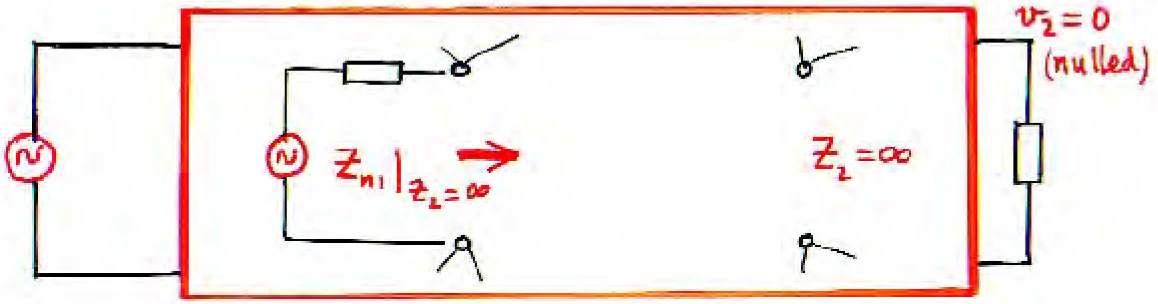
The Z_n's and Z_d's are the driving point impedances "seen" by the "extra elements" Z_1 and Z_2, calculated under various conditions as follows:



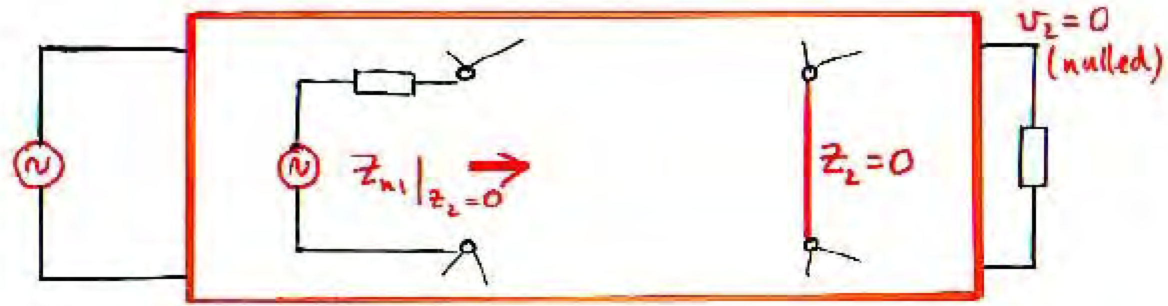
Single injection



Single injection



Null double injection



Null double injection

There are four Z_n's and one redundancy constraint. The result can be expressed in terms of any three of the Z_n's.

The same statement holds for the four Z_d's.

For the 2EET in the above form, the reference gain is $A\Big|_{\substack{Z_1=\infty \\ Z_2=\infty}}$. The Theorem can be expressed in terms of any three other reference gains,

$$A\Big|_{\substack{Z_1=0 \\ Z_2=0}}, \quad A\Big|_{\substack{Z_1=\infty \\ Z_2=0}}, \quad A\Big|_{\substack{Z_1=0 \\ Z_2=\infty}}$$

as follows:

$$A = A\Big|_{\substack{Z_1=0\\Z_2=0}} \frac{1 + \dfrac{Z_1}{Z_{n1}|_{Z_2=0}} + \dfrac{Z_2}{Z_{n2}|_{Z_1=0}} + K_n \dfrac{Z_1}{Z_{n1}|_{Z_2=0}} \dfrac{Z_2}{Z_{n2}|_{Z_1=0}}}{1 + \dfrac{Z_1}{Z_{d1}|_{Z_2=0}} + \dfrac{Z_2}{Z_{d2}|_{Z_1=0}} + K_d \dfrac{Z_1}{Z_{d1}|_{Z_2=0}} \dfrac{Z_2}{Z_{d2}|_{Z_1=0}}}$$

$$A = A\Big|_{\substack{Z_1=\infty\\Z_2=0}} \frac{1 + \dfrac{Z_{n1}|_{Z_2=0}}{Z_1} + \dfrac{Z_2}{Z_{n2}|_{Z_1=\infty}} + \dfrac{1}{K_n} \dfrac{Z_{n1}|_{Z_2=0}}{Z_1} \dfrac{Z_2}{Z_{n2}|_{Z_1=\infty}}}{1 + \dfrac{Z_{d1}|_{Z_2=0}}{Z_1} + \dfrac{Z_2}{Z_{d2}|_{Z_1=\infty}} + \dfrac{1}{K_d} \dfrac{Z_{d1}|_{Z_2=0}}{Z_1} \dfrac{Z_2}{Z_{d2}|_{Z_1=\infty}}}$$

$$A = A\Big|_{\substack{Z_1=0\\Z_2=\infty}} \frac{1 + \dfrac{Z_1}{Z_{n1}|_{Z_2=\infty}} + \dfrac{Z_{n2}|_{Z_1=0}}{Z_2} + \dfrac{1}{K_n} \dfrac{Z_1}{Z_{n1}|_{Z_2=\infty}} \dfrac{Z_{n2}|_{Z_1=0}}{Z_2}}{1 + \dfrac{Z_1}{Z_{d1}|_{Z_2=\infty}} + \dfrac{Z_{d2}|_{Z_1=0}}{Z_2} + \dfrac{1}{K_d} \dfrac{Z_1}{Z_{d1}|_{Z_2=\infty}} \dfrac{Z_{d2}|_{Z_1=0}}{Z_2}}$$

Note the symmetry in all four versions of the theorem

In each version, the dpi's for each element are calculated with the other element at its reference value, except to find the K's.

If there is no interaction between the two extra elements, the dpi's seen by each element are independent of the other element, so the "interaction parameters" K_n and K_d are each equal to unity:

$$\text{Interaction Parameter } K_n \equiv \frac{Z_{n1}|_{Z_2=0}}{Z_{n1}|_{Z_2=\infty}} = \frac{Z_{n2}|_{Z_1=0}}{Z_{n2}|_{Z_1=\infty}}$$

$$= 1 \text{ if no interaction}$$

$$\text{Interaction Parameter } K_d \equiv \frac{Z_{d1}|_{Z_2=0}}{Z_{d1}|_{Z_2=\infty}} = \frac{Z_{d2}|_{Z_1=0}}{Z_{d2}|_{Z_1=\infty}}$$

$$= 1 \text{ if no interaction}$$

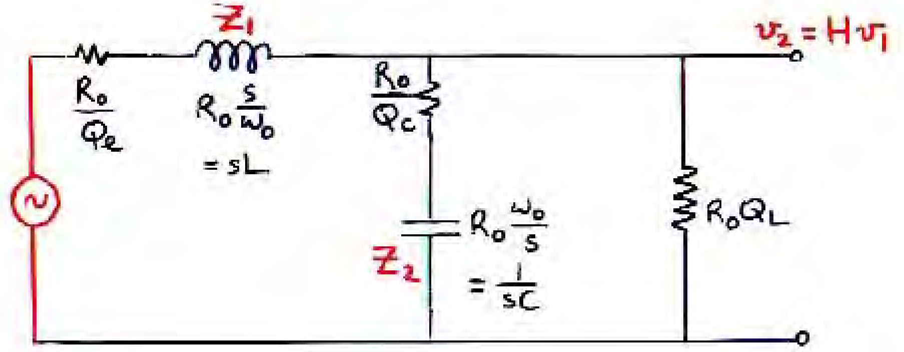
Then, if $K_n = 1$ and $K_d = 1$, the 2EET factors exactly.
For example, for the version in which the reference gain is $A\big|_{\substack{Z_1=\infty \\ Z_2=\infty}}$:

$$A = A\Big|_{\substack{Z_1=\infty \\ Z_2=\infty}} \frac{1 + \dfrac{Z_{n1}|_{Z_2=\infty}}{Z_1} + \dfrac{Z_{n2}|_{Z_1=\infty}}{Z_2} + K_n \dfrac{Z_{n1}|_{Z_2=\infty}}{Z_1}\dfrac{Z_{n2}|_{Z_1=\infty}}{Z_2}}{1 + \dfrac{Z_{d1}|_{Z_2=\infty}}{Z_1} + \dfrac{Z_{d2}|_{Z_1=\infty}}{Z_2} + K_d \dfrac{Z_{d1}|_{Z_2=\infty}}{Z_1}\dfrac{Z_{d2}|_{Z_1=\infty}}{Z_2}}$$

$$\overset{K_n=1}{\underset{K_d=1}{=}} A\Big|_{\substack{Z_1=\infty \\ Z_2=\infty}} \frac{1 + \dfrac{Z_{n1}|_{Z_2=\infty}}{Z_1}}{1 + \dfrac{Z_{d1}|_{Z_2=\infty}}{Z_1}} \; \frac{1 + \dfrac{Z_{n2}|_{Z_1=\infty}}{Z_2}}{1 + \dfrac{Z_{d2}|_{Z_1=\infty}}{Z_2}}$$

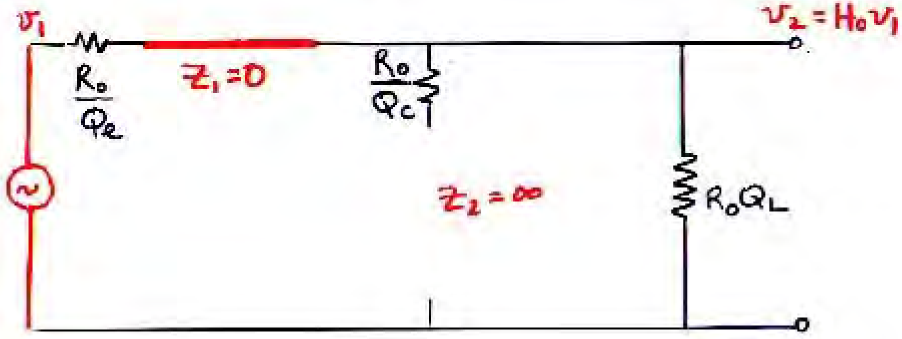
This result is the same as would be obtained by applying the single EET successively for each extra element.

Both the Extra Element Theorem and the Two Extra Element Theorem can be used to advantage to reduce the work in analysis of a system, by identification of one or more elements as "extra." The advantage is especially great when the extra elements are reactances and the circuit is purely resistive when the extra elements have their reference values: the Z_n's and Z_d's are then resistive, and the correction factor gives the corner frequencies directly.

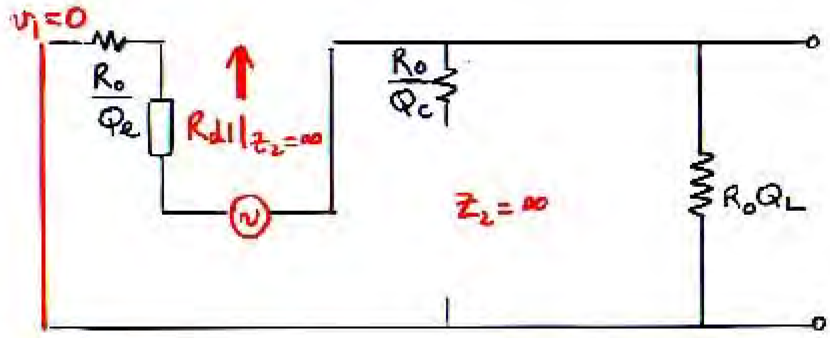


Identify L and C as "extra elements", with reference values $Z_1 = 0$ and $Z_2 = \infty$, so that the reference transfer function is the low-frequency value $H_o = H\Big|_{\substack{Z_1=0 \\ Z_2=\infty}}$.

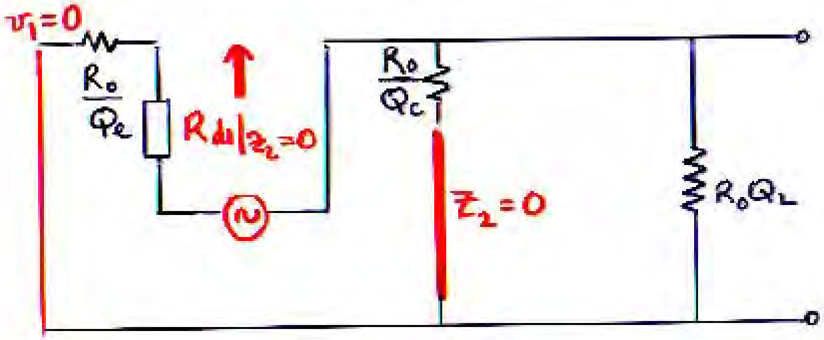
All the dpi's are resistances.



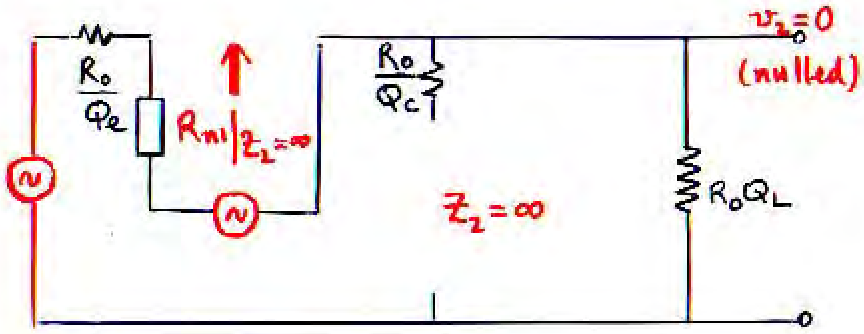
$$H_o = H\Big|_{\substack{Z_1=0\\ Z_2=\infty}} = \frac{Q_L}{\frac{1}{Q_e} + Q_L} = \frac{1}{1 + 1/Q_e Q_L}$$



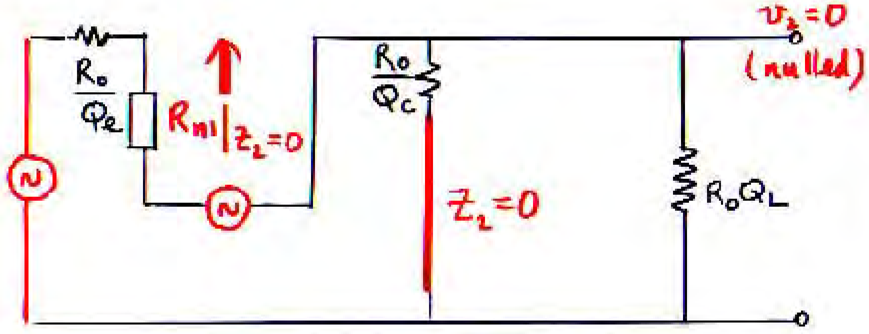
$$R_{d1}\Big|_{Z_2=\infty} = \frac{R_o}{Q_e} + R_o Q_L = R_o Q_L \left(1 + \frac{1}{Q_e Q_L}\right)$$



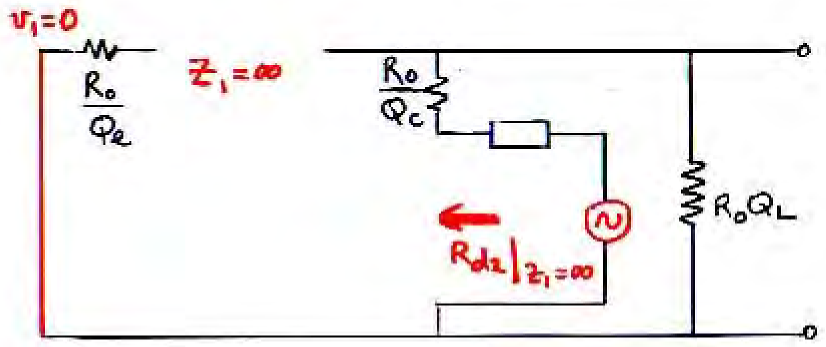
$$R_{d1}\Big|_{z_2=0} = R_o\left(\frac{1}{Q_e} + \frac{\frac{1}{Q_c}Q_L}{\frac{1}{Q_c}+Q_L}\right) = R_o\left(\frac{1}{Q_e} + \frac{1}{Q_c}\,\frac{1}{1+1/Q_cQ_L}\right)$$



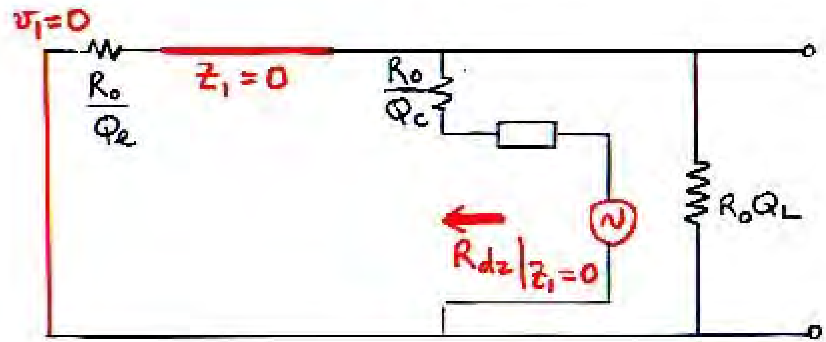
$$R_{n1}\Big|_{Z_2=\infty} = \infty$$



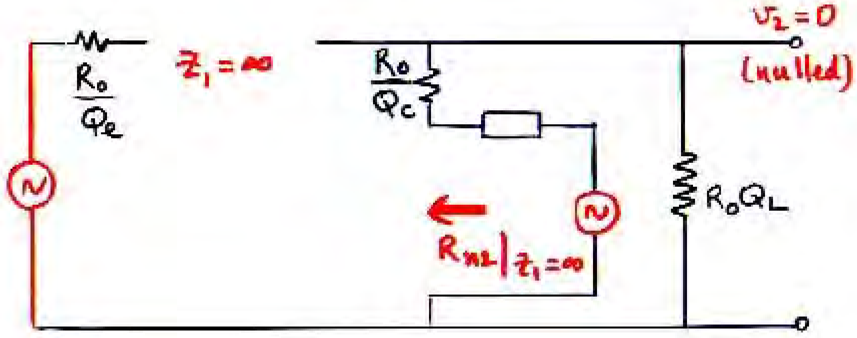
$$R_{n1}\Big|_{z_2=0} = \infty$$



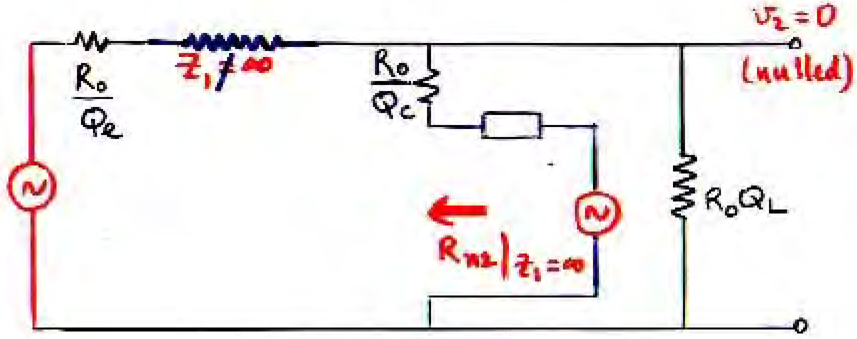
$$R_{d2}\Big|_{z_1=\infty} = R_o\left(\frac{1}{Q_c}+Q_L\right) = R_oQ_L\left(1+\frac{1}{Q_cQ_L}\right)$$



$$R_{dz}\Big|_{z_1=0} = R_o\left(\frac{1}{Q_c} + \frac{\frac{1}{Q_e}Q_L}{\frac{1}{Q_e}+Q_L}\right) = R_o\left(\frac{1}{Q_c} + \frac{1}{Q_e}\,\frac{1}{1+1/Q_eQ_L}\right)$$



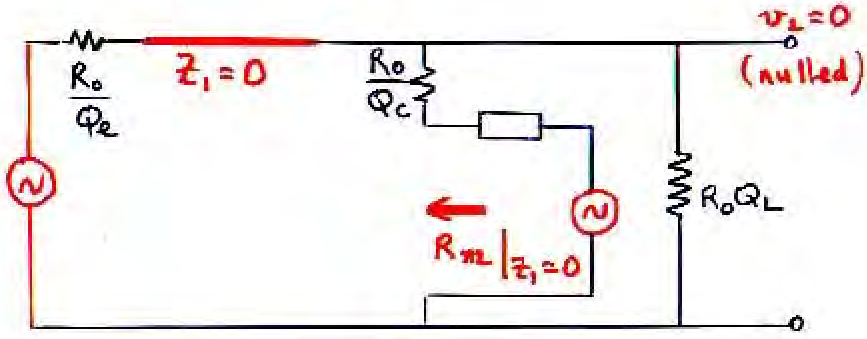
$$R_{n2}\big|_{Z_1=\infty} = ?$$



$$R_{n2}\big|_{Z_1=\infty} = ?$$

When a driving point impedance is indeterminate, replace the extra element by an arbitrary impedance. Hence:

$$R_{n2}\big|_{Z_1\to\infty} = \frac{R_o}{Q_c}$$



$$R_{n2}\big|_{Z_1=0} = \frac{R_o}{Q_c}$$

Interaction parameters:

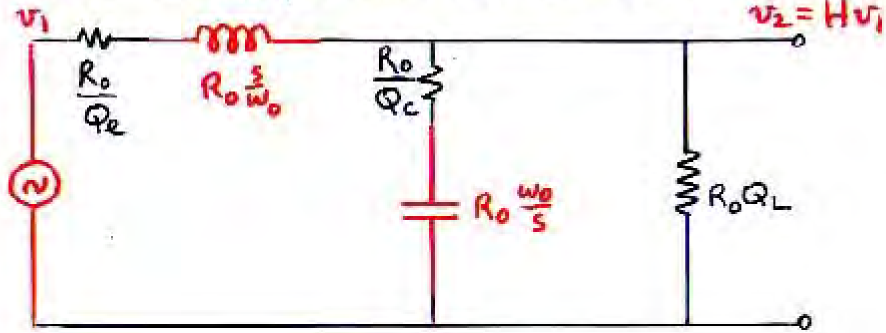
$$K_n = \frac{R_{n1}|_{Z_2=0}}{R_{n1}|_{Z_2=\infty}} = \frac{\infty}{\infty} = ? \qquad K_n = \frac{R_{n2}|_{Z_1=0}}{R_{n2}|_{Z_1=\infty}} = \frac{R_o/Q_c}{R_o/Q_c} = 1$$

$$K_d = \frac{R_{d1}|_{Z_2=0}}{R_{d1}|_{Z_2=\infty}} = \frac{\frac{1}{Q_e} + \frac{1}{Q_c}\,\frac{1}{1+1/Q_cQ_L}}{Q_L\left(1+\frac{1}{Q_eQ_L}\right)}$$

These are the same

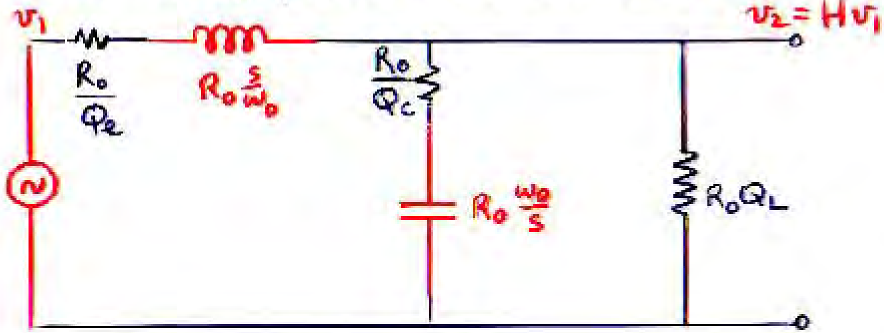
$$K_d = \frac{R_{d2}|_{Z_1=0}}{R_{d2}|_{Z_1=\infty}} \qquad \frac{\frac{1}{Q_c} + \frac{1}{Q_e}\,\frac{1}{1+1/Q_eQ_L}}{Q_L\left(1+\frac{1}{Q_cQ_L}\right)}$$

The result in the presence of L and C is



$$H = H\bigg|_{\substack{Z_1=0\\ Z_2=\infty}} \frac{1 + \dfrac{Z_1}{R_{n1}\big|_{Z_2=\infty}} + \dfrac{R_{n2}\big|_{Z_1=0}}{Z_2} + \dfrac{1}{K_n}\dfrac{Z_1}{R_{n1}\big|_{Z_2=\infty}}\dfrac{R_{n2}\big|_{Z_1=0}}{Z_2}}{1 + \dfrac{Z_1}{R_{d1}\big|_{Z_2=\infty}} + \dfrac{R_{d2}\big|_{Z_1=0}}{Z_2} + \dfrac{1}{K_d}\dfrac{Z_1}{R_{d1}\big|_{Z_2=\infty}}\dfrac{R_{d2}\big|_{Z_1=0}}{Z_2}}$$

The result in the presence of L and C is



$$H = H\Big|_{\substack{Z_1=0\\ Z_2=\infty}} \frac{1 + \dfrac{Z_1}{R_{n1}|_{Z_2=\infty}} + \dfrac{R_{n2}|_{Z_1=0}}{Z_2} + \dfrac{1}{K_n}\dfrac{Z_1}{R_{n1}|_{Z_2=\infty}}\dfrac{R_{n2}|_{Z_1=0}}{Z_2}}{1 + \dfrac{Z_1}{R_{d1}|_{Z_2=\infty}} + \dfrac{R_{d2}|_{Z_1=0}}{Z_2} + \dfrac{1}{K_d}\dfrac{Z_1}{R_{d1}|_{Z_2=\infty}}\dfrac{R_{d2}|_{Z_1=0}}{Z_2}}$$

$$= \frac{1}{1+\frac{1}{Q_eQ_L}} \frac{1 + \dfrac{Z_1}{\infty} + \dfrac{R_o}{Q_C Z_2} + \dfrac{Z_1}{\infty}\dfrac{R_o}{Q_C Z_2}}{1 + \dfrac{Z_1}{R_oQ_L\left(1+\frac{1}{Q_eQ_L}\right)} + \dfrac{R_o\left(\frac{1}{Q_C} + \frac{1}{Q_e}\frac{1}{1+1/Q_eQ_L}\right)}{Z_2} + \dfrac{1+\frac{1}{Q_eQ_C}}{1+\frac{1}{Q_eQ_L}}\dfrac{Z_1}{Z_2}}$$

$$\frac{1}{Q_cQ_L}$$

$$= \frac{1}{1+\frac{1}{Q_eQ_L}} \; \frac{1+\frac{1}{Q_c}\left(\frac{s}{\omega_o}\right)}{1+\left(\frac{1}{Q_L\left(1+\frac{1}{Q_eQ_L}\right)}+\frac{1}{Q_c}+\frac{1}{Q_e}\frac{1}{1+\frac{1}{Q_eQ_L}}\right)\left(\frac{s}{\omega_o}\right)+\frac{1+\frac{1}{Q_eQ_c}}{1+\frac{1}{Q_eQ_L}}\left(\frac{s}{\omega_o}\right)^2}$$

$$\frac{1}{Q_cQ_L}$$

$$= \frac{1}{1+\frac{1}{Q_eQ_L}} \; \frac{1+\frac{1}{Q_c}\left(\frac{s}{\omega_o}\right)}{1+\frac{\frac{1}{Q_e}+\frac{1}{Q_L}+\frac{1+1/Q_eQ_L}{Q_c}}{1+\frac{1}{Q_eQ_L}}\left(\frac{s}{\omega_o}\right)+\frac{1+\frac{1}{Q_eQ_c}}{1+\frac{1}{Q_eQ_L}}\left(\frac{s}{\omega_o}\right)^2}$$

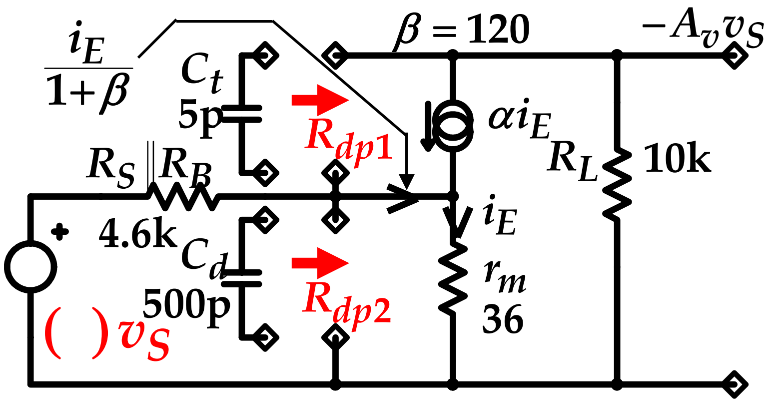
Q_cQ_L

For $Q_eQ_c \gg 1$, $Q_eQ_L \gg 1$, the result reduces to that previously obtained by extrapolation of the result for $Q_c = \infty$:

$$H \approx \frac{1+\frac{1}{Q_c}\left(\frac{s}{\omega_o}\right)}{1+\left(\frac{1}{Q_e}+\frac{1}{Q_c}+\frac{1}{Q_L}\right)\left(\frac{s}{\omega_o}\right)+\left(\frac{s}{\omega_o}\right)^2}$$

1CE with C_t and C_d

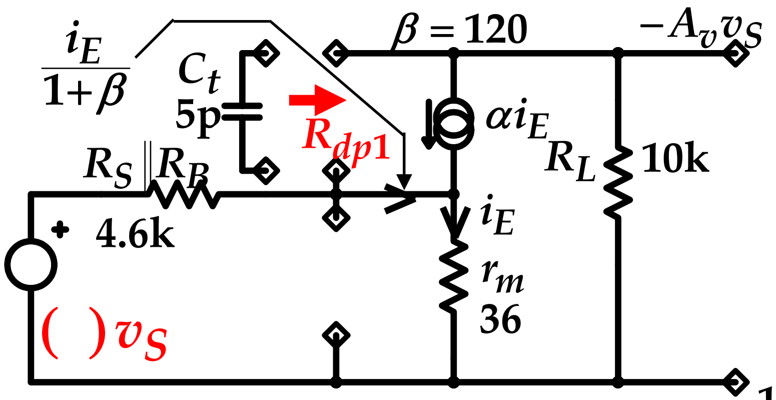
Use the 2EET to add C_t and C_d



R_S and R_B are already absorbed into a Thevenin equivalent

$$A_{vm} \equiv \frac{R_B}{R_S + R_B} \frac{\alpha R_L}{r_m + \frac{R_S \| R_B}{1+\beta}} = 62 \Rightarrow 36dB$$

For C_t alone, results are already known:



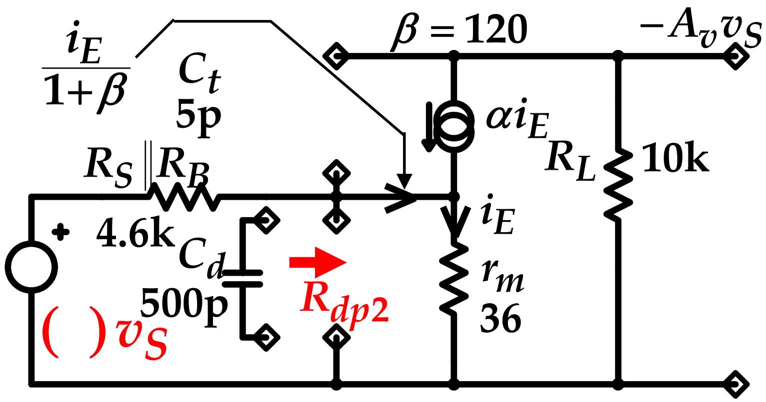
$$A_v = A_{vm}\frac{1-\frac{s}{\omega_z}}{1+\frac{s}{\omega_{p1}}} = 36dB\,\frac{1-\frac{s/2\pi}{880MHz}}{1+\frac{s/2\pi}{51kHz}}$$

$$A_{vm} \equiv \frac{R_B}{R_S+R_B}\,\frac{\alpha R_L}{r_m+\frac{R_S\|R_B}{1+\beta}} = 62 \Rightarrow 36dB$$

$$R_{n1}^{Z_2=\infty} = r_m/\alpha = 36\Omega \qquad R_{d1}^{Z_2=\infty} = mR_L = 620k$$

$$\omega_z \equiv \frac{1}{C_t R_{n1}^{Z_2=\infty}} \qquad \omega_{p1} \equiv \frac{1}{C_t R_{d1}^{Z_2=\infty}} \qquad m \equiv \frac{R_S\|R_B\|(1+\beta)r_m}{R_S\|R_B\|r_m\|R_L} = 62$$

For C_d alone:



$$A_v = A_{vm}\frac{1}{1+\frac{s}{\omega_{p2}}} = 36dB\frac{1}{1+\frac{s/2\pi}{140kHz}}$$

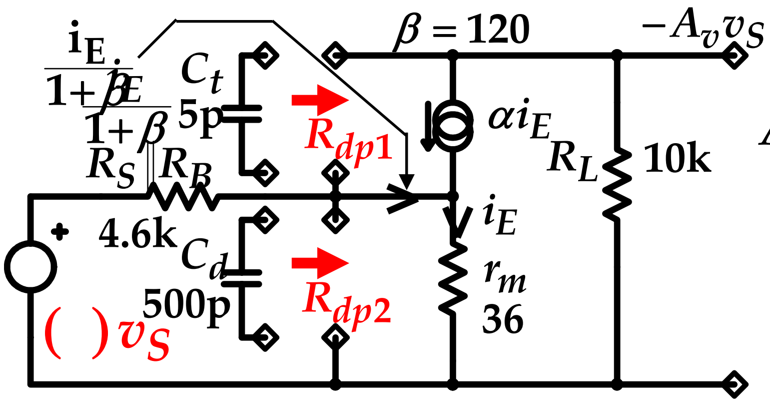
$$A_{vm} \equiv \frac{R_B}{R_S+R_B}\frac{\alpha R_L}{r_m+\frac{R_S\|R_B}{1+\beta}} = 62 \Rightarrow 36dB$$

$$R_{n2}^{Z_1=\infty} = 0 \qquad R_{d2}^{Z_1=\infty} = R_S\|R_B\|(1+\beta)\,r_m = 2.2k$$

$$\omega_{p2} \equiv \frac{1}{C_d R_{d2}^{Z_1=\infty}} \qquad m \equiv \frac{R_S\|R_B\|(1+\beta)r_m}{R_S\|R_B\|r_m\|R_L} = 62$$

These results are obtainable by inspection;

no algebra is required.



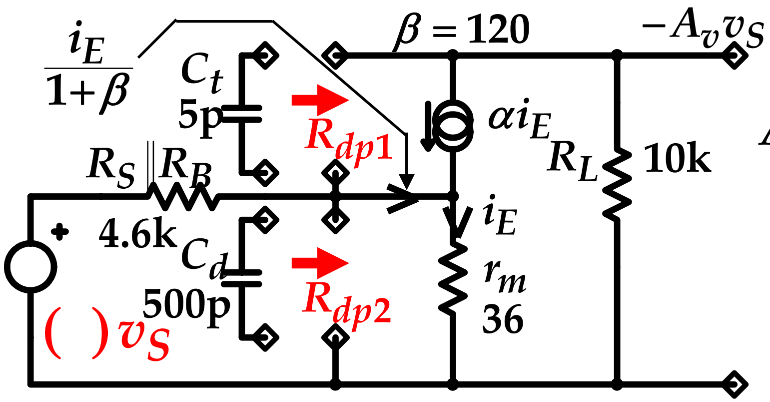
$$A_v = A_{vm} \frac{1 - \frac{s}{\omega_z}}{1 + \frac{s}{\omega_{p1}}} \qquad A_v = A_{vm} \frac{1}{1 + \frac{s}{\omega_{p2}}}$$

$$m \equiv \frac{R_S \| R_B \| (1+\beta) r_m}{R_S \| R_B \| r_m \| R_L} = 62$$

With the corner frequencies for C_t and C_d separately now known, only K_n and K_d remain to be found.

Since C_d does not contribute a zero, K_n is irrelevant, and

For C_t and C_d:



$$A_v = A_{vm} \frac{1 - \frac{s}{\omega_z}}{1 + \frac{s}{\omega_{p1}}}$$

$$A_v = A_{vm} \frac{1}{1 + \frac{s}{\omega_{p2}}}$$

$$m \equiv \frac{R_S \| R_B \| (1+\beta) r_m}{R_S \| R_B \| r_m \| R_L} = 62$$

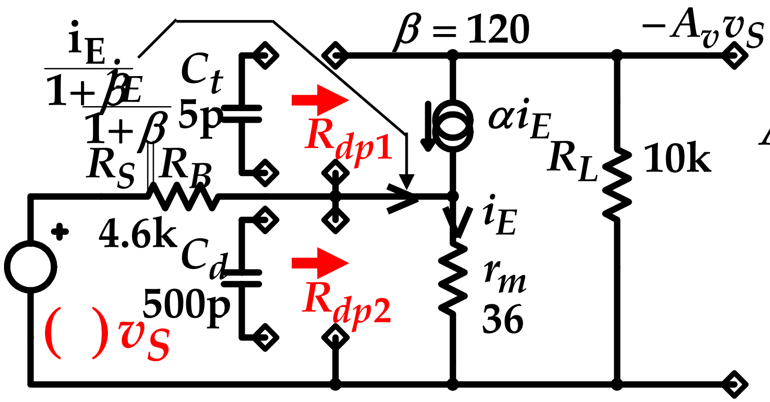
$$A_v = A_{vm} \frac{1 - \frac{s}{\omega_z}}{1 + \frac{s}{\omega_{p1}} + \frac{s}{\omega_{p2}} + K_d \frac{s}{\omega_{p1}} \frac{s}{\omega_{p2}}}$$

where

$$K_d \equiv \frac{R_{d1}^{Z_2=0}}{R_{d1}^{Z_2=\infty}} = \frac{R_L}{mR_L} = \frac{1}{m} \quad \text{or} \quad K_d \equiv \frac{R_{d2}^{Z_1=0}}{R_{d2}^{Z_1=\infty}} = \frac{R_S \| R_B \| r_m \| R_L}{R_S \| R_B \| (1+\beta) r_m} = \frac{1}{m}$$

(Redundancy check)

For C_t and C_d:



$$A_v = A_{vm}\frac{1-\frac{s}{\omega_z}}{1+\frac{s}{\omega_{p1}}} \qquad A_v = A_{vm}\frac{1}{1+\frac{s}{\omega_{p2}}}$$

$$K_d = \frac{1}{m} \qquad m \equiv \frac{R_S \| R_B \| (1+\beta) r_m}{R_S \| R_B \| r_m \| R_L} = 62$$

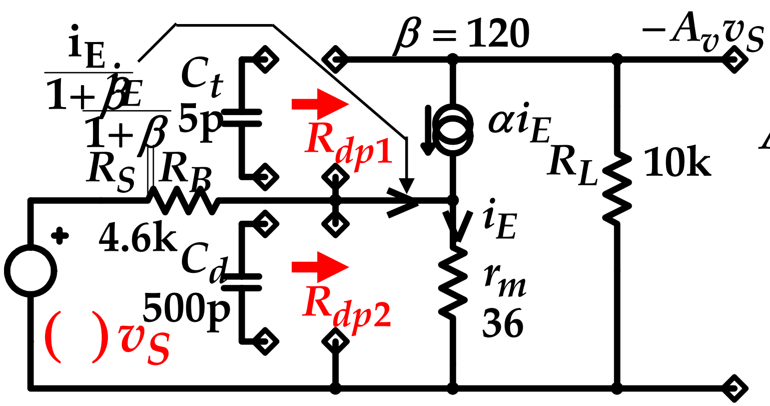
$$A_v = A_{vm}\frac{1-\frac{s}{\omega_z}}{1+\left(\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}\right)s+\left(\frac{1}{m\omega_{p1}\omega_{p2}}\right)s^2}$$

The Q of the quadratic is

$$Q = \frac{\sqrt{\frac{1}{m\omega_{p1}\omega_{p2}}}}{\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}} = \frac{1}{\sqrt{m}}\frac{\sqrt{\omega_{p1}\omega_{p2}}}{\omega_{p1}+\omega_{p2}}$$

In general, $Q_{max} = \frac{0.5}{\sqrt{m}}$, and since $m \geq 1$ the quadratic always has real roots.

For C_t and C_d:



$$A_v = A_{vm} \frac{1 - \frac{s}{\omega_z}}{1 + \frac{s}{\omega_{p1}}} \qquad A_v = A_{vm} \frac{1}{1 + \frac{s}{\omega_{p2}}}$$

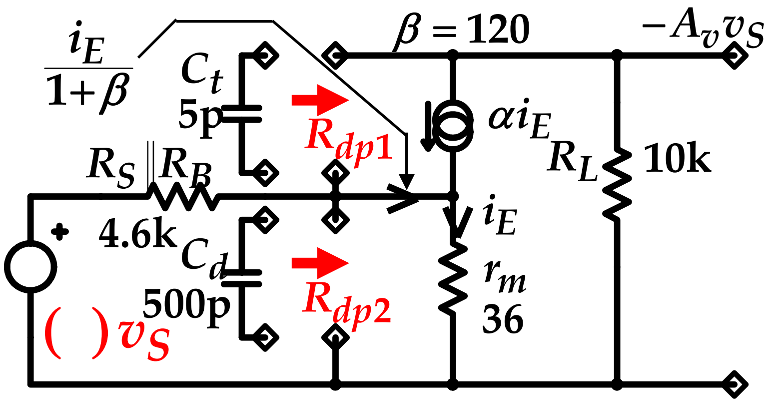
$$K_d = \frac{1}{m} \qquad m \equiv \frac{R_S \| R_B \| (1+\beta) r_m}{R_S \| R_B \| r_m \| R_L} = 62$$

$$A_v = A_{vm} \frac{1 - \frac{s}{\omega_z}}{1 + \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} \right) s + \left(\frac{1}{m \omega_{p1} \omega_{p2}} \right) s^2}$$

Here, $m = 62$ so the real root approximation is extremely good, and the result can be written

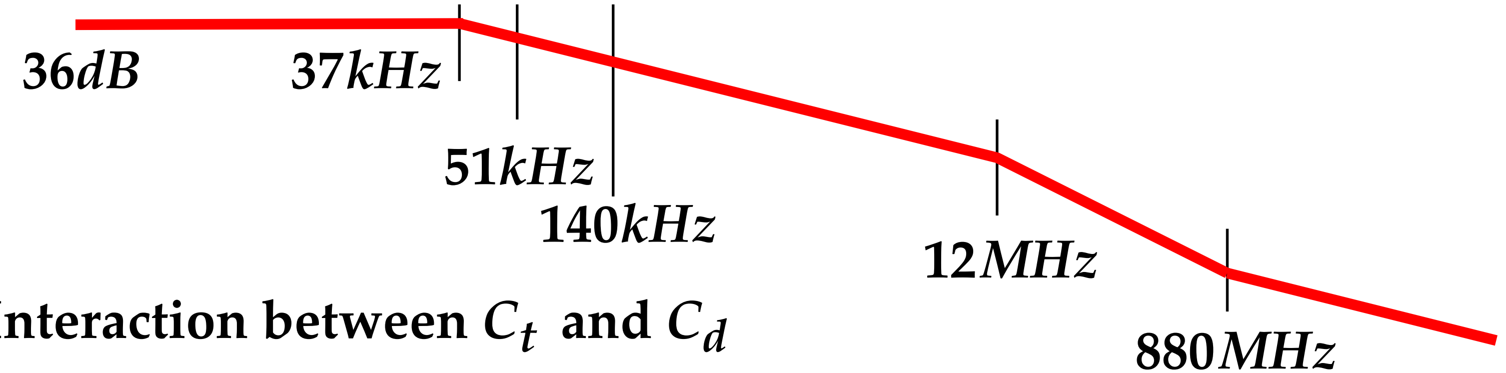
$$A_v = A_{vm} \frac{1 - \frac{s}{\omega_z}}{\left(1 + \frac{s}{\omega_{p1} \| \omega_{p2}} \right) \left(1 + \frac{s}{m(\omega_{p1} + \omega_{p2})} \right)}$$

For C_t and C_d:



$$m \equiv \frac{R_S \| R_B \| (1+\beta) r_m}{R_S \| R_B \| r_m \| R_L} = 62$$

$$A_v = A_{vm} \frac{1 - \frac{s}{\omega_z}}{\left(1 + \frac{s}{\omega_{p1} \| \omega_{p2}}\right)\left(1 + \frac{s}{m\left(\omega_{p1} + \omega_{p2}\right)}\right)} = 36dB \frac{1 - \frac{s/2\pi}{880MHz}}{\left(1 + \frac{s/2\pi}{37kHz}\right)\left(1 + \frac{s/2\pi}{12MHz}\right)}$$



Interaction between C_t and C_d causes "pole splitting."

The Extra Element Theorem (EET):

$$A\Big|_Z = A\Big|_{Z=\infty}\frac{1+\frac{Z_n}{Z}}{1+\frac{Z_d}{Z}} = A\Big|_{Z=0}\frac{1+\frac{Z}{Z_n}}{1+\frac{Z}{Z_d}}$$

The EET can be extended to the Two Extra Element Theorem (2EET):

$$A\Big|_{Z_1,Z_2} = A\Big|_{\substack{Z_1=\infty\\Z_2=\infty}}\frac{1+\frac{Z_{n1}}{Z_1}+\frac{Z_{n2}}{Z_2}+K_n\frac{Z_{n1}}{Z_1}\cdot\frac{Z_{n2}}{Z_2}}{1+\frac{Z_{d1}}{Z_1}+\frac{Z_{d2}}{Z_2}+K_d\frac{Z_{d1}}{Z_1}\cdot\frac{Z_{d2}}{Z_2}}$$

and its dual forms.

and, ultimately, to the N Extra Element Theorem (NEET).

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The N Extra Element Theorem

R. David Middlebrook, *Life Fellow, IEEE*, Vatché Vorpérian, *Senior Member, IEEE*, and John Lindal

It's Really NEET!

"Basic" NEET Version, for $N = 3$ and All Ref States Short

$$H = H_{\text{ref}} \frac{Num}{Denom} \tag{30a}$$

where

$$\begin{aligned} Num = 1 &+ \left[\left(\frac{Z_1}{Z_{n1}} + \frac{Z_2}{Z_{n2}} \right) + \frac{Z_3}{Z_{n3}} \right] \\ &+ \left[\left\{ \frac{Z_1}{Z_{n1}} \frac{Z_2}{Z_{n2}^{(1)}} \right\} + \left(\frac{Z_1}{Z_{n1}} \frac{Z_3}{Z_{n3}^{(1)}} + \frac{Z_2}{Z_{n2}} \frac{Z_3}{Z_{n3}^{(2)}} \right) \right] \\ &+ \left[\left\{ \frac{Z_1}{Z_{n1}} \frac{Z_2}{Z_{n2}^{(1)}} \frac{Z_3}{Z_{n3}^{(1,2)}} \right\} \right] \end{aligned} \tag{30b}$$

Denom = [same as *Num* with sub d instead of sub n].

NEET Version for $N \geq 4$, EE3 Ref State Open

$$
\begin{aligned}
Num = 1 &+ \left[\left(\frac{Z_1}{Z_{n1}} + \frac{Z_2}{Z_{n2}} + \frac{Y_3}{Y_{n3}}\right) + \frac{Z_4}{Z_{n4}} + \cdots\right] \\
&+ \left[\left\{\frac{Z_1}{Z_{n1}}\frac{Z_2}{Z_{n2}^{(1)}} + \frac{Z_1}{Z_{n1}}\frac{Y_3}{Y_{n3}^{(1)}} + \frac{Z_2}{Z_{n2}}\frac{Y_3}{Y_{n3}^{(2)}}\right\}\right. \\
&+ \left.\left(\frac{Z_1}{Z_{n1}}\frac{Z_4}{Z_{n4}^{(1)}} + \frac{Z_2}{Z_{n2}}\frac{Z_4}{Z_{n4}^{(2)}} + \frac{Y_3}{Y_{n3}}\frac{Z_4}{Z_{n4}^{(3)}}\right) + \cdots\right] \\
&+ \left[\left\langle \frac{Z_1}{Z_{n1}}\frac{Z_2}{Z_{n2}^{(1)}}\frac{Y_3}{Y_{n3}^{(1,2)}} \right\rangle + \left\{\frac{Z_1}{Z_{n1}}\frac{Z_2}{Z_{n2}^{(1)}}\frac{Z_4}{Z_{n4}^{(1,2)}}\right.\right. \\
&+ \left.\left.\frac{Z_1}{Z_{n1}}\frac{Y_3}{Y_{n3}^{(1)}}\frac{Z_4}{Z_{n4}^{(1,3)}} + \frac{Z_2}{Z_{n2}}\frac{Y_3}{Y_{n3}^{(2)}}\frac{Z_4}{Z_{n4}^{(2,3)}}\right\} + \cdots\right]
\end{aligned} \tag{80a}
$$

$$
+ \left[\left\langle \frac{Z_1}{Z_{n1}}\frac{Z_2}{Z_{n2}^{(1)}}\frac{Y_3}{Y_{n3}^{(1,2)}}\frac{Z_4}{Z_{n4}^{(1,2,3)}} \right\rangle + \cdots\right] + \cdots
$$

Denom **=** [same as *Num* with sub d instead of sub n]. (80b)

13. DNTI AND THE 2GFT:

Double Null Triple Injection and the Two General Feedback Theorem

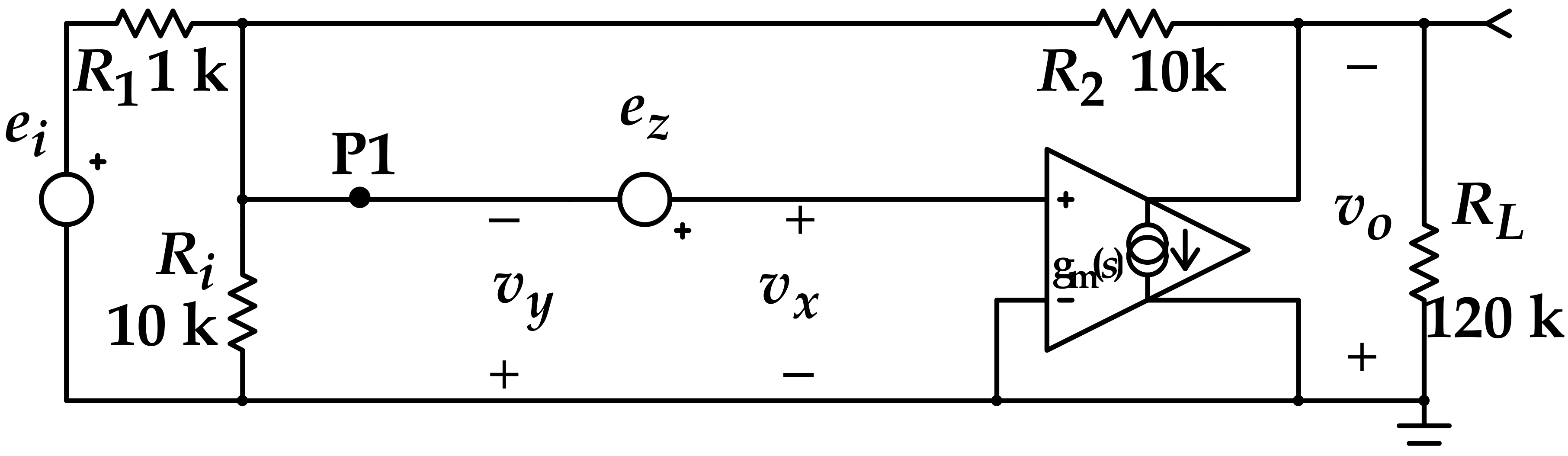
Why two injected test signals are needed in the general case

The GFT with a single injected test signal, as a special case of the Dissection Theorem, incorporates all nonidealities, but still requires an ideal test signal injection point.

Why does a nonideal test signal injection point give "wrong" answers?

Consider the example circuit already discussed:

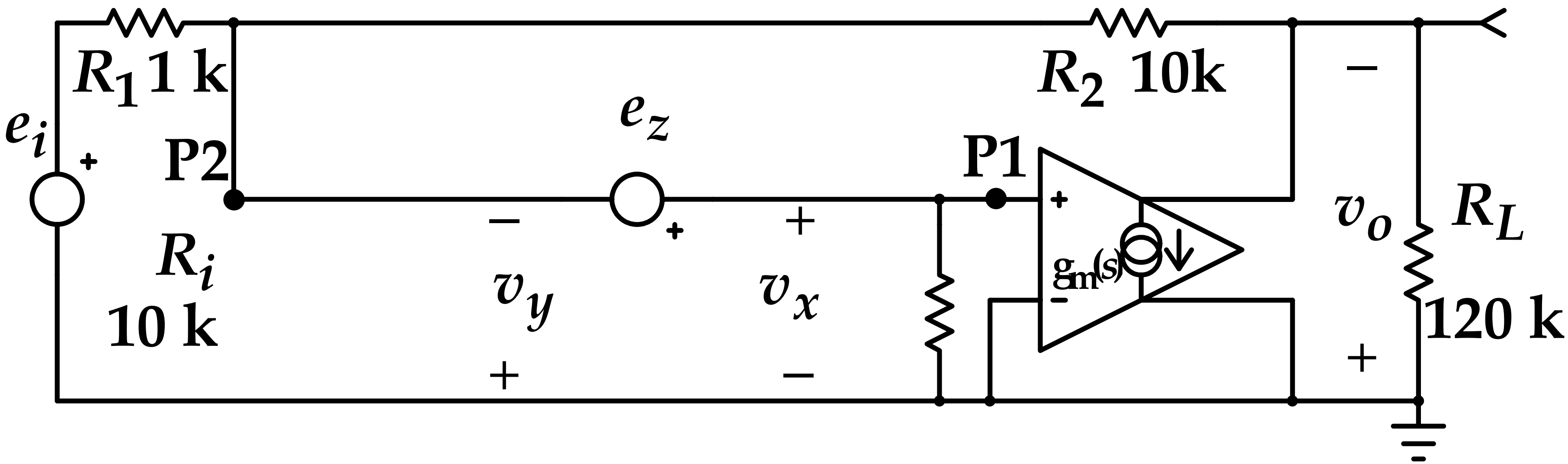
Ideal voltage injection point



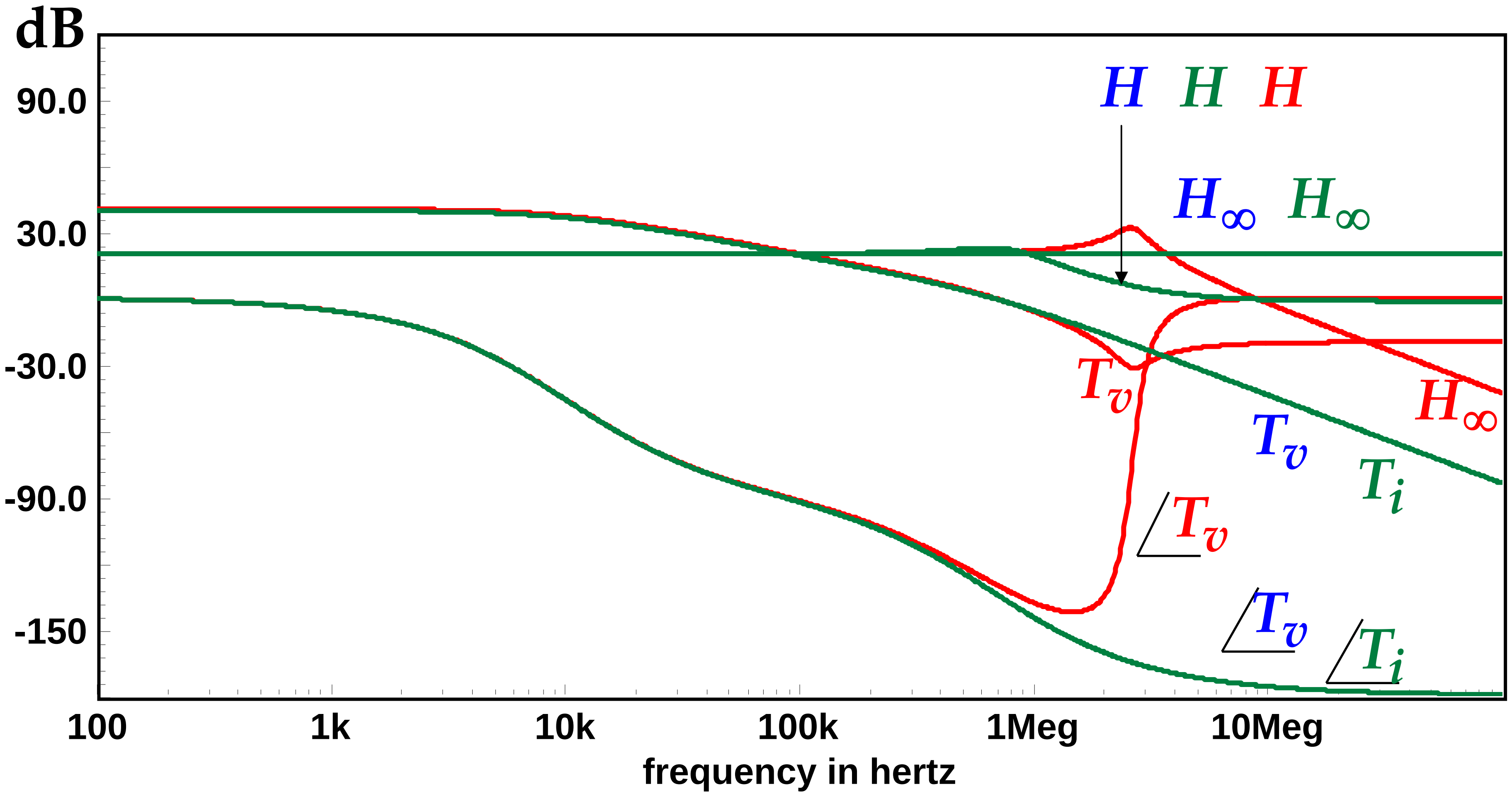
An ideal voltage injection point is where v_y comes from an ideal (zero impedance) voltage generator,

or **where v_x looks into an infinite impedance.**

Test voltage injection on the other side of R_i :

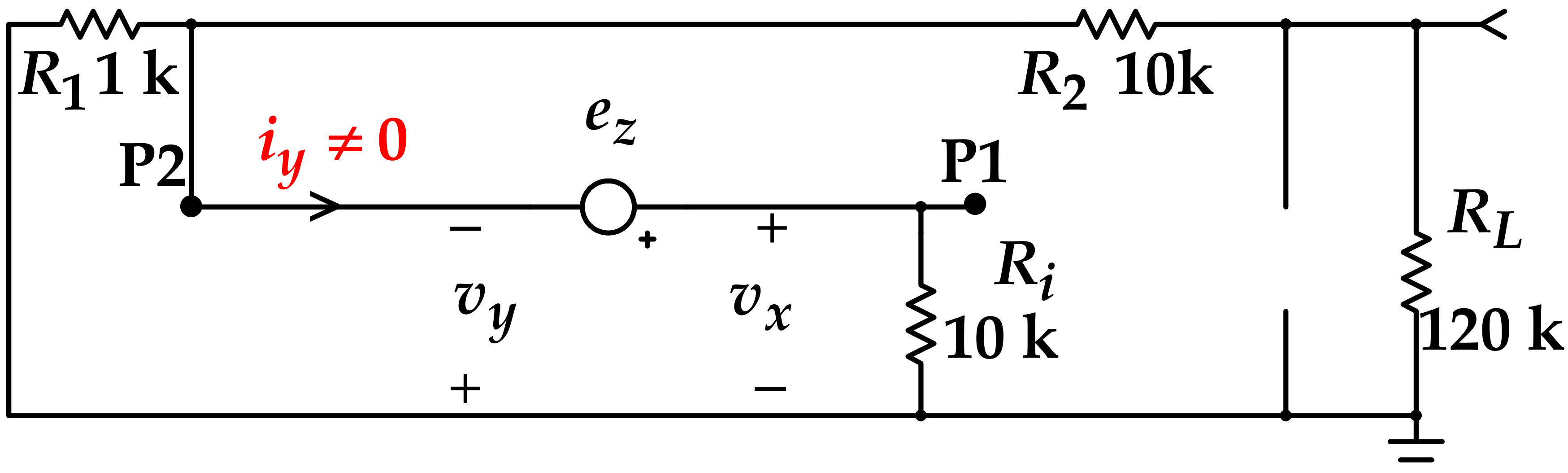


This is a nonideal voltage injection point, and the results for T_v are quite different:



dB
90.0
30.0
-30.0
-90.0
-150
100
1k
10k
100k
1Meg
10Meg
frequency in hertz
H
H
H
H_∞
H_∞
T_v
H_∞
T_v
T_i
$\angle T_v$
$\angle T_v$
$\angle T_i$

Reason for the difference in T_v:



Consider $T_v(\omega \to \infty)$: opamp gain goes to zero

$$v_y = [R_1 \| (R_2 + R_L)] i_y \approx R_1 i_y$$

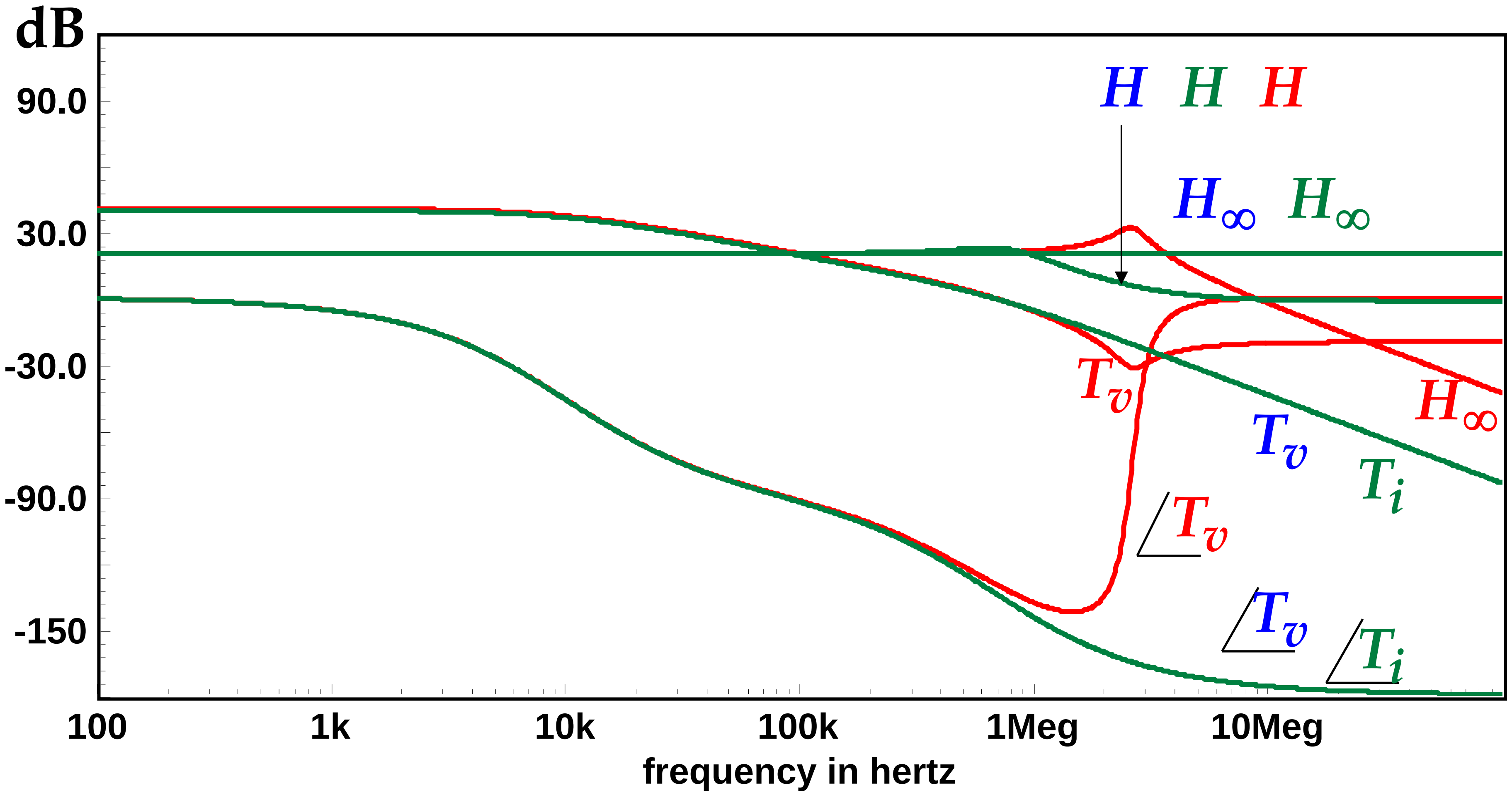
$$v_x = R_i i_y$$

so $$T_v = \frac{v_y}{v_x} = \frac{R_1}{R_i} \neq 0$$

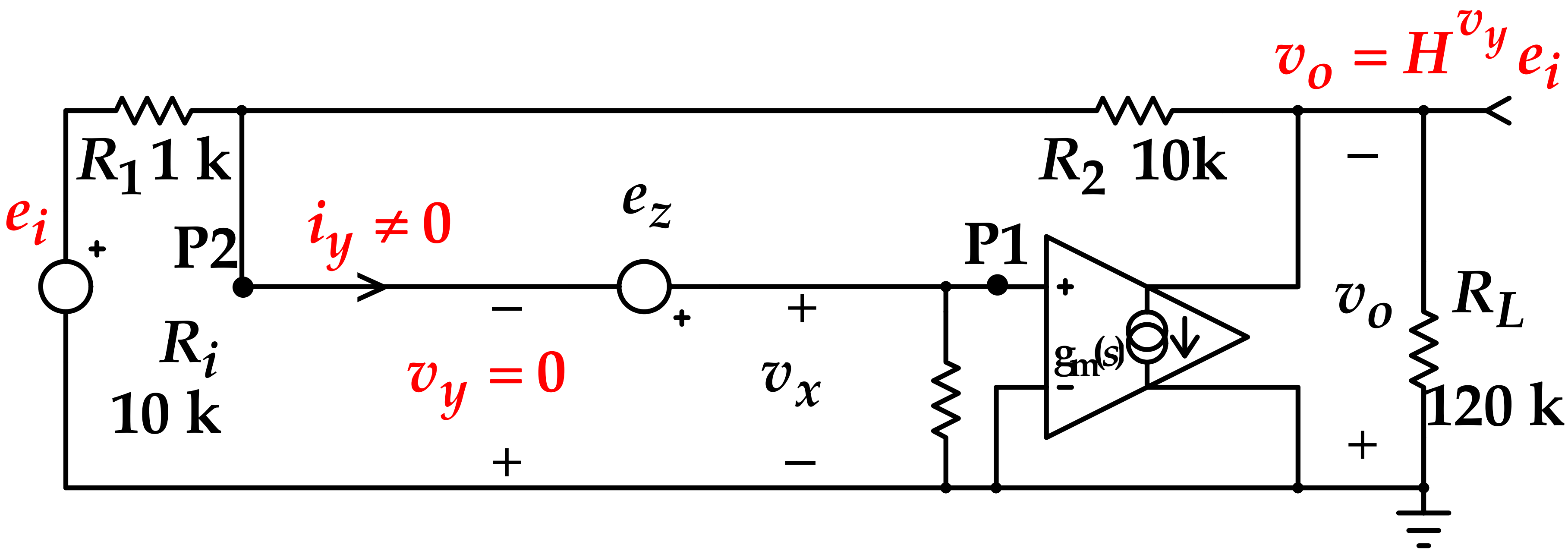
A nonideal injection point gives a u_y / u_x that depends not only on T, but also on the impedance ratio (looking forward and backward) from the injection point.

A nonideal injection point gives a u_y / u_x that depends not only on T, but also on the impedance ratio (looking forward and backward) from the injection point.

A nonideal injection point also gives a H^{u_y} that is different from the desired H_∞ :


dB
90.0
30.0
-30.0
-90.0
-150
100
1k
10k
100k
1Meg
10Meg
frequency in hertz
H
H
H
H_∞
H_∞
T_v
H_∞
T_v
T_i
$\angle T_v$
$\angle T_v$
$\angle T_i$

Reason for the difference in H^{v_y}:



Again, $i_y \neq 0$ **so** $H^{v_y} = \left.\dfrac{v_o}{e_i}\right|_{v_y=0} \neq \dfrac{R_2}{R_1}$,

but depends also on the opamp gain.

An extension of the Basic Feedback Theorem is needed:

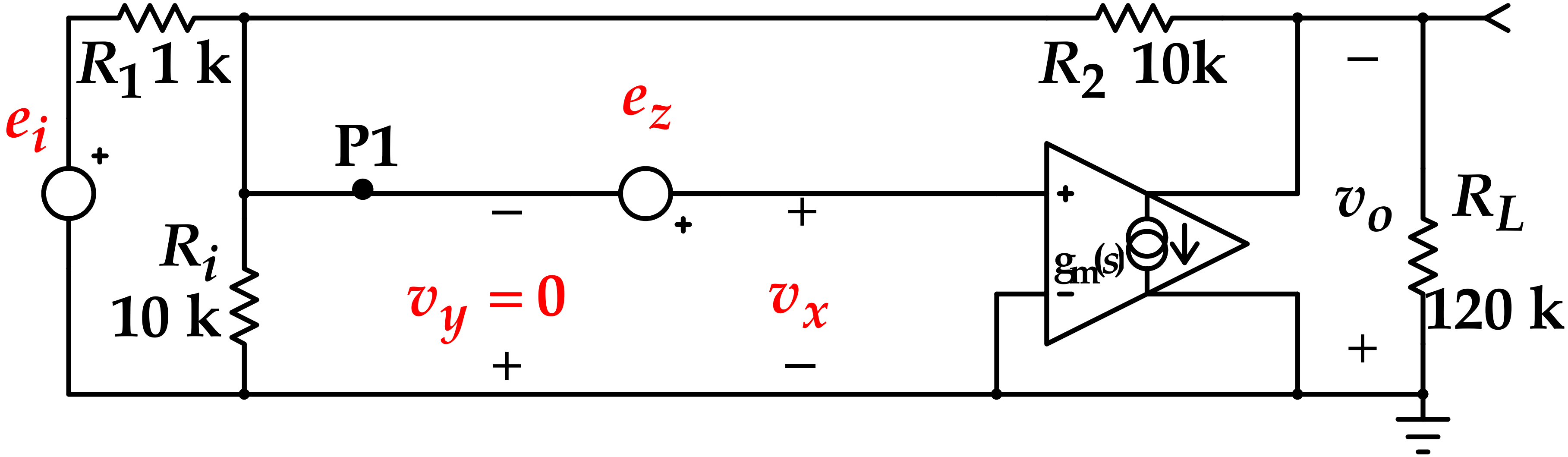
To set up $H_\infty = 1/K$, the reciprocal of the feedback ratio, both the error voltage v_y and the error current i_y must be nulled.

In the preceding example with injected test voltage e_z, the desired result $H_\infty = 1/K$ was obtained because nulled error voltage automatically implied nulled error current:

$$H_\infty = \left.\frac{v_o}{e_i}\right|_{v_y=0} \equiv H^{v_y}$$

A superscript indicates the signal being nulled

$v_o = H^{v_y} e_i$

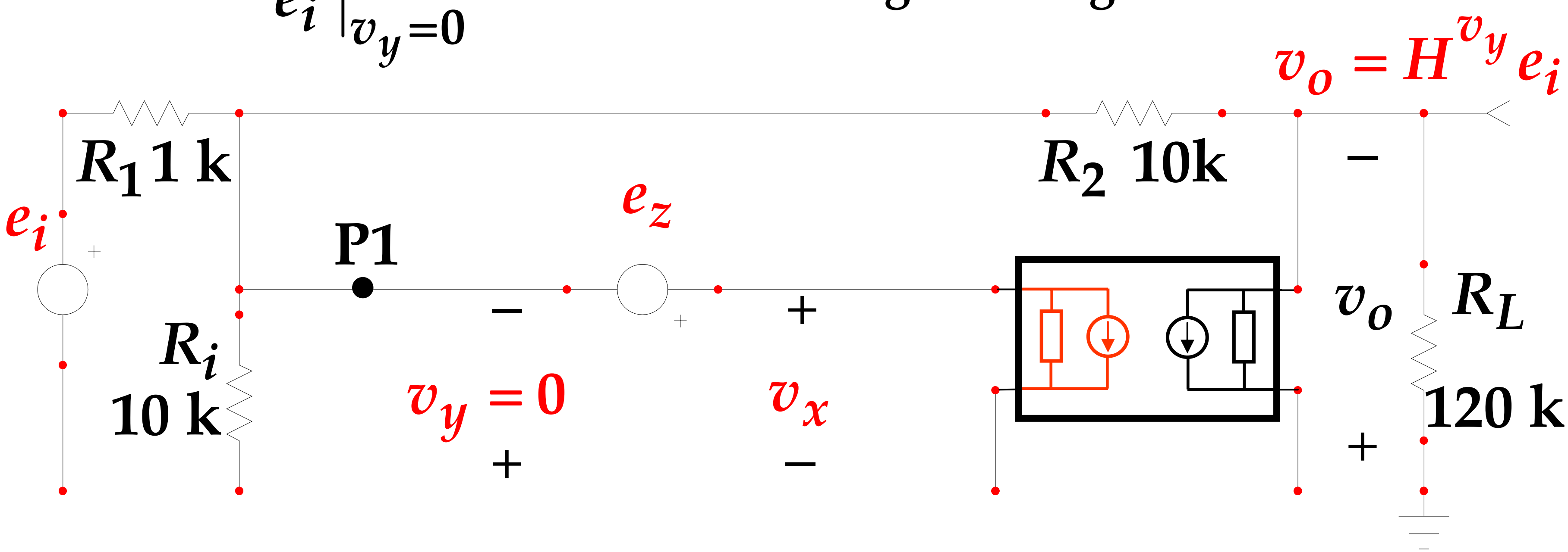


$$H^{v_y} = \frac{R_2}{R_1} = 10 \Rightarrow 20\ \text{dB}$$

In a realistic circuit model, the injection point is not ideal and the X terminal looks into a 2-port model that exhibits noninfinite input impedance and nonzero reverse transmission:

$$H_\infty = \left.\frac{v_o}{e_i}\right|_{v_y=0} \equiv H^{v_y}$$

A superscript indicates the signal being nulled



When v_y is nulled, the error current is *not* nulled, and H_∞ is not equal to R_2 / R_1.

A nonideal voltage injection point implies that when v_y is nulled, i_y is not nulled.

A nonideal current injection point implies that when i_y is nulled, v_y is not nulled.

The 2GFT: The Final Solution

$$H = H_\infty \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}} = H_\infty D D_n$$

To make H_∞ equal $1/K$:

What is needed is a way to null simultaneously both the error voltage v_y and the error current i_y.

The answer is obvious:

Since

One signal can be nulled by mutual adjustment of two independent sources (ndi, null double injection)

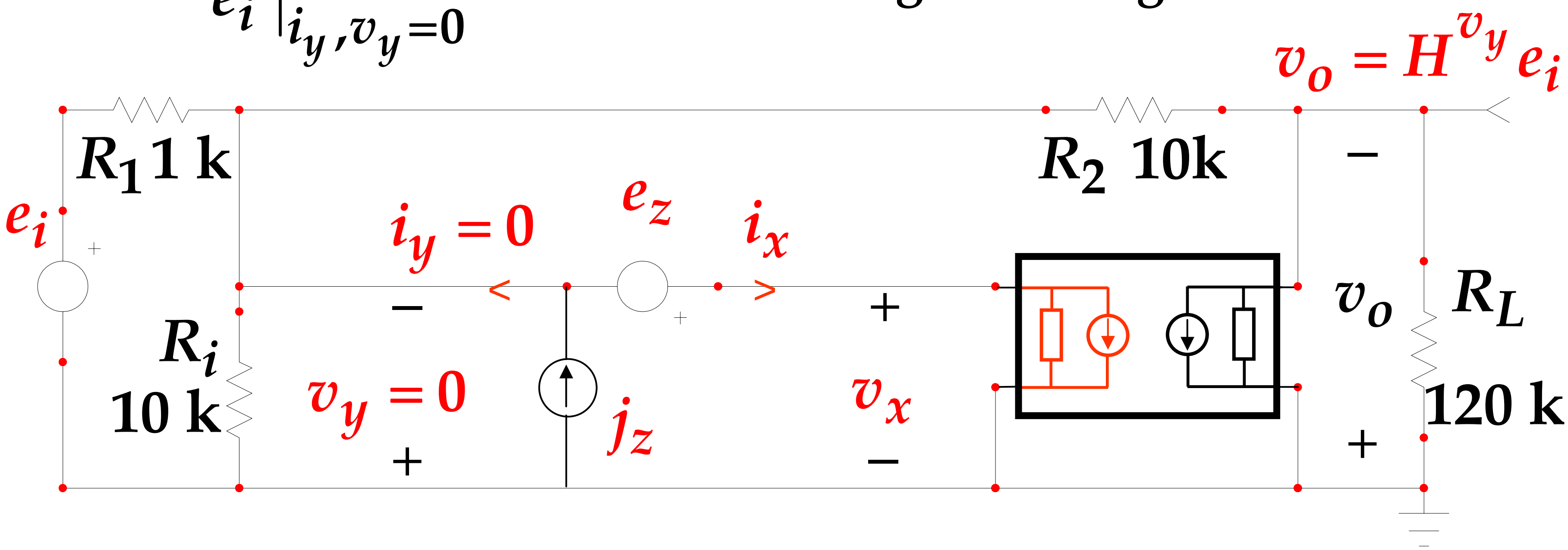
Two signals can be nulled by mutual adjustment of three independent sources (dnti, double null triple injection).

Therefore:

In the presence of the input signal e_i, inject both voltage and current test signals so that both the error voltage v_y and the error current i_y can be nulled.

$$H_\infty = \left.\frac{v_o}{e_i}\right|_{i_y, v_y = 0} \equiv H^{i_y v_y}$$

Superscripts indicate the signals being nulled



When v_y and i_y are both nulled, H_∞ is again equal to R_2 / R_1, the reciprocal of the feedback ratio K.

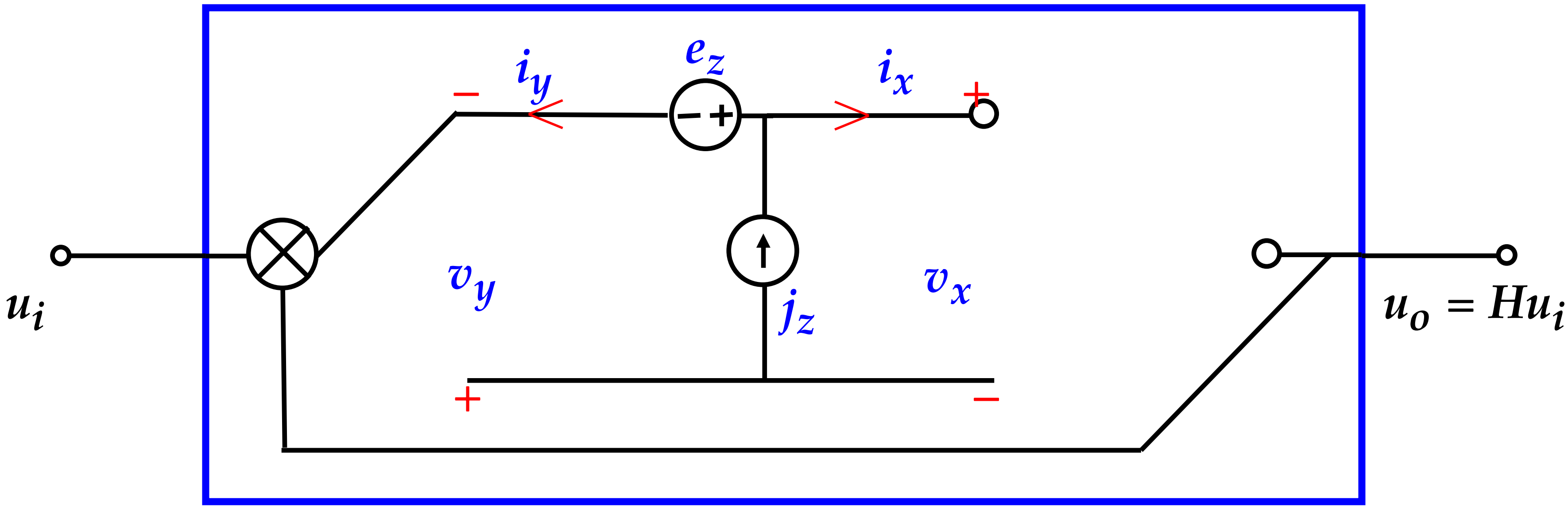
The format of the GFT remains the same:

$$H = H_{\infty} \frac{1 + \frac{1}{T_n}}{1 + \frac{1}{T}}$$

where now $H_{\infty} = H^{i_y v_y}$, and T and T_n contain both voltage and current loop gains.

Since the GFT and the EET are special cases of the Dissection Theorem for a single injected test signal, and the 2EET is a special case of the 2DT, it is to be expected that the 2GFT for two injected test signals would have the same format as the 2EET with impedance ratios replaced by return ratios.

The 2GFT tells how to find T when both current and voltage loop gains are present.

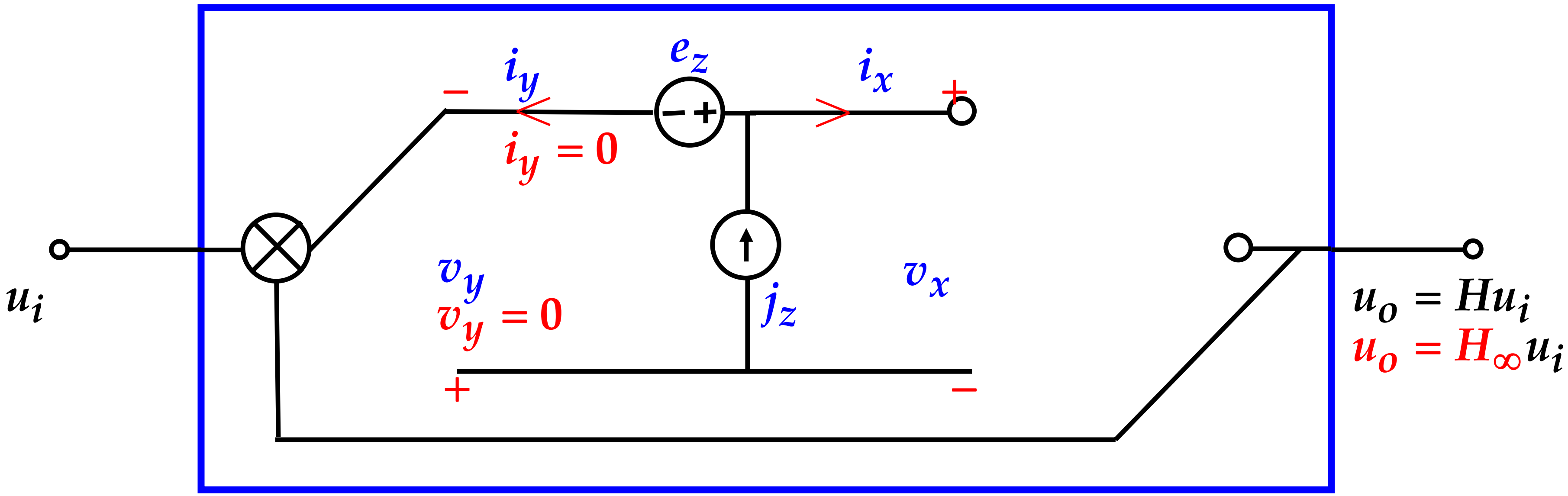


$$H = H^{i_y v_y} \frac{1 + \frac{1}{T_{ni}^{v_y}} + \frac{1}{T_{nv}^{i_y}} + K_n \frac{1}{T_{ni}^{v_y}} \frac{1}{T_{nv}^{i_y}}}{1 + \frac{1}{T_i^{v_y}} + \frac{1}{T_v^{i_y}} + K_d \frac{1}{T_i^{v_y}} \frac{1}{T_v^{i_y}}}$$

where K_d and K_n are interaction parameters:

$$K_d \equiv \frac{T_i^{v_y}}{T_i^{v_x}} = \frac{T_v^{i_y}}{T_v^{i_x}} \qquad K_n \equiv \frac{T_{ni}^{v_y}}{T_{ni}^{v_x}} = \frac{T_{nv}^{i_y}}{T_{nv}^{i_x}}$$

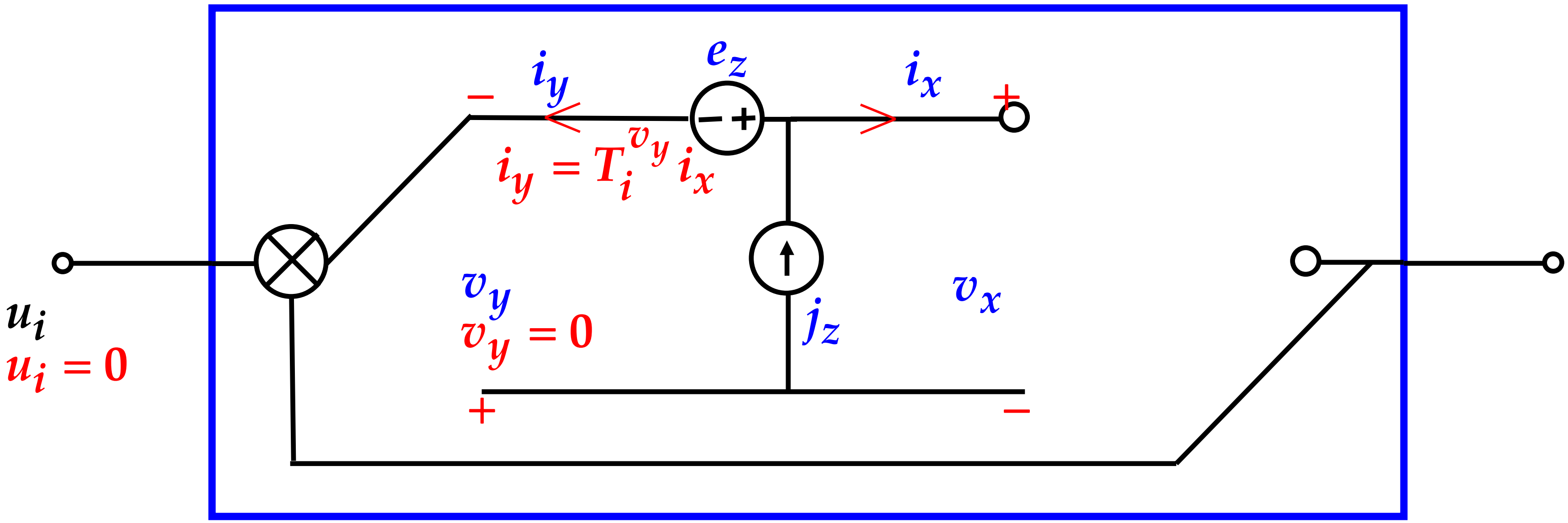
Interpretations:



$$H_\infty \equiv H^{i_y v_y} \equiv \left.\frac{u_o}{u_i}\right|_{i_y, v_y = 0} = \text{ideal closed-loop gain}$$

$H^{i_y v_y}$ **is a double null triple injection (dnti) calculation, which is even simpler and shorter than an ndi calculation.**

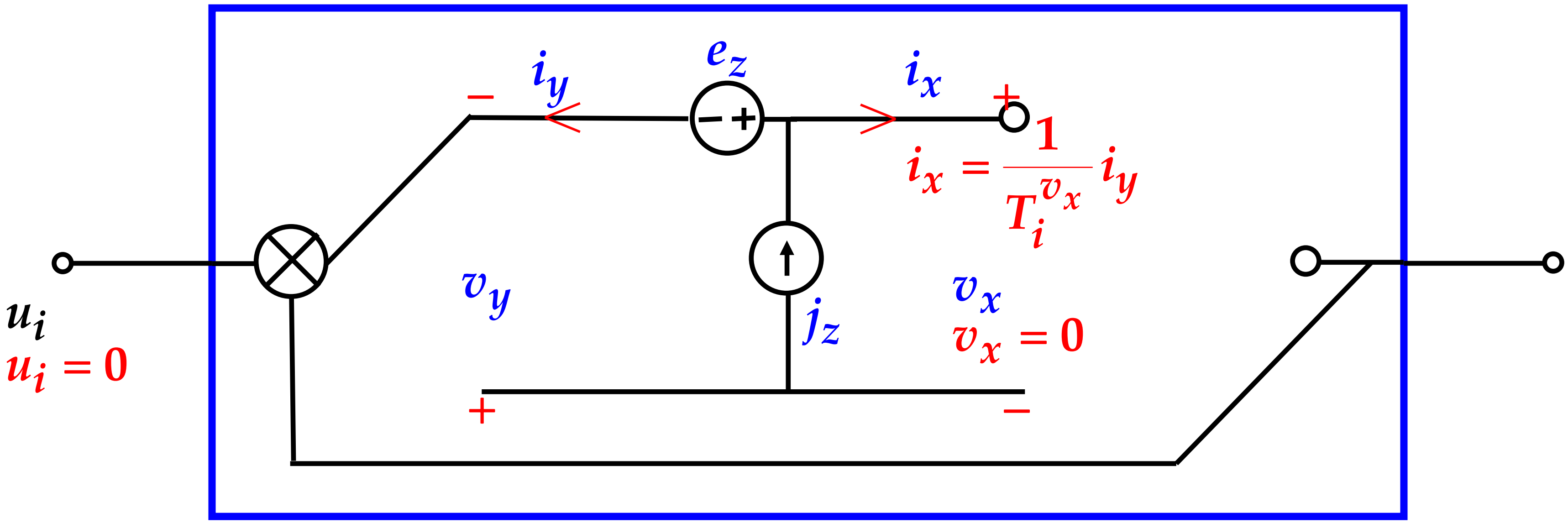
Interpretations:



$$T_i^{v_y} \equiv \left. \frac{i_y}{i_x} \right|_{v_y=0} = \textbf{short-circuit current loop gain}$$

$T_i^{v_y}$ **is a null double injection (ndi) calculation, which is simpler and shorter than an si calculation.**

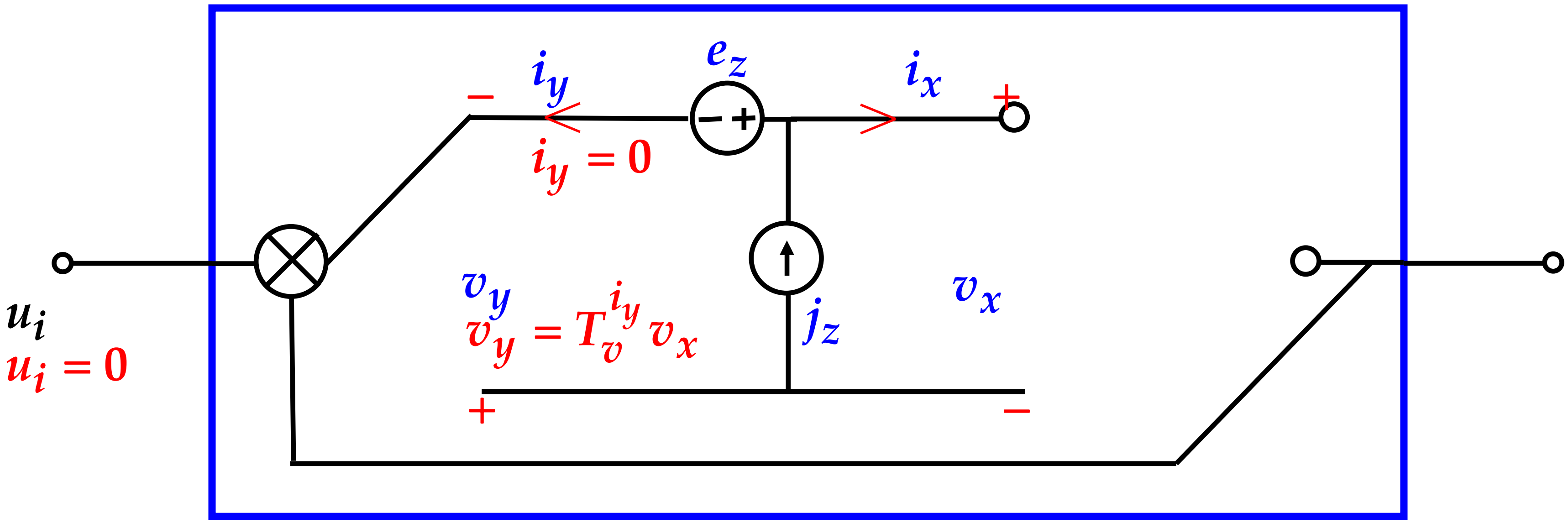
Interpretations:



$$\frac{1}{T_i^{v_x}} \equiv \left.\frac{i_x}{i_y}\right|_{v_x=0}$$ = **short-circuit reverse current loop gain**

$T_i^{v_x}$ **is a null double injection (ndi) calculation.**

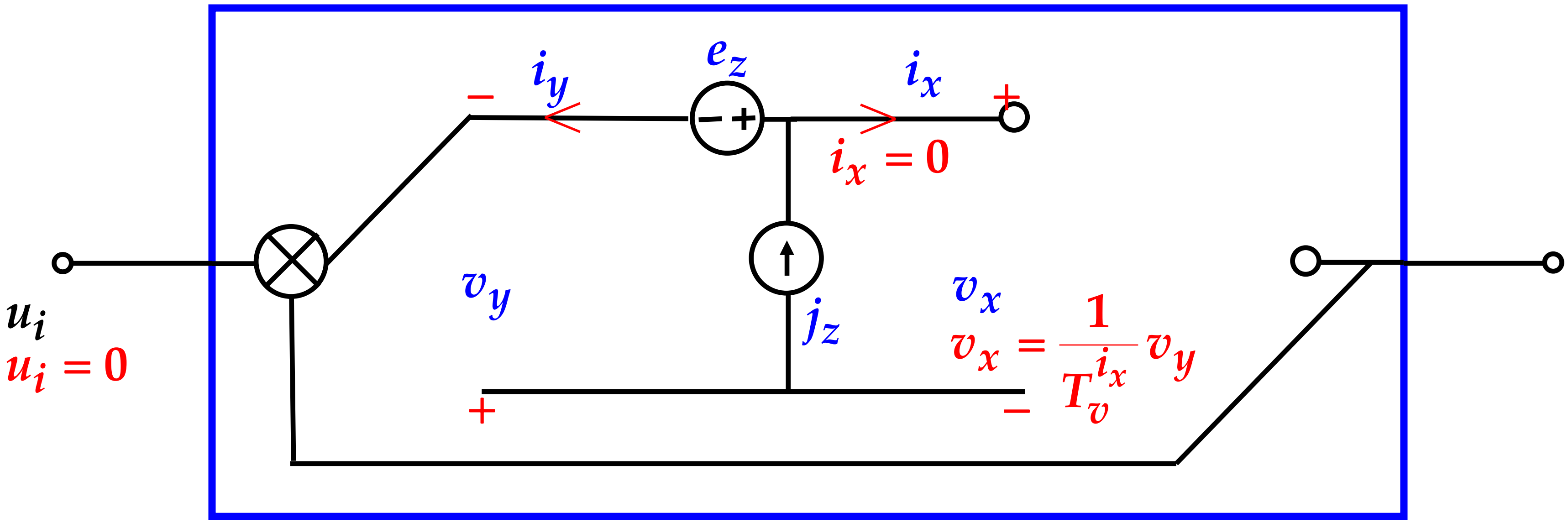
Interpretations:



$$T_v^{i_y} \equiv \left.\frac{v_y}{v_x}\right|_{i_y=0}$$ **= open-circuit voltage loop gain**

$T_v^{i_y}$ **is a null double injection (ndi) calculation.**

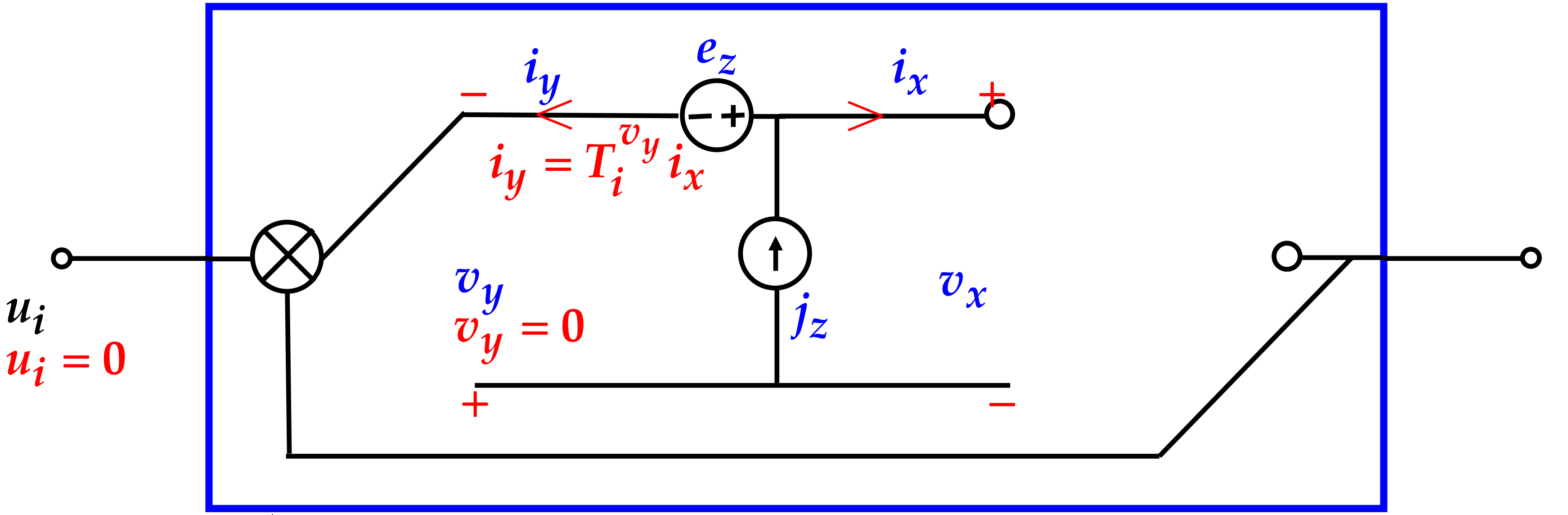
Interpretations:



$$\frac{1}{T_v^{i_x}} \equiv \left.\frac{v_x}{v_y}\right|_{i_x=0}$$ **= open-circuit reverse voltage loop gain**

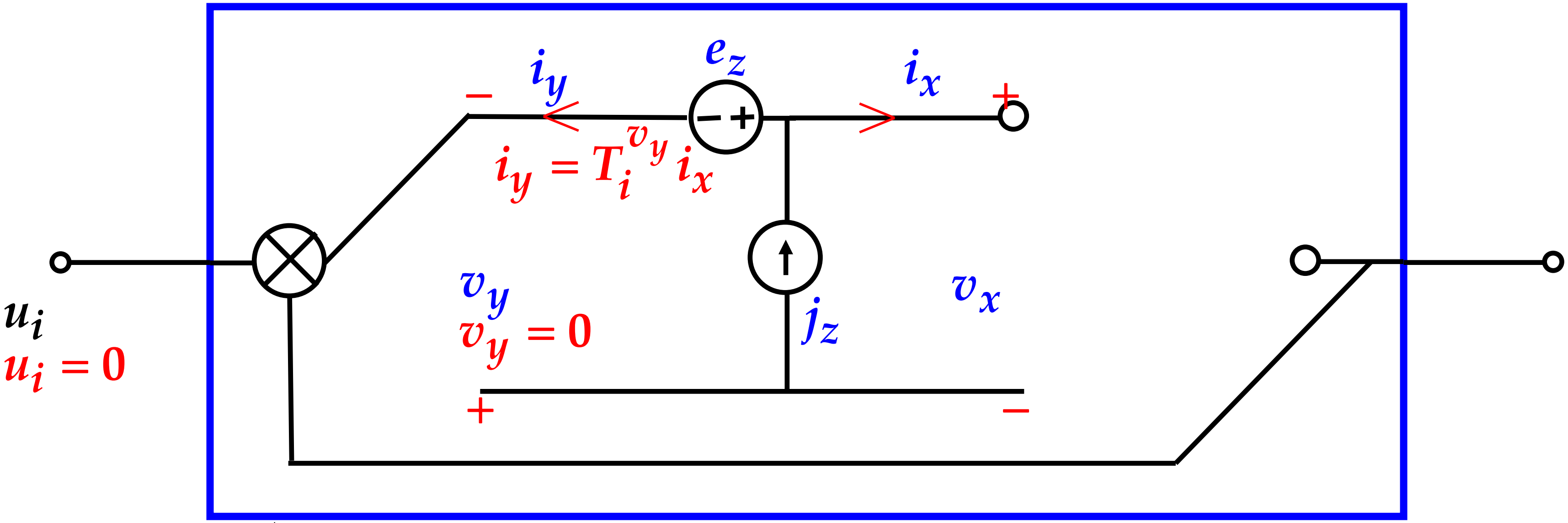
$T_v^{i_x}$ **is a null double injection (ndi) calculation.**

Definitions:

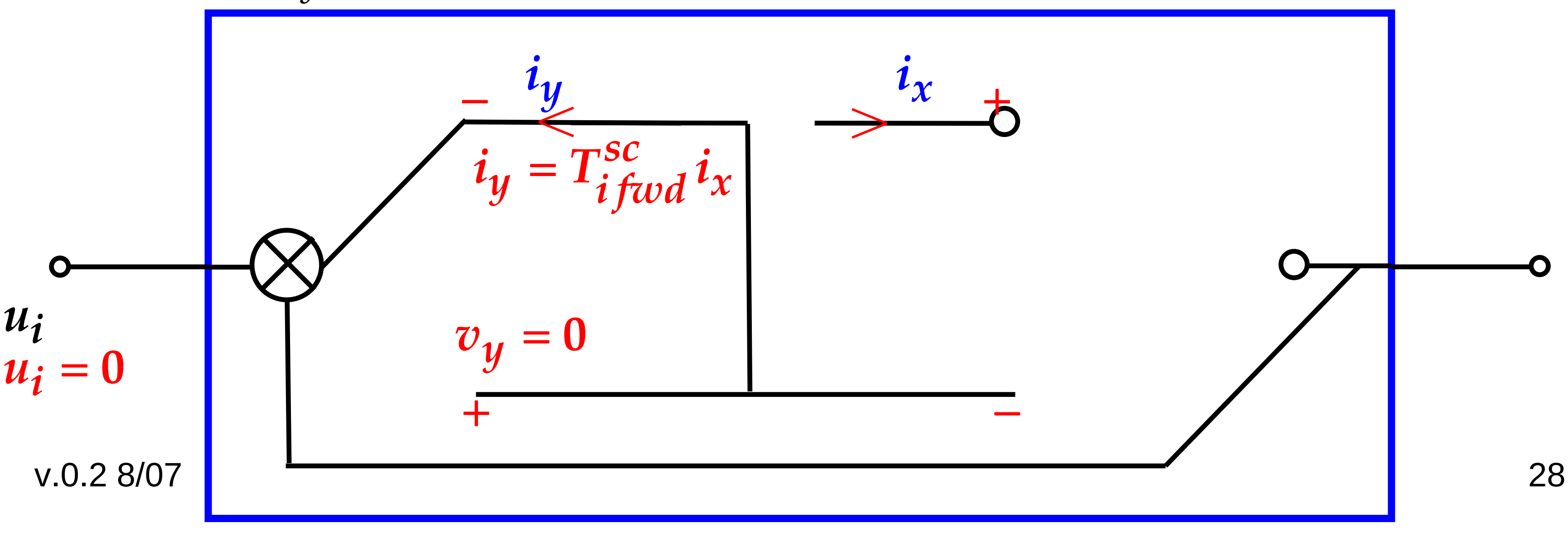


$$T_i^{v_y} \equiv \left. \frac{i_y}{i_x} \right|_{v_y = 0}$$ = **short-circuit forward current loop gain**

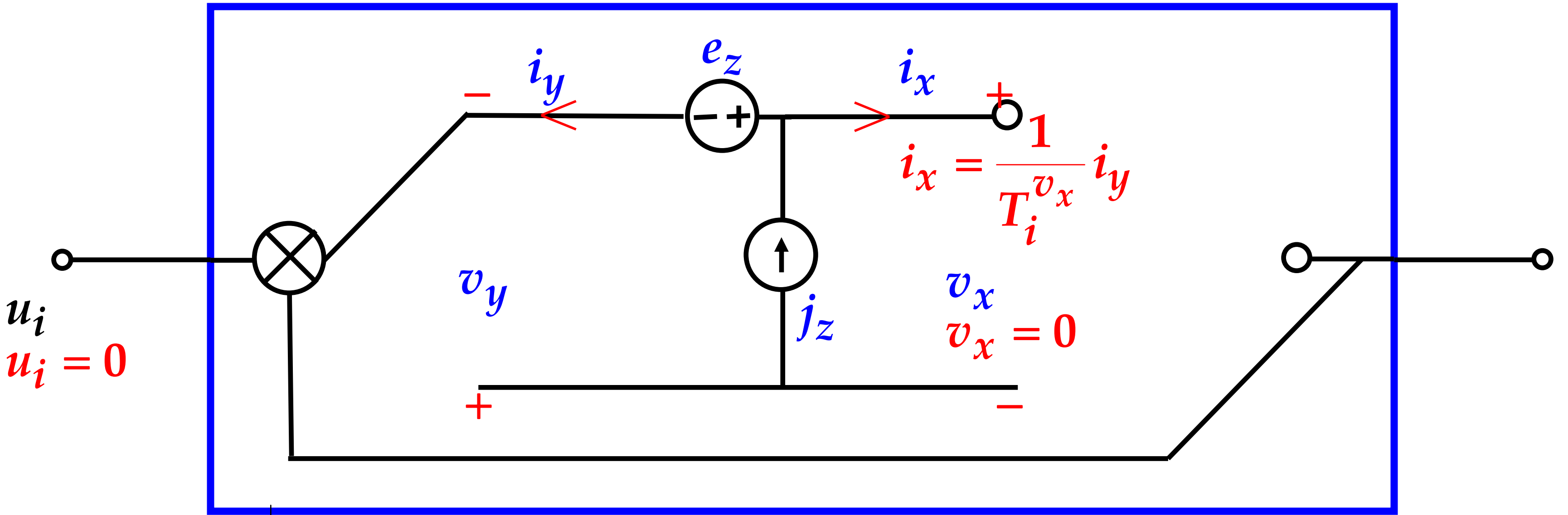
Definitions:



$$T_i^{v_y} \equiv \left.\frac{i_y}{i_x}\right|_{v_y=0} = \text{short-circuit forward current loop gain} \equiv T_{i\,fwd}^{sc}$$

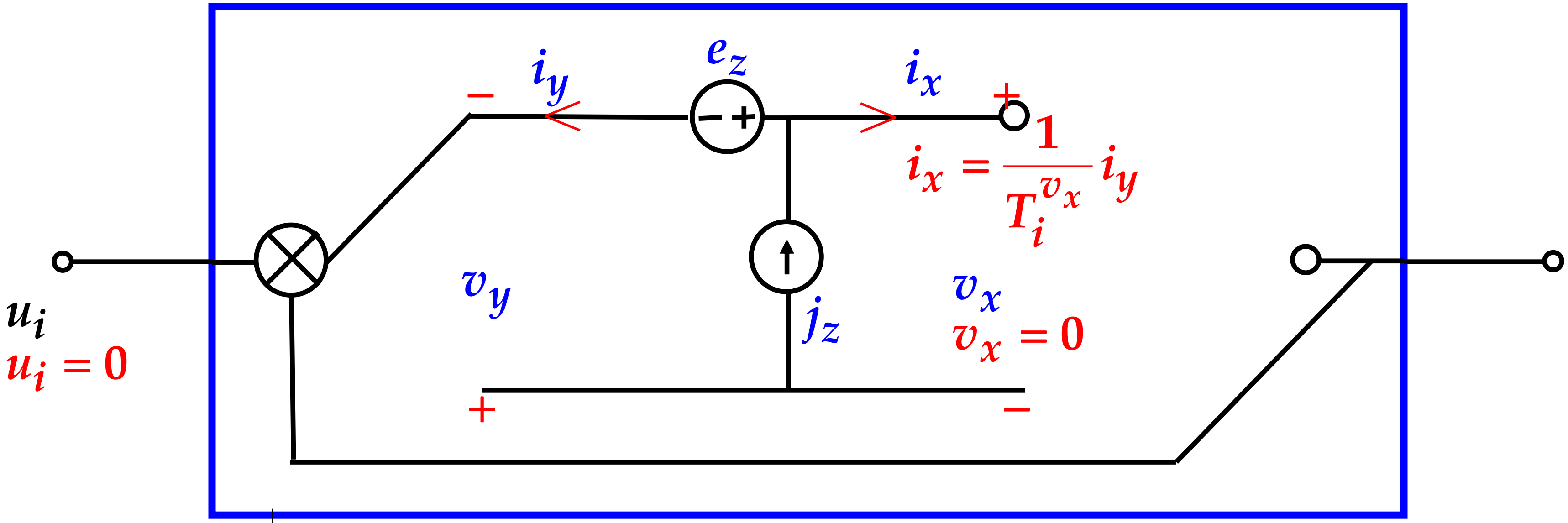


Definitions:

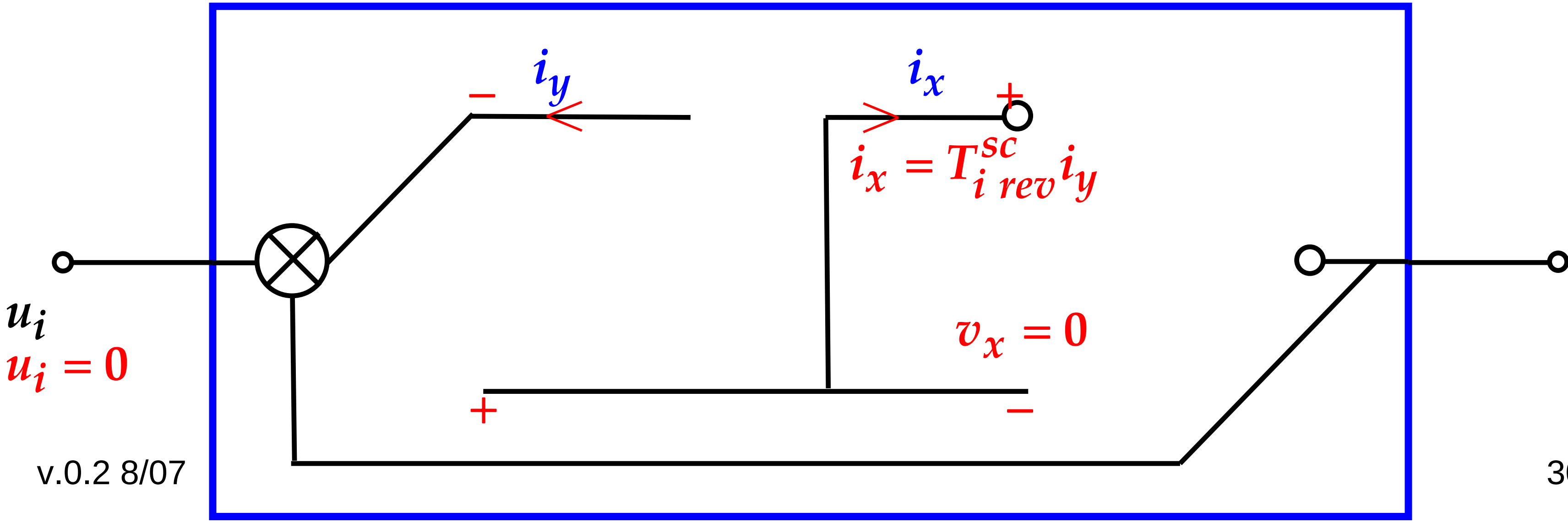


$$\frac{1}{T_i^{v_x}} \equiv \left.\frac{i_x}{i_y}\right|_{v_x=0}$$ = **short-circuit reverse current loop gain**

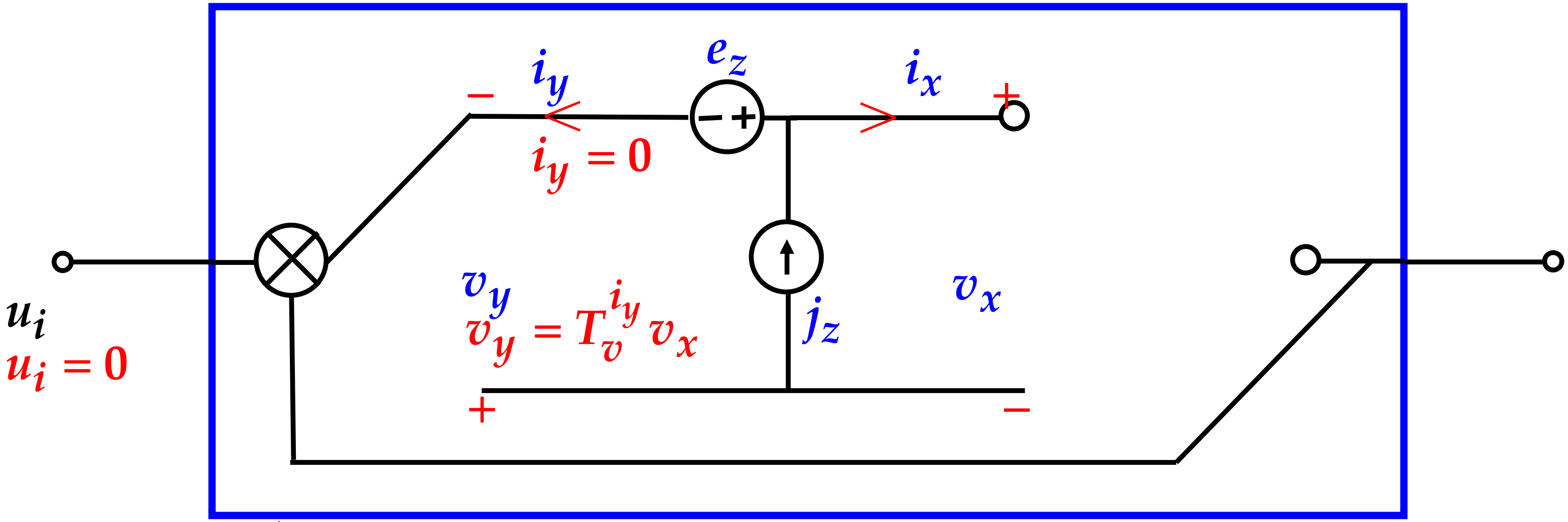
Definitions:



$$\frac{1}{T_i^{v_x}} \equiv \left.\frac{i_x}{i_y}\right|_{v_x=0}$$ = short-circuit reverse current loop gain $\equiv T_{i\ rev}^{sc}$

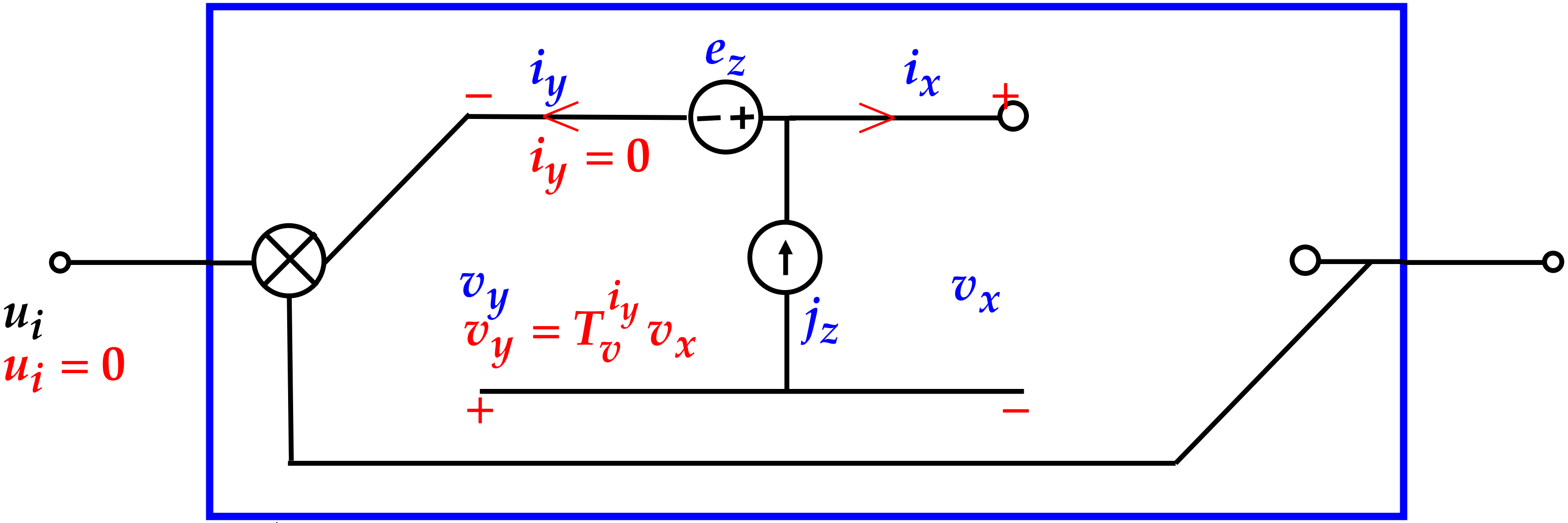


Definitions:

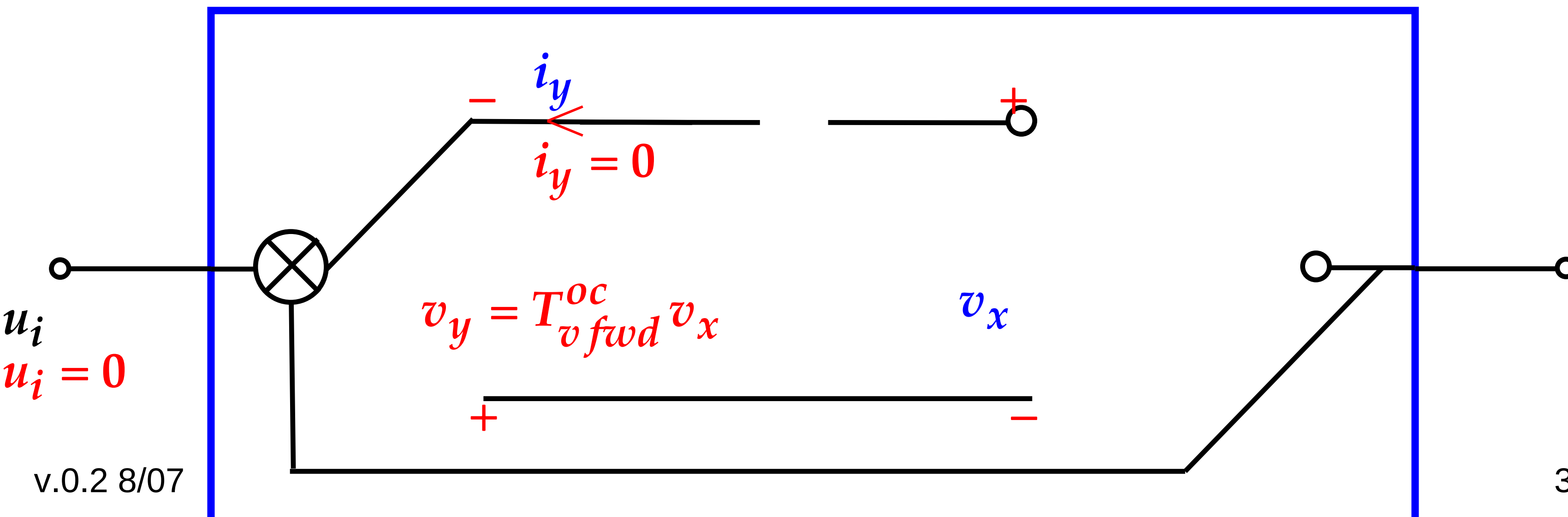


$$T_v^{i_y} \equiv \left.\frac{v_y}{v_x}\right|_{i_y=0}$$ = **open-circuit forward voltage loop gain**

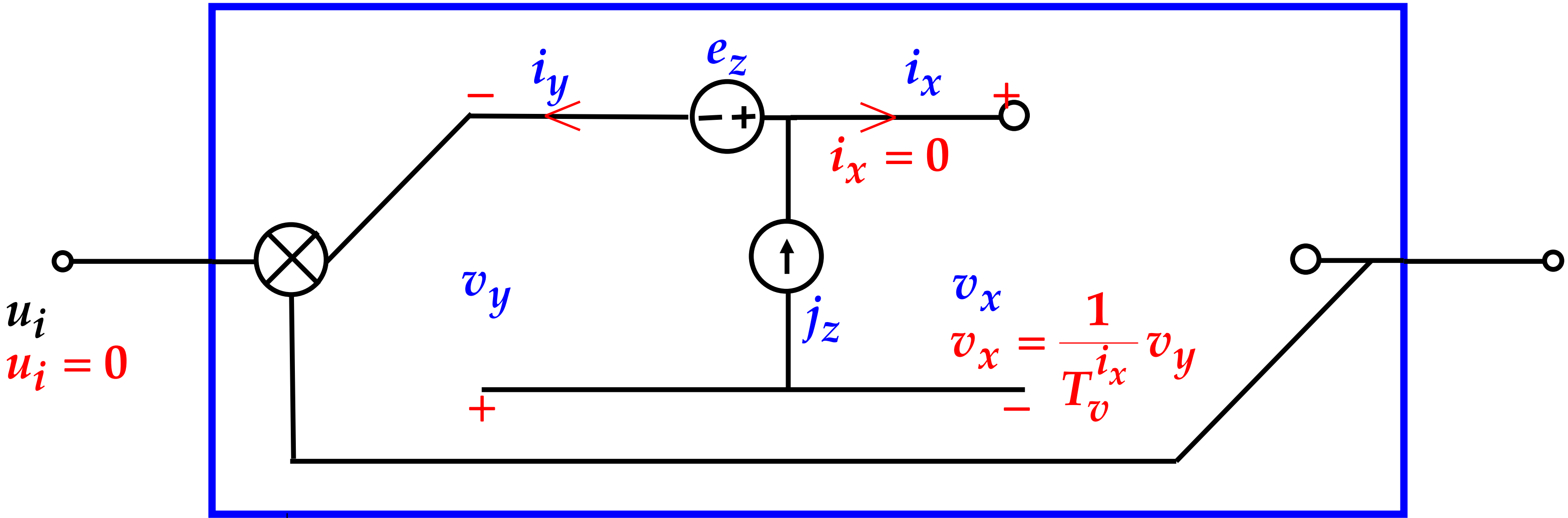
Definitions:



$$T_v^{i_y} \equiv \left.\frac{v_y}{v_x}\right|_{i_y=0}$$ = open-circuit **forward** voltage loop gain $\equiv T_{v\,fwd}^{oc}$

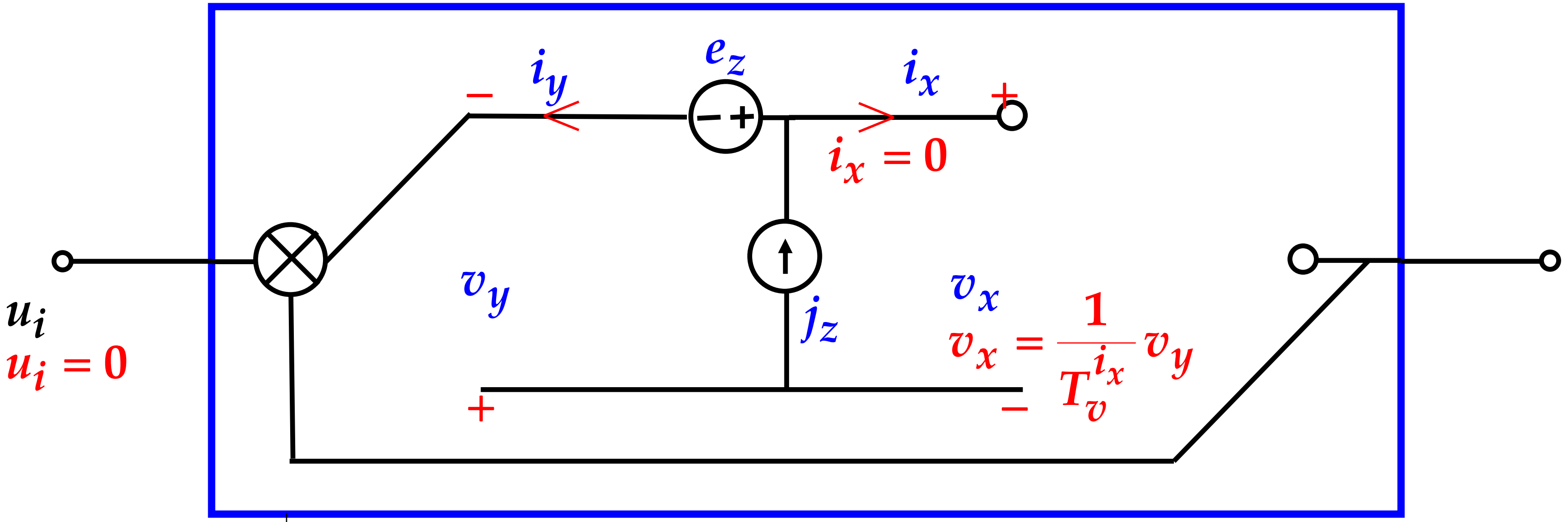


Definitions:

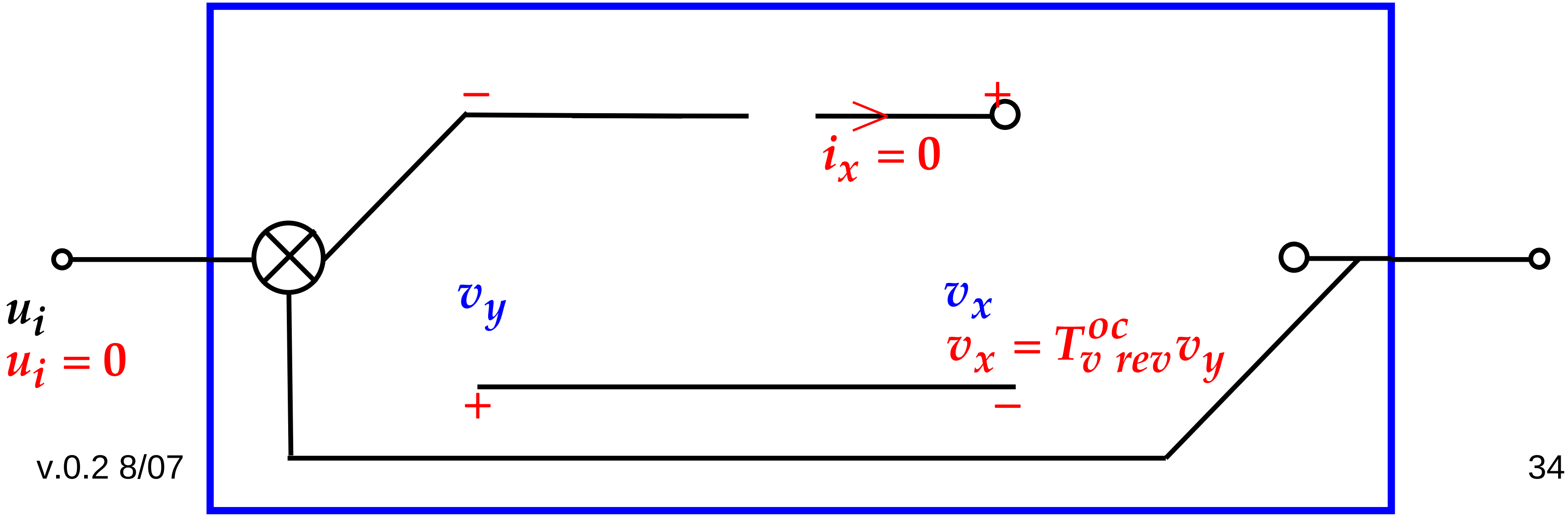


$$\frac{1}{T_v^{i_x}} \equiv \left. \frac{v_x}{v_y} \right|_{i_x=0}$$ = open-circuit reverse voltage loop gain

Definitions:



$$\frac{1}{T_v^{i_x}} \equiv \left.\frac{v_x}{v_y}\right|_{i_x=0}$$ = **open-circuit reverse voltage loop gain** $\equiv T_{v\ rev}^{oc}$



Summary so far:

The 2GFT: $$H = H_{\infty} \frac{1 + \frac{1}{T_n}}{1 + \frac{1}{T}}$$

$$\frac{1}{T} \equiv \frac{1}{T_i^{v_y}} + \frac{1}{T_v^{i_y}} + K_d \frac{1}{T_i^{v_y}} \frac{1}{T_v^{i_y}}$$

in which the second level TF T, the total loop gain, can be written in terms of the newly defined third level TFs:

$$T_i^{v_y} \equiv \left.\frac{i_y}{i_x}\right|_{v_y=0}$$ = **short-circuit forward current loop gain** $\equiv T_{i\,fwd}^{sc}$

$$T_v^{i_y} \equiv \left.\frac{v_y}{v_x}\right|_{i_y=0}$$ = **open-circuit forward voltage loop gain** $\equiv T_{v\,fwd}^{oc}$

$$\frac{1}{T_i^{v_x}} \equiv \left.\frac{i_x}{i_y}\right|_{v_x=0}$$ = **short-circuit reverse current loop gain** $\equiv T_{i\,rev}^{sc}$

$$\frac{1}{T_v^{i_x}} \equiv \left.\frac{v_x}{v_y}\right|_{i_x=0}$$ = **open-circuit reverse voltage loop gain** $\equiv T_{v\,rev}^{oc}$

$$\frac{1}{T} \equiv \frac{1}{T_{i\,fwd}^{sc}} + \frac{1}{T_{v\,fwd}^{oc}} + K_d \frac{1}{T_{i\,fwd}^{sc}} \frac{1}{T_{v\,fwd}^{oc}}$$

and the redundant interaction parameter K_d is

$$K_d \equiv \frac{T_i^{v_y}}{T_i^{v_x}} = \frac{T_v^{i_y}}{T_v^{i_x}} \qquad \text{or} \qquad K_d \equiv T_{i\,fwd}^{sc} T_{i\,rev}^{sc} = T_{v\,fwd}^{oc} T_{v\,rev}^{oc}$$

In words, the redundancy relation says that

K_d = **product of short-circuit forward and reverse current loop gains** = **product of open-circuit forward and reverse voltage loop gains**

The expression for T discloses that nonzero reverse loop gain enters only through K_d.

In the absence of reverse loop gain, T reduces to the forward loop gain T_{fwd} given by

$$\frac{1}{T_{fwd}} = \frac{1}{T_{i\,fwd}^{sc}} + \frac{1}{T_{v\,fwd}^{oc}} \qquad \text{or} \qquad T_{fwd} = T_{i\,fwd}^{sc} \,\Big\|\, T_{v\,fwd}^{oc}$$

In words, T_{fwd} is the parallel combination of the short-circuit forward current gain and the open circuit forward voltage gain, and is therefore dominated by whichever one is smaller.

Now that reverse current and voltage loop gains have been identified, a reverse loop gain T_{rev} can be defined:

$$\frac{1}{T_{rev}} = \frac{1}{T_{i\,rev}^{sc}} + \frac{1}{T_{v\,rev}^{oc}} \qquad \textbf{or} \qquad T_{rev} = T_{i\,rev}^{sc} \| T_{v\,rev}^{oc}$$

From the redundancy relation $K_d \equiv T_{i\,fwd}^{sc} T_{i\,rev}^{sc} = T_{v\,fwd}^{oc} T_{v\,rev}^{oc}$,

T_{rev} **can be written in terms of** K_d **as**

$$T_{rev} = \frac{K_d}{T_{i\,fwd}^{sc} + T_{v\,fwd}^{oc}}$$

or

$$K_d \frac{1}{T_{i\,fwd}^{sc}} \frac{1}{T_{v\,fwd}^{oc}} = T_{rev} \left(\frac{1}{T_{i\,fwd}^{sc}} + \frac{1}{T_{v\,fwd}^{oc}} \right)$$

Finally, by substitution for K_d in terms of T_{rev}, the total loop gain T

$$\frac{1}{T} \equiv \frac{1}{T_{i\,fwd}^{sc}} + \frac{1}{T_{v\,fwd}^{oc}} + K_d \frac{1}{T_{i\,fwd}^{sc}} \frac{1}{T_{v\,fwd}^{oc}}$$

becomes

$$\frac{1}{T} = \left(\frac{1}{T_{i\,fwd}^{sc}} + \frac{1}{T_{v\,fwd}^{oc}} \right) + T_{rev} \left(\frac{1}{T_{i\,fwd}^{sc}} + \frac{1}{T_{v\,fwd}^{oc}} \right) = \frac{1 + T_{rev}}{T_{fwd}}$$

or

$$\frac{1}{T} = \frac{1 + T_{rev}}{T_{fwd}}$$

or

$$T = \frac{T_{fwd}}{1 + T_{rev}}$$

A 1975 paper* showed how the loop gain could be found by combining the current and voltage loop gains obtained by simultaneous injection of current and voltage test signals at a nonideal injection point. That result is actually $T_{fwd} = T_{i\,fwd}^{sc} \| T_{v\,fwd}^{oc}$ obtained above.

However, the 1975 paper was based on a model that excluded nonzero reverse loop gain, and so the exact 2GFT result can be considered an extension of the previous limited result.

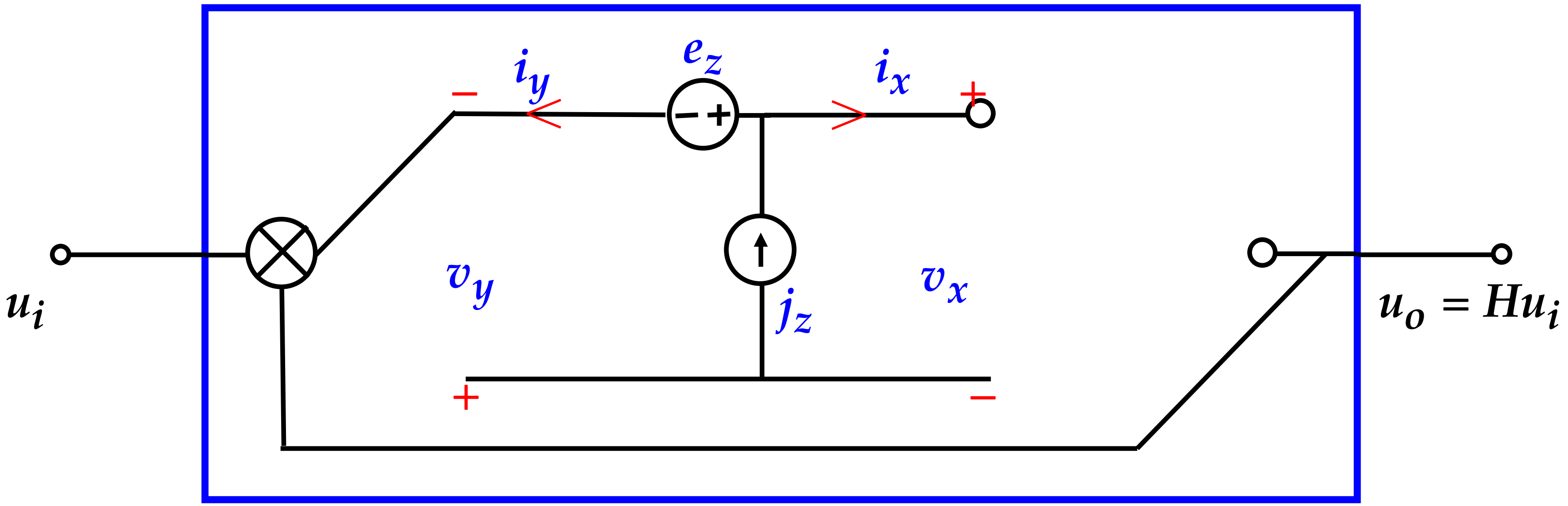
In summary, the total loop gain T given by the 2GFT is

$$T = \frac{T_{fwd}}{1 + T_{rev}}$$

where $$T_{fwd} = T_{i\,fwd}^{sc} \| T_{v\,fwd}^{oc} \quad \text{and} \quad T_{rev} = T_{i\,rev}^{sc} \| T_{v\,rev}^{oc}$$

*** R.D.Middlebrook, "Measurement of loop gain in feedback systems," Int. J. Electronics, 1975, vol. 38, No. 4, pp. 485 – 512.**

The 2GFT also tells how to find T_n when both current and voltage loop gains are present.

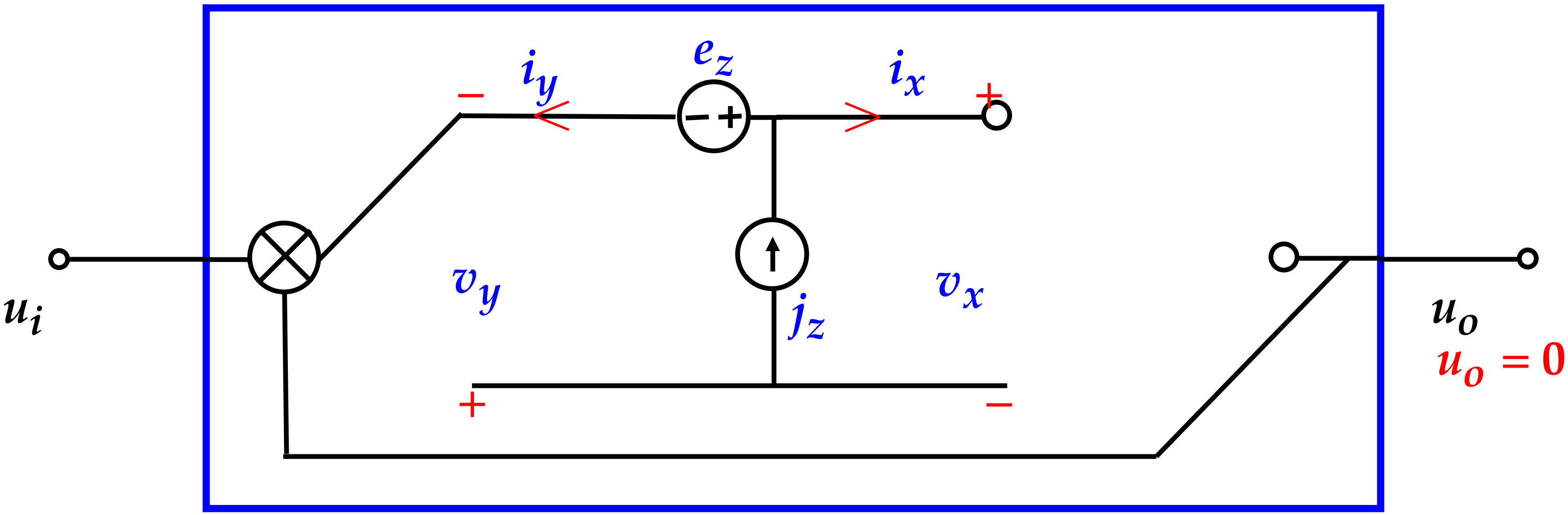


$$H = H^{i_y v_y} \frac{1 + \dfrac{1}{T_{ni}^{v_y}} + \dfrac{1}{T_{nv}^{i_y}} + K_n \dfrac{1}{T_{ni}^{v_y}} \dfrac{1}{T_{nv}^{i_y}}}{1 + \dfrac{1}{T_i^{v_y}} + \dfrac{1}{T_v^{i_y}} + K_d \dfrac{1}{T_i^{v_y}} \dfrac{1}{T_v^{i_y}}}$$

where K_d and K_n are interaction parameters:

$$K_d \equiv \frac{T_i^{v_y}}{T_i^{v_x}} = \frac{T_v^{i_y}}{T_v^{i_x}} \qquad K_n \equiv \frac{T_{ni}^{v_y}}{T_{ni}^{v_x}} = \frac{T_{nv}^{i_y}}{T_{nv}^{i_x}}$$

The four T_n's are defined in the same way as the four T's, except that the input u_i is restored and the output u_o is also nulled.



The four T_n's are double null triple injection (dnti) calculations, as is $H^{i_y v_y}$, which are even simpler and shorter than ndi calculations.

Since the structure of the null loop gain T_n is exactly the same as that of the loop gain T, the discussion for T can simply be repeated for T_n with the extra modifier "null", as follows:

Discussion of the total null loop gain T_n :

The 2GFT: $$H = H_\infty \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}}$$

$$\frac{1}{T_n} = \frac{1}{T^{sc}_{ni\,fwd}} + \frac{1}{T^{oc}_{nv\,fwd}} + K_n \frac{1}{T^{sc}_{ni\,fwd} T^{oc}_{nv\,fwd}}$$

in which the second level TF T_n, the total null loop gain, can be written in terms of the newly defined third level TFs:

$$T^{v_y}_{ni} \equiv \left.\frac{i_y}{i_x}\right|_{u_o, v_y=0}$$ = **short-circuit forward null current loop gain** $\equiv T^{sc}_{ni\,fwd}$

$$T^{i_y}_{nv} \equiv \left.\frac{v_y}{v_x}\right|_{u_o, i_y=0}$$ = **open-circuit forward null voltage loop gain** $\equiv T^{oc}_{nv\,fwd}$

$$\frac{1}{T^{v_x}_{ni}} \equiv \left.\frac{i_x}{i_y}\right|_{u_o, v_x=0}$$ = **short-circuit reverse null current loop gain** $\equiv T^{sc}_{ni\,rev}$

$$\frac{1}{T^{i_x}_{nv}} \equiv \left.\frac{v_x}{v_y}\right|_{u_o, i_x=0}$$ = **open-circuit reverse null voltage loop gain** $\equiv T^{oc}_{nv\,rev}$

$$\frac{1}{T_n} = \frac{1}{T^{sc}_{ni\,fwd}} + \frac{1}{T^{oc}_{nv\,fwd}} + K_n \frac{1}{T^{sc}_{ni\,fwd} T^{oc}_{nv\,fwd}}$$

and the redundant interaction parameter K_n is

$$K_n \equiv \frac{T^{v_y}_{ni}}{T^{v_x}_{ni}} = \frac{T^{i_y}_{nv}}{T^{i_x}_{nv}} \qquad \text{or} \qquad K_n = T^{sc}_{ni\,fwd} T^{sc}_{ni\,rev} = T^{oc}_{nv\,fwd} T^{oc}_{nv\,rev}$$

In words, the redundancy relation says that

K_n = **product of short-circuit forward and reverse null current loop gains** = **product of open-circuit forward and reverse null voltage loop gains**

The expression for T_n discloses that nonzero reverse null loop gain enters only through K_n.

In the absence of reverse null loop gain, T_n reduces to the forward null loop gain $T_{n\,fwd}$ given by

$$\frac{1}{T_{n\,fwd}} = \frac{1}{T^{sc}_{ni\,fwd}} + \frac{1}{T^{oc}_{nv\,fwd}} \quad \text{or} \quad T_{n\,fwd} = T^{sc}_{ni\,fwd} \,\|\, T^{oc}_{nv\,fwd}$$

In words, $T_{n\,fwd}$ is the parallel combination of the short-circuit forward null current loop gain and the open circuit forward null voltage loop gain, and is therefore dominated by whichever one is smaller.

Now that reverse current and voltage null loop gains have been identified, a reverse null loop gain $T_{n\,rev}$ can be defined:

$$\frac{1}{T_{n\,rev}} = \frac{1}{T_{ni\,rev}^{sc}} + \frac{1}{T_{nv\,rev}^{oc}} \qquad \textbf{or} \qquad T_{n\,rev} = T_{ni\,rev}^{sc} \,\|\, T_{nv\,rev}^{oc}$$

From the redundancy relation $K_n = T_{ni\,fwd}^{sc}\,T_{ni\,rev}^{sc} = T_{nv\,fwd}^{oc}\,T_{nv\,rev}^{oc}$**,** $T_{n\,rev}$ **can be written in terms of** K_n **as**

$$T_{n\,rev} = \frac{K_n}{T_{ni\,fwd}^{sc} + T_{nv\,fwd}^{oc}}$$

or

$$K_n \frac{1}{T_{ni\,fwd}^{sc}} \frac{1}{T_{nv\,fwd}^{oc}} = T_{n\,rev}\left(\frac{1}{T_{ni\,fwd}^{sc}} + \frac{1}{T_{nv\,fwd}^{oc}}\right)$$

Finally, by substitution for K_n in terms of $T_{n\,rev}$, the total null loop gain T_n

$$\frac{1}{T_n} = \frac{1}{T_{ni\,fwd}^{sc}} + \frac{1}{T_{nv\,fwd}^{oc}} + K_n \frac{1}{T_{ni\,fwd}^{sc} T_{nv\,fwd}^{oc}}$$

becomes

$$\frac{1}{T_n} = \left(\frac{1}{T_{ni\,fwd}^{sc}} + \frac{1}{T_{nv\,fwd}^{oc}} \right) + T_{n\,rev} \left(\frac{1}{T_{ni\,fwd}^{sc}} + \frac{1}{T_{nv\,fwd}^{oc}} \right) = \frac{1 + T_{n\,rev}}{T_{n\,fwd}}$$

or

$$\frac{1}{T_n} = \frac{1 + T_{n\,rev}}{T_{n\,fwd}}$$

or

$$T_n = \frac{T_{n\,fwd}}{1 + T_{n\,rev}}$$

In summary, the 2GFT in terms of the newly defined TFs is

$$H = H_{\infty} \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}}$$ **where**

$$H_{\infty} = H^{i_y v_y}$$

$$T = \frac{T_{fwd}}{1+T_{rev}}$$ **in which** $T_{fwd} = T^{sc}_{i\,fwd} \| T^{oc}_{v\,fwd}$ **and** $T_{rev} = T^{sc}_{i\,rev} \| T^{oc}_{v\,rev}$

$$T_n = \frac{T_{n\,fwd}}{1+T_{n\,rev}}$$ **in which** $T_{n\,fwd} = T^{sc}_{ni\,fwd} \| T^{oc}_{nv\,fwd}$ **and** $T_{n\,rev} = T^{sc}_{ni\,rev} \| T^{oc}_{nv\,rev}$

Reminder:

$$H = H_\infty \frac{T}{1+T} + H_0 \frac{1}{1+T} = H_\infty \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}} = H_\infty D D_n$$

Redundancy Relation: $$\frac{H_\infty}{H_0} = \frac{T_n}{T}$$

In order for the *definitions* of the four second-level TFs to have the following *interpretations*

$H_\infty =$ **ideal closed loop gain**

$T =$ **principal loop gain**

$H_0 =$ **direct forward transmission**

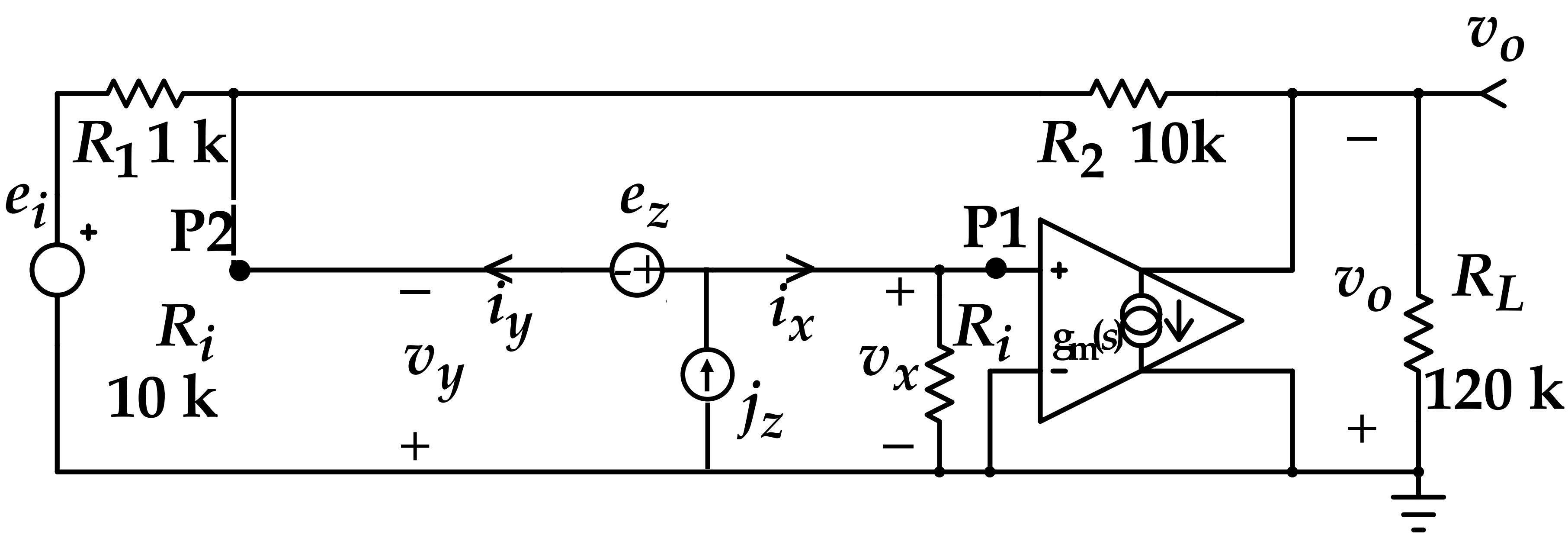
$T_n =$ **null loop gain**

it is necessary that the Test Signal Injection Configuration satisfy two conditions:

1. **The test signal u_z (e_z and/or j_z) must be injected so that u_y (i_y and/or v_y) is the error signal.**

2. **The test signal must be injected inside the major loop, but outside any minor loops.**

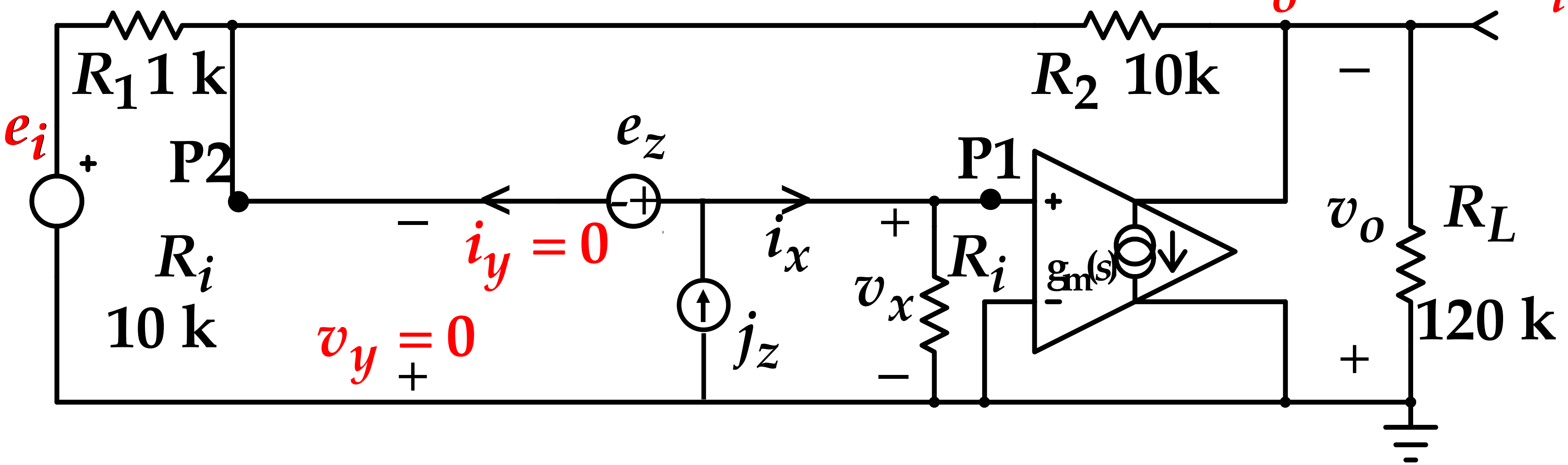
Inverting Opamp with nonideal injection point



Inject both a test voltage and a test current

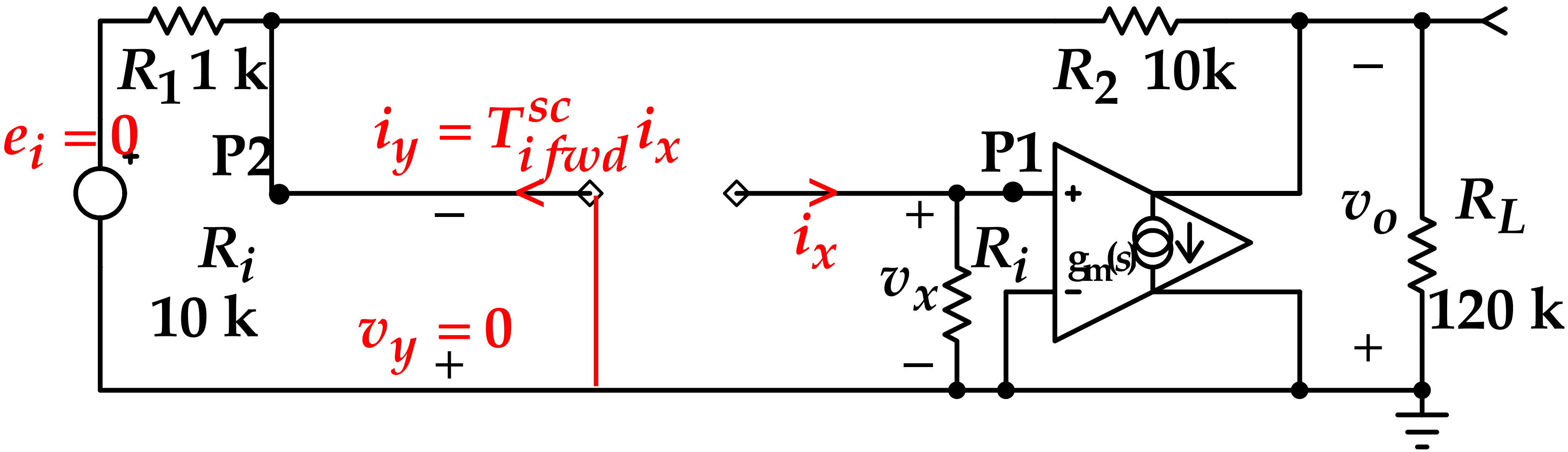
$$H = H_\infty \frac{1 + \frac{1}{T_n}}{1 + \frac{1}{T}}$$

$$H_{\infty} = \left.\frac{v_o}{e_i}\right|_{i_y, v_y = 0} \equiv H^{i_y v_y}$$



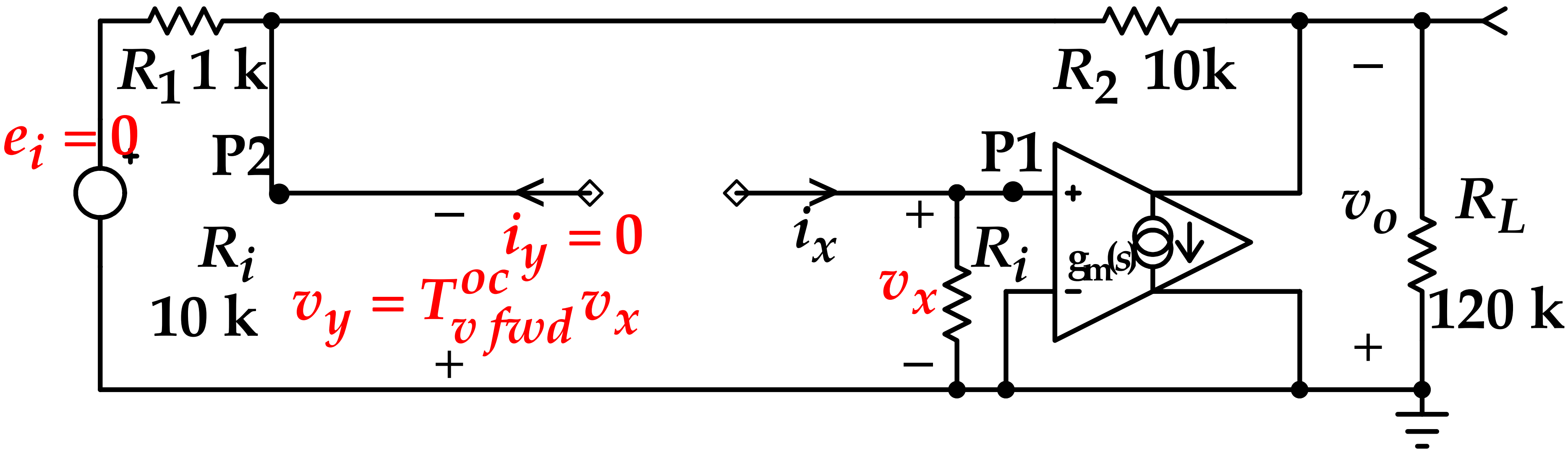
$$H^{i_y v_y} = \frac{R_2}{R_1} = 10 \Rightarrow 20 \text{ dB}$$

$$T_i^{v_y} = \left.\frac{i_y}{i_x}\right|_{e_i, v_y = 0} \equiv T_{i\,fwd}^{sc}$$



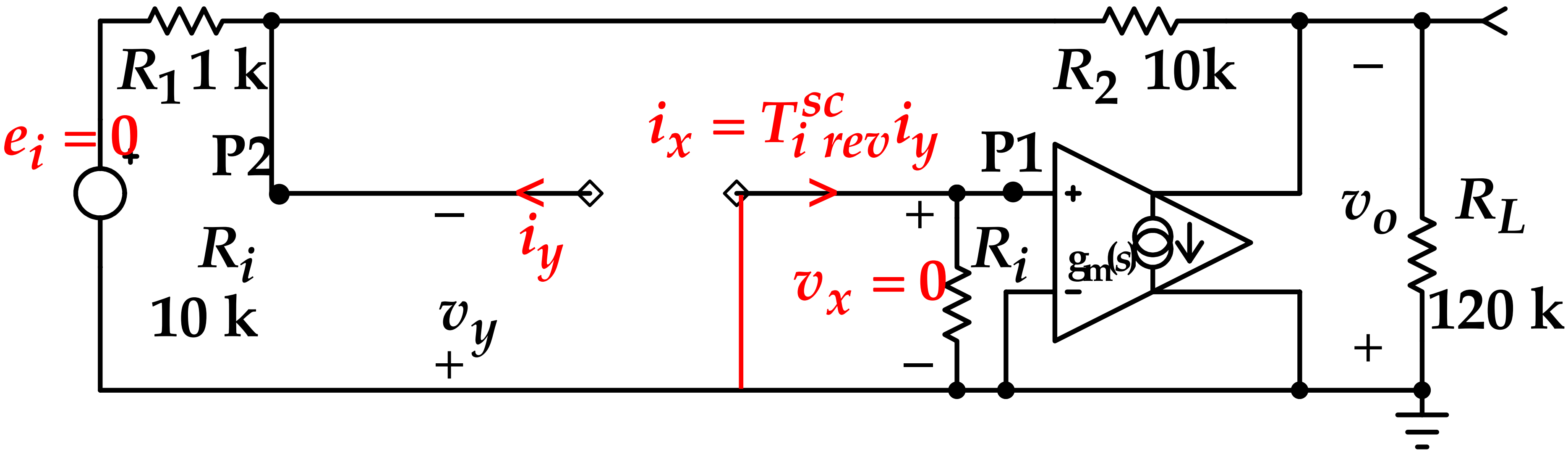
$$T_{i\,fwd}^{sc} = R_i g_m(s) \frac{R_L}{R_L + R_2}$$

$$T_v^{i_y} = \left.\frac{v_y}{v_x}\right|_{e_i, i_y = 0} \equiv T_{v\,fwd}^{oc}$$



$$T_{v\,fwd}^{oc} = g_m(s)\frac{R_L}{R_L + R_2 + R_1} R_1$$

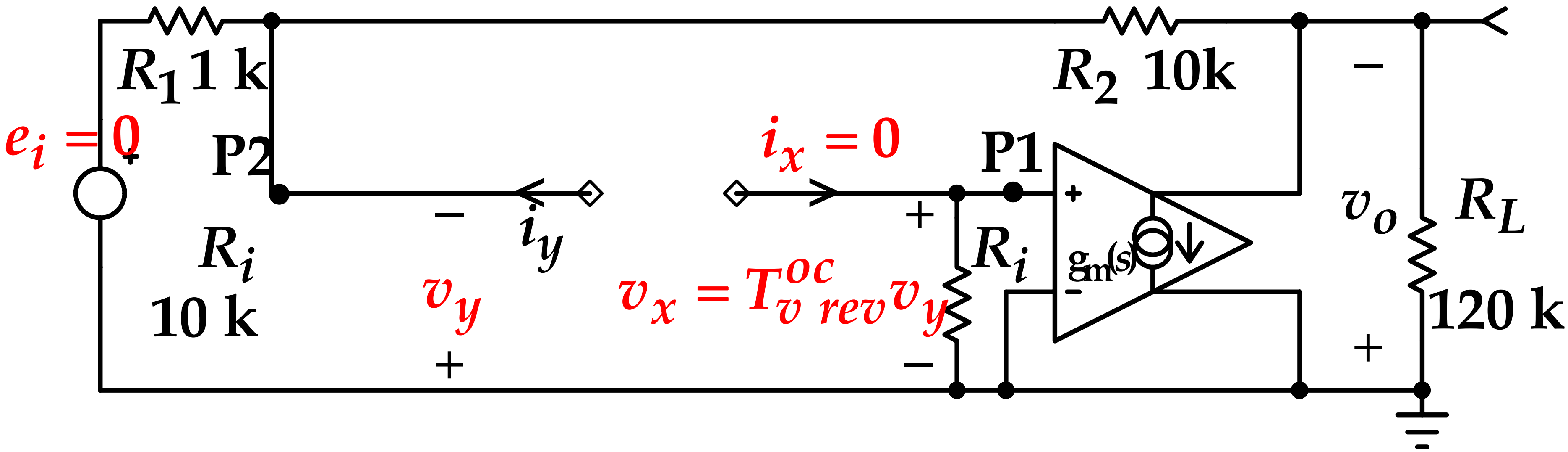
$$\frac{1}{T_i^{v_x}} \equiv \left.\frac{i_x}{i_y}\right|_{v_x=0} \equiv T_{i\,rev}^{sc}$$



$v_x = 0,$ and because there is no reverse current loop gain,

$i_x = 0$ so $T_{i\,rev}^{sc} = 0.$

$$\frac{1}{T_v^{i_x}} \equiv \left.\frac{v_x}{v_y}\right|_{i_x=0} \equiv T_{v\ rev}^{oc}$$



$i_x = 0$, **and because there is no reverse voltage loop gain,**

$v_x = 0$ **so** $T_{v\ rev}^{oc} = 0.$

Finally,

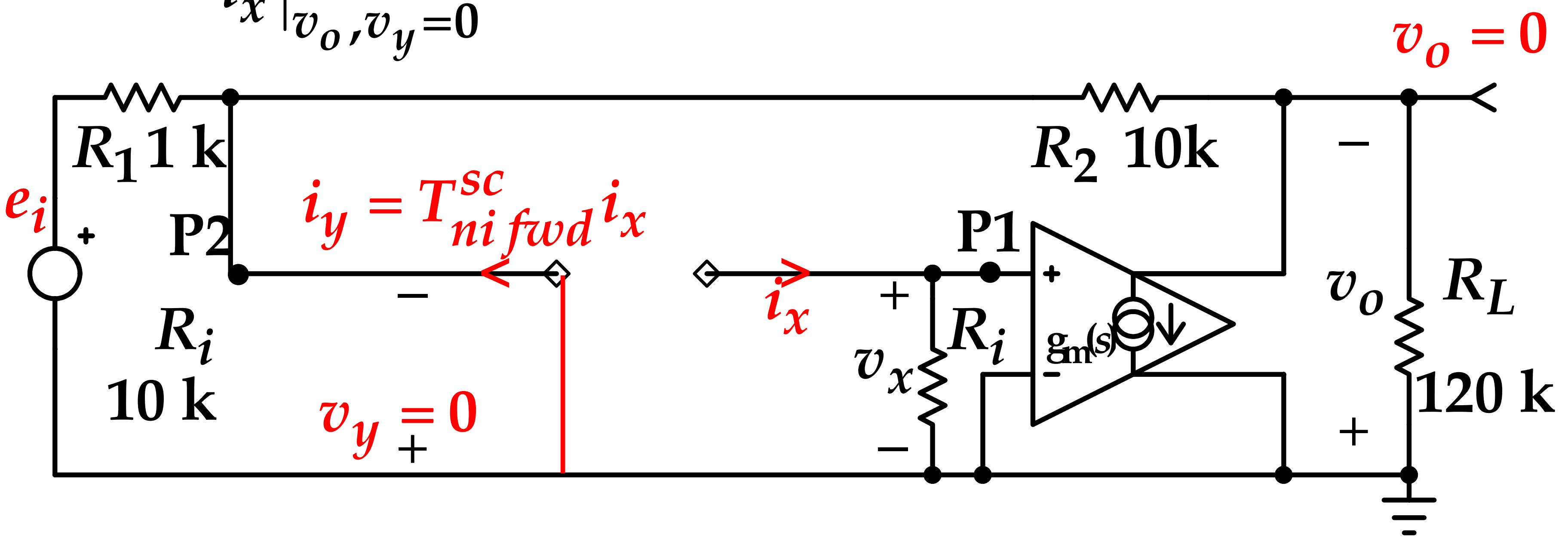
$$T = \frac{T_{fwd}}{1 + T_{rev}} \quad \textbf{where} \quad T_{rev} = T_{i\,rev}^{sc} \| T_{v\,rev}^{oc} = 0,$$

so $T = T_{fwd} = T_{i\,fwd}^{sc} \| T_{v\,fwd}^{oc}$

$$T(0) = g_m(0) \frac{R_L}{R_L + R_2 + R_1 \| R_i} \left(R_1 \| R_i \right) = 92 \Rightarrow 39.2 \text{ dB}$$

which is the same as $T_v(0)$ **obtained from single voltage injection at the ideal voltage injection point P1.**

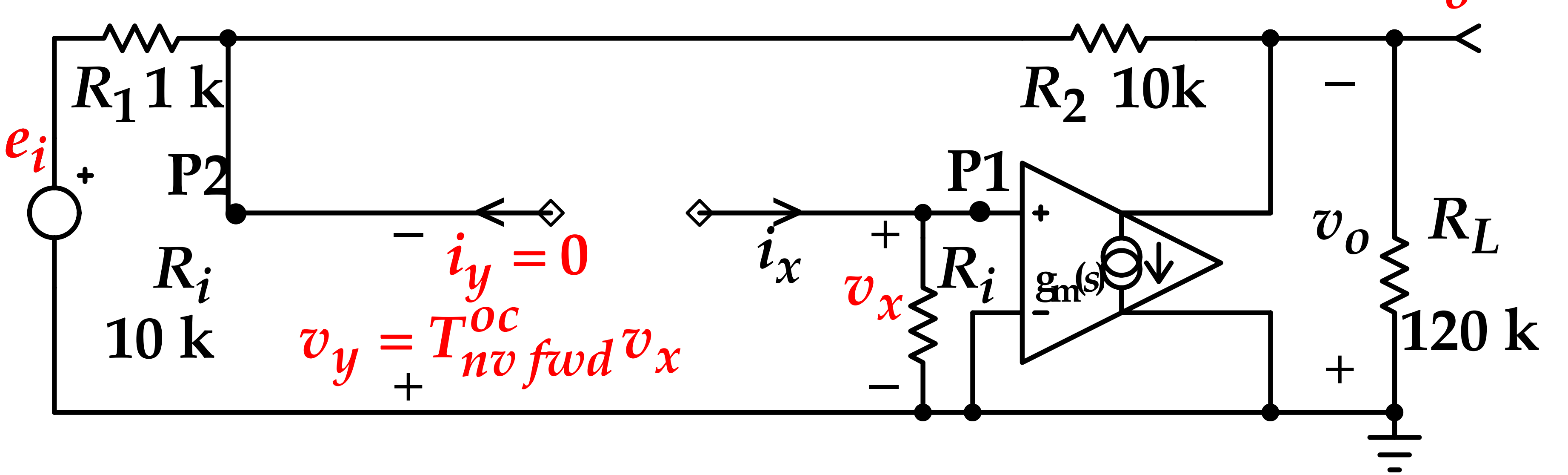
$$T_{ni}^{v_y} = \left. \frac{i_y}{i_x} \right|_{v_o, v_y = 0} \equiv T_{ni\,fwd}^{sc}$$



Because v_y and v_o are both nulled, there is no current in R_2 or R_L, so $i_x = 0$ and

$$T_{ni\,fwd}^{sc} = \infty$$

$$T_{nv}^{i_y} = \left.\frac{v_y}{v_x}\right|_{v_o, i_y = 0} \equiv T_{nv\,fwd}^{oc}$$



$$T_{nv\,fwd}^{oc} = g_m(s) R_2$$

$T_{ni\,rev}^{sc}$ and $T_{nv\,rev}^{oc}$ are both zero for the same reason that $T_{i\,rev}^{sc}$ and $T_{v\,rev}^{oc}$ are zero: there is no reverse current or voltage loop gain.

Finally,

$$T_n = \frac{T_{n\,fwd}}{1 + T_{n\,rev}} \textbf{ where } \quad T_{n\,rev} = T^{sc}_{ni\,rev} \Big\| T^{oc}_{nv\,rev} = 0$$

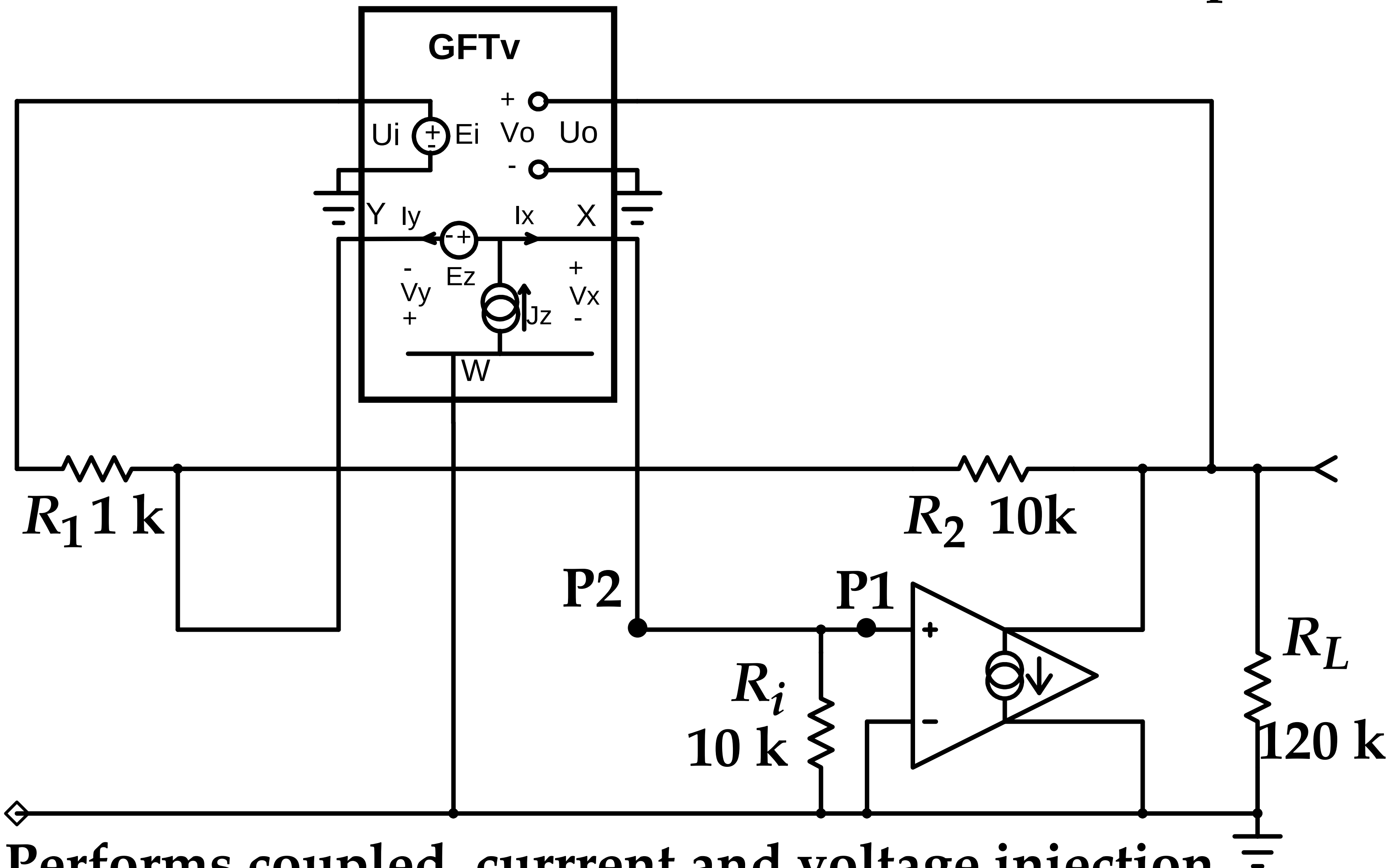
so $T_n = T_{n\,fwd} = T^{sc}_{ni\,fwd} \Big\| T^{oc}_{nv\,fwd}$

However, $T^{sc}_{ni\,fwd} = \infty$, so $T_n = T^{oc}_{nv\,fwd}$

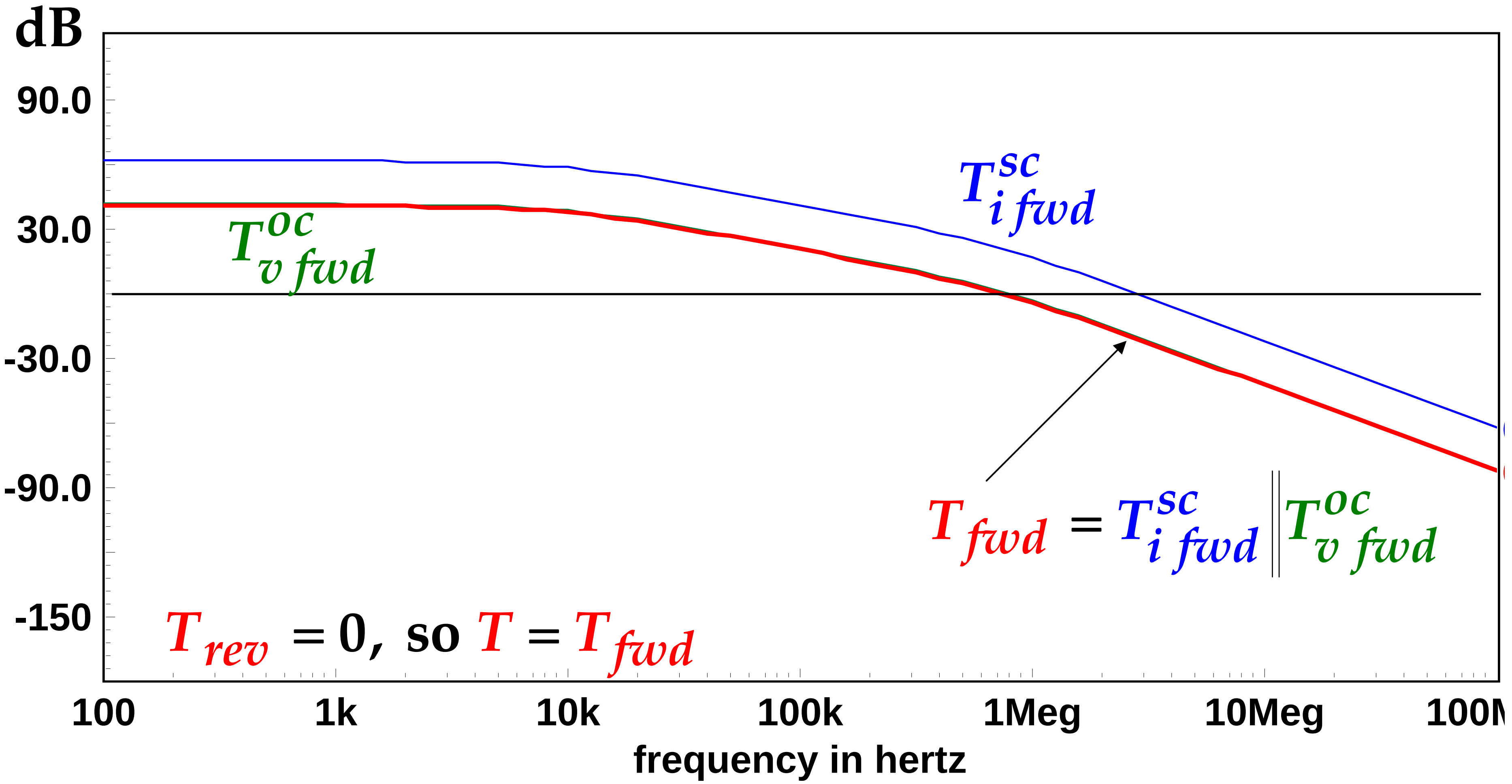
$$T_n = g_m(s)R_2$$

which is the same as T_{nv} obtained from single voltage injection at the ideal voltage injection point P1.

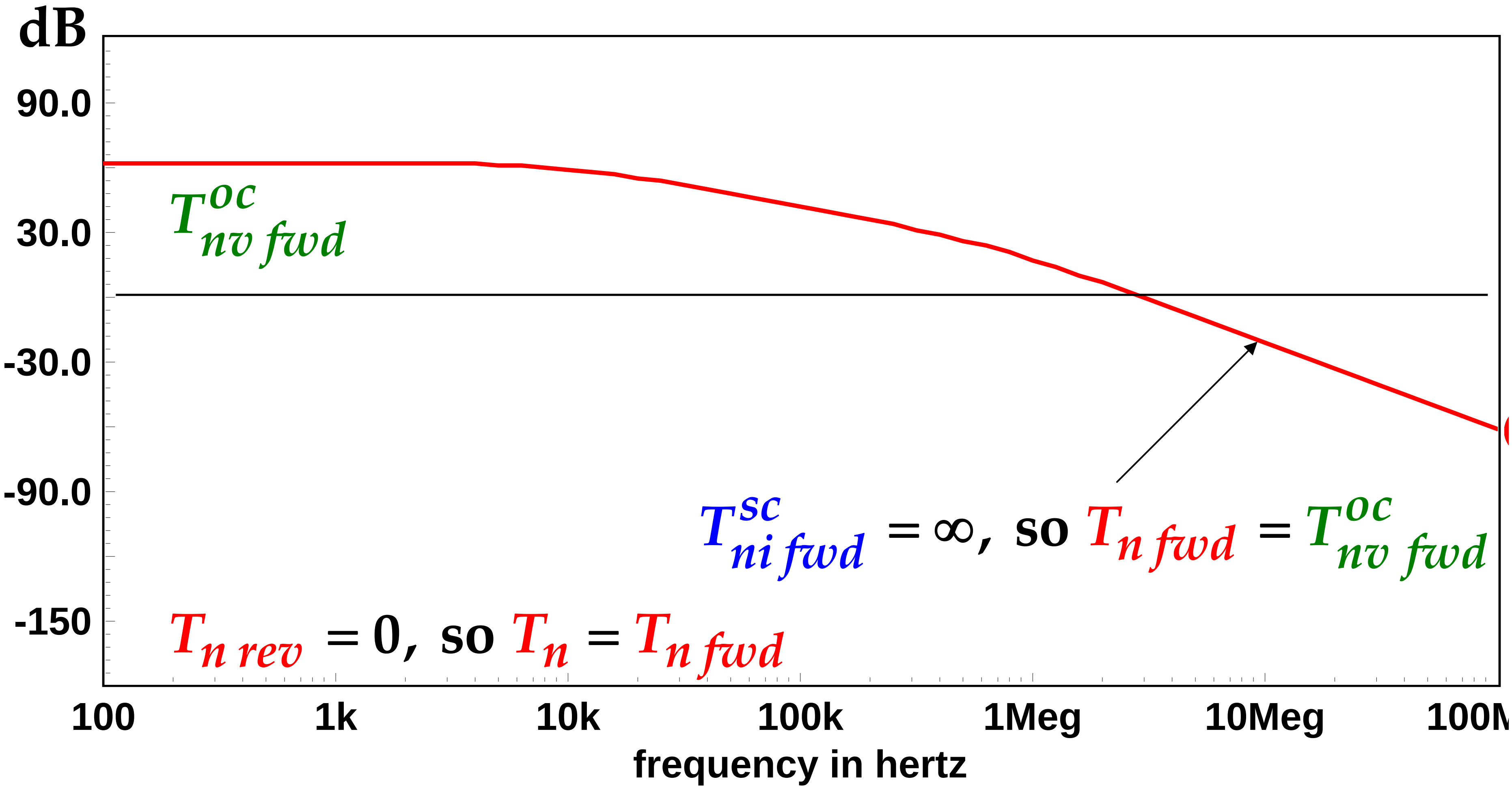
Intusoft ICAP/4 Circuit Simulator with GFT Template

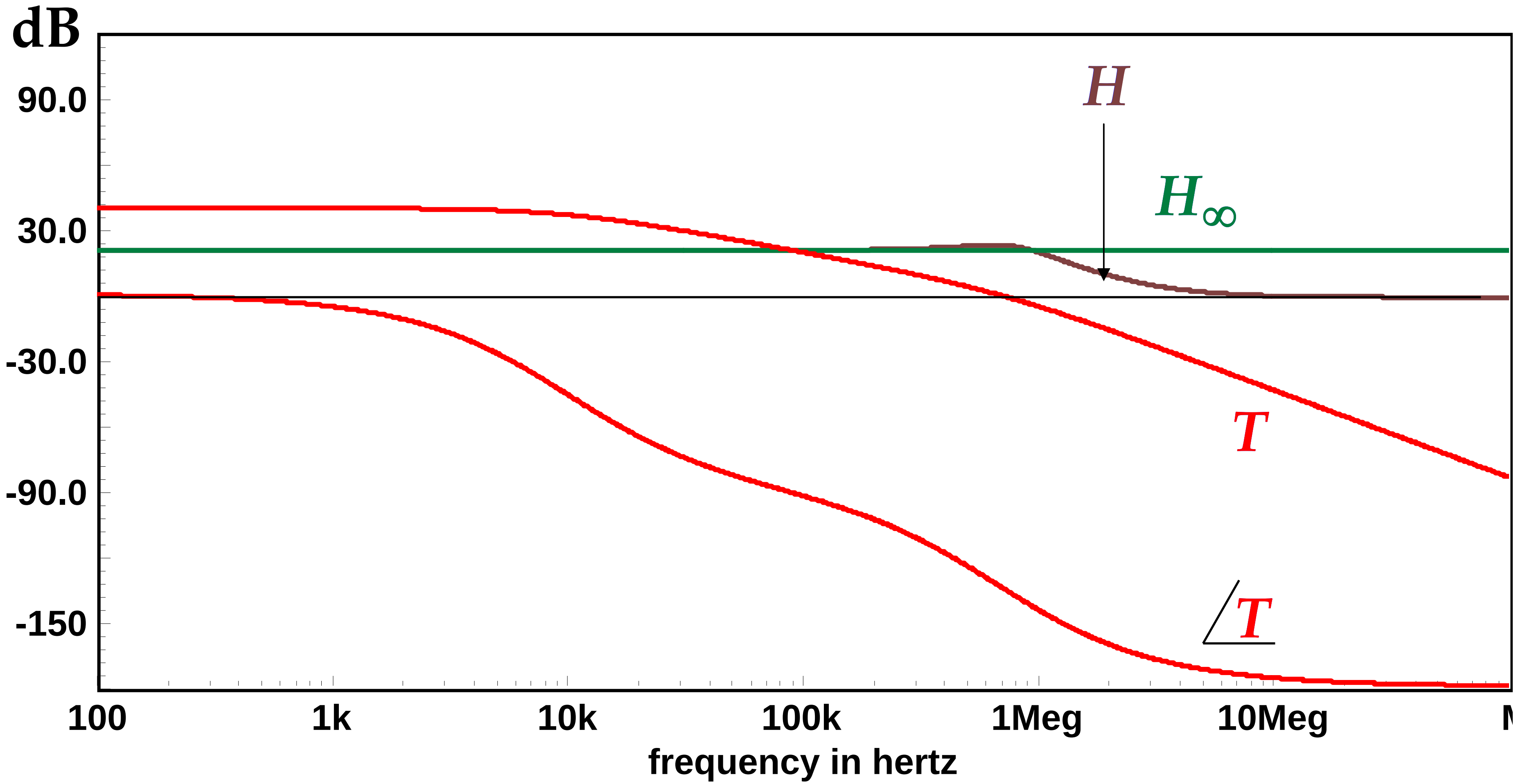


Performs coupled currrent and voltage injection at the same nonideal point P2



dB
90.0
30.0
-30.0
-90.0
-150
$T^{sc}_{i\,fwd}$
$T^{oc}_{v\,fwd}$
$T_{fwd} = T^{sc}_{i\,fwd} \| T^{oc}_{v\,fwd}$
$T_{rev} = 0$, so $T = T_{fwd}$
100
1k
10k
100k
1Meg
10Meg
100M
frequency in hertz


dB
90.0
30.0
-30.0
-90.0
-150
$T^{oc}_{nv\,fwd}$
$T^{sc}_{ni\,fwd} = \infty$, so $T_{n\,fwd} = T^{oc}_{nv\,fwd}$
$T_{n\,rev} = 0$, so $T_n = T_{n\,fwd}$
100
1k
10k
100k
1Meg
10Meg
frequency in hertz



Results are the same as for single voltage injection at the ideal point P1

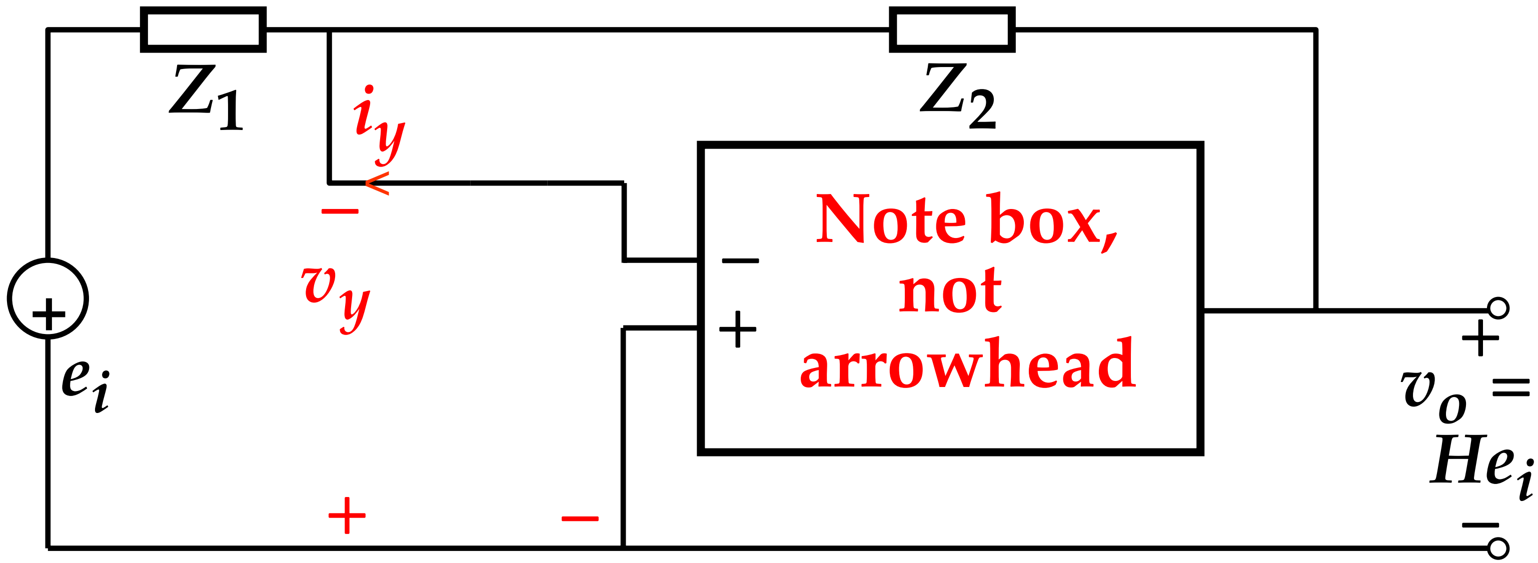
An *injection configuration* is defined by *which* test signals are adopted, and *where* they are injected.

Different injection configurations produce different sets of second-level TFs, but each set combines to give the *same* first-level TF H.

Therefore, the choice of Test Signal Injection Configuration is crucial in making the second-level TFs H_∞, T, T_n have the desired interpretations.

Basic Inverting Opamp

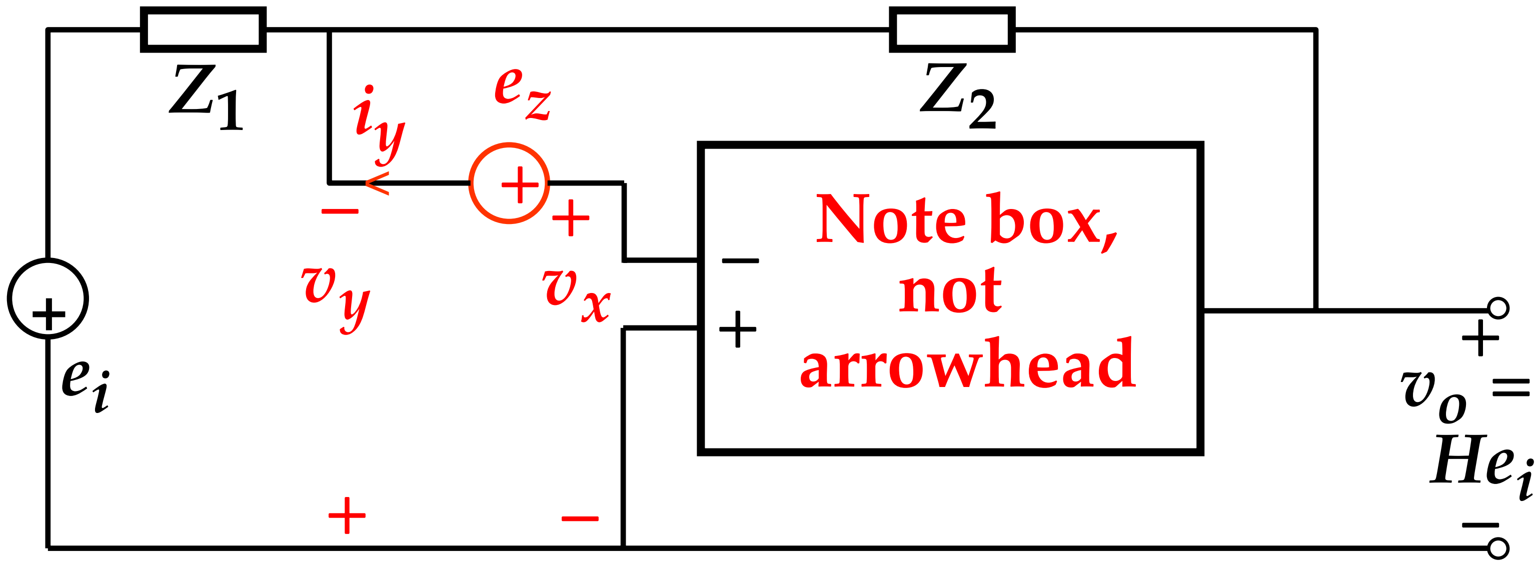
Identify error voltage v_y and error current i_y :



$$H_\infty \equiv \text{ideal closed-loop gain} = H^{i_y v_y} = \frac{Z_2}{Z_1}$$

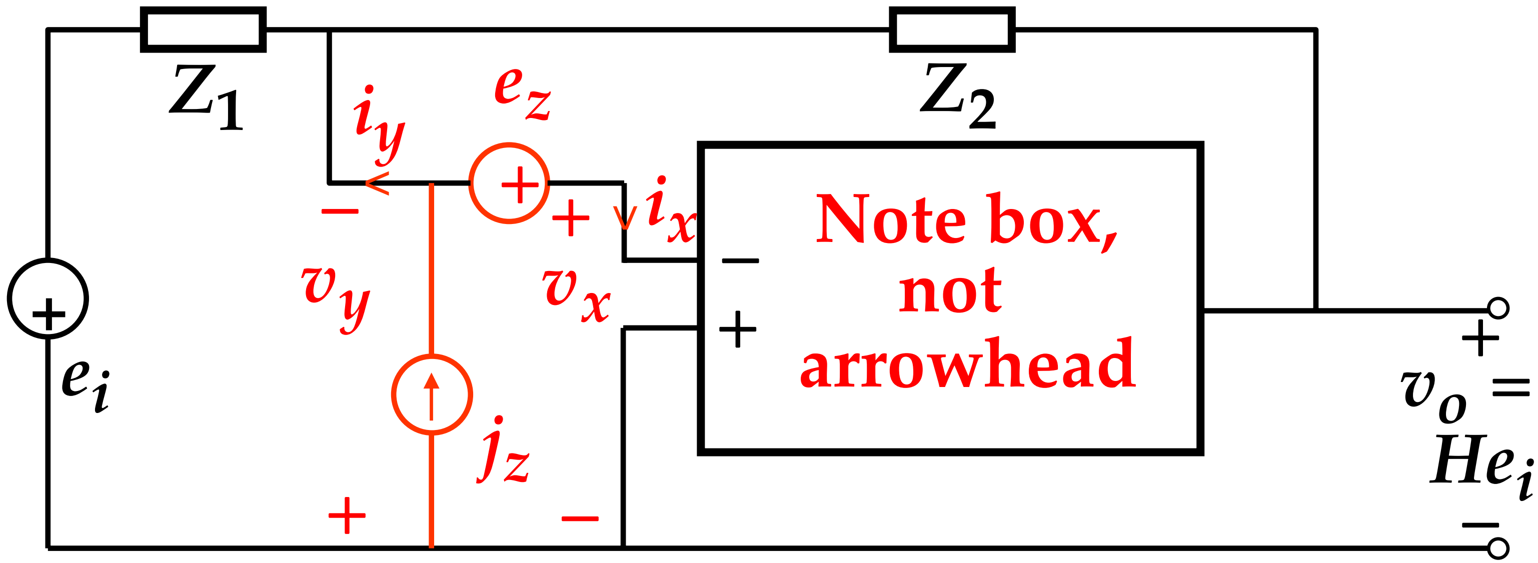
(Requirement 1)

Identify v_x drive to the forward path, and hence where e_z is to be injected (Requirement 2):



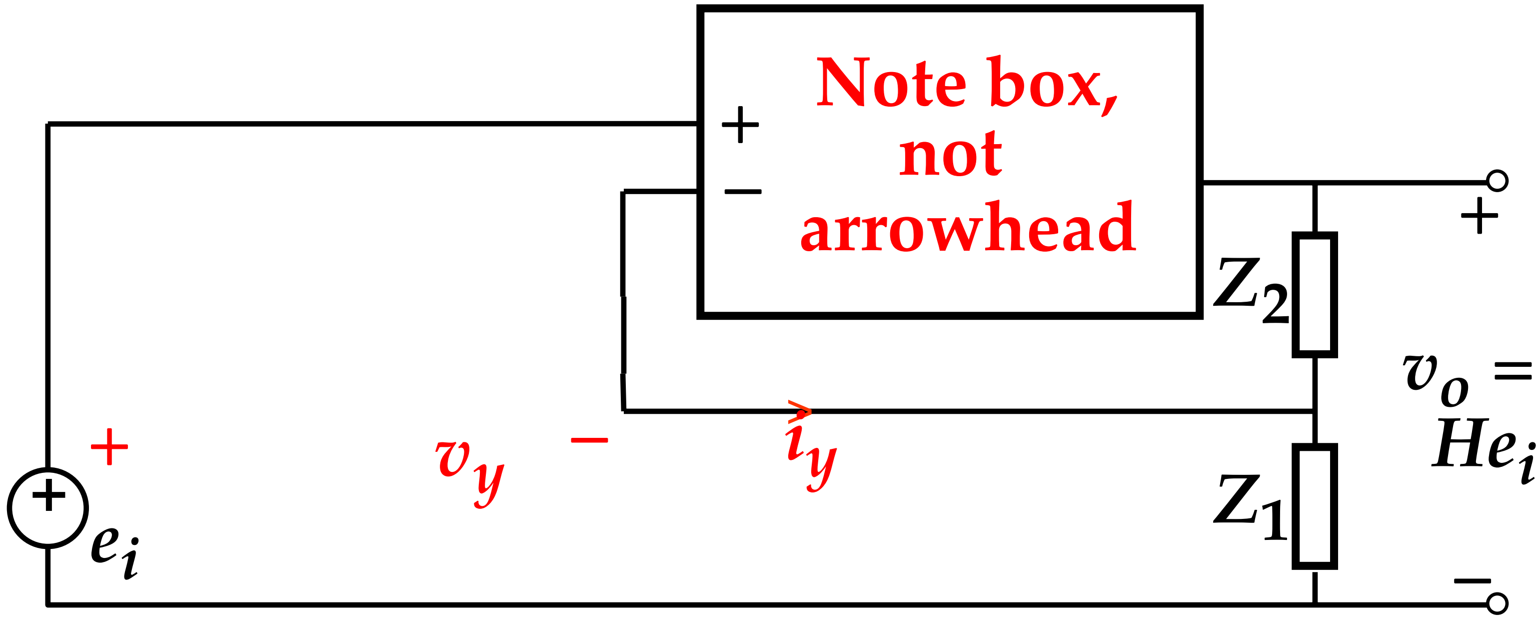
When v_y is nulled, is i_y automatically nulled?

If not, identify i_x drive to the forward path, and hence where j_z is to be injected:



Basic Noninverting Opamp

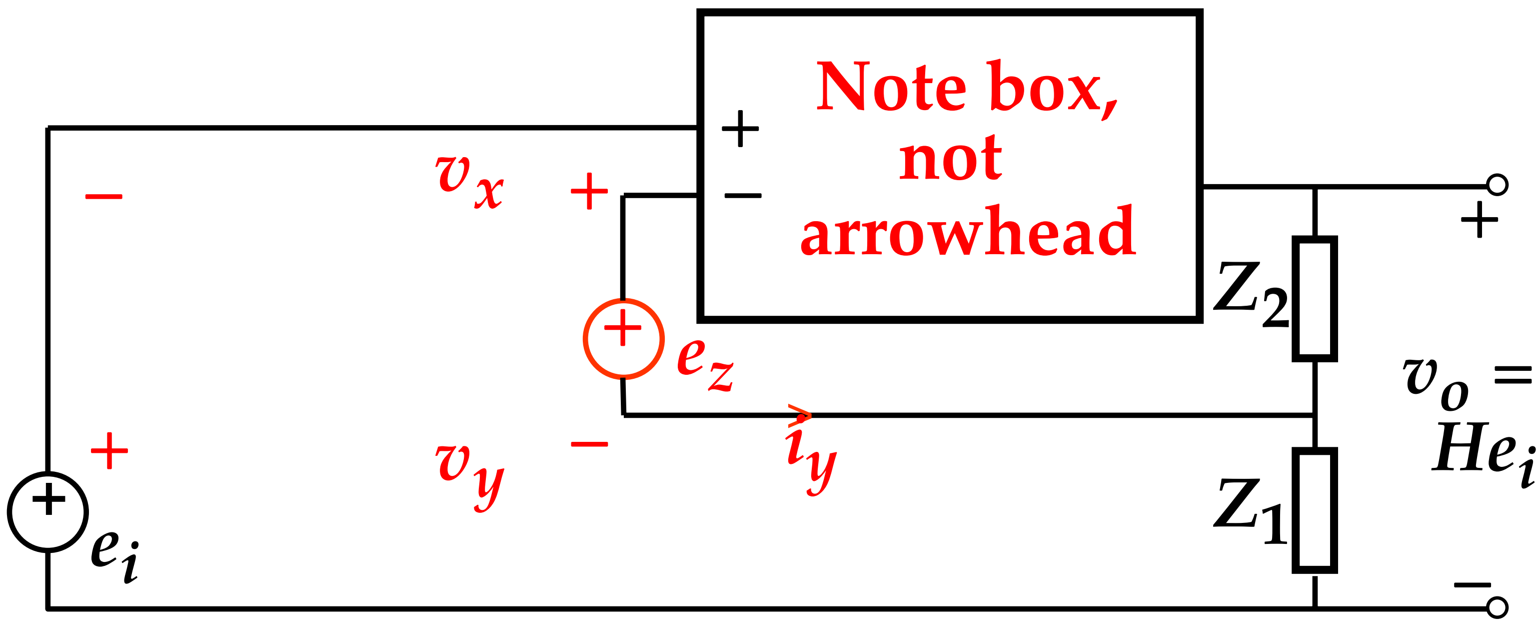
Identify error voltage v_y and error current i_y :



$$H_\infty \equiv \textbf{ideal closed-loop gain} = H^{i_y v_y} = \frac{Z_1 + Z_2}{Z_1}$$

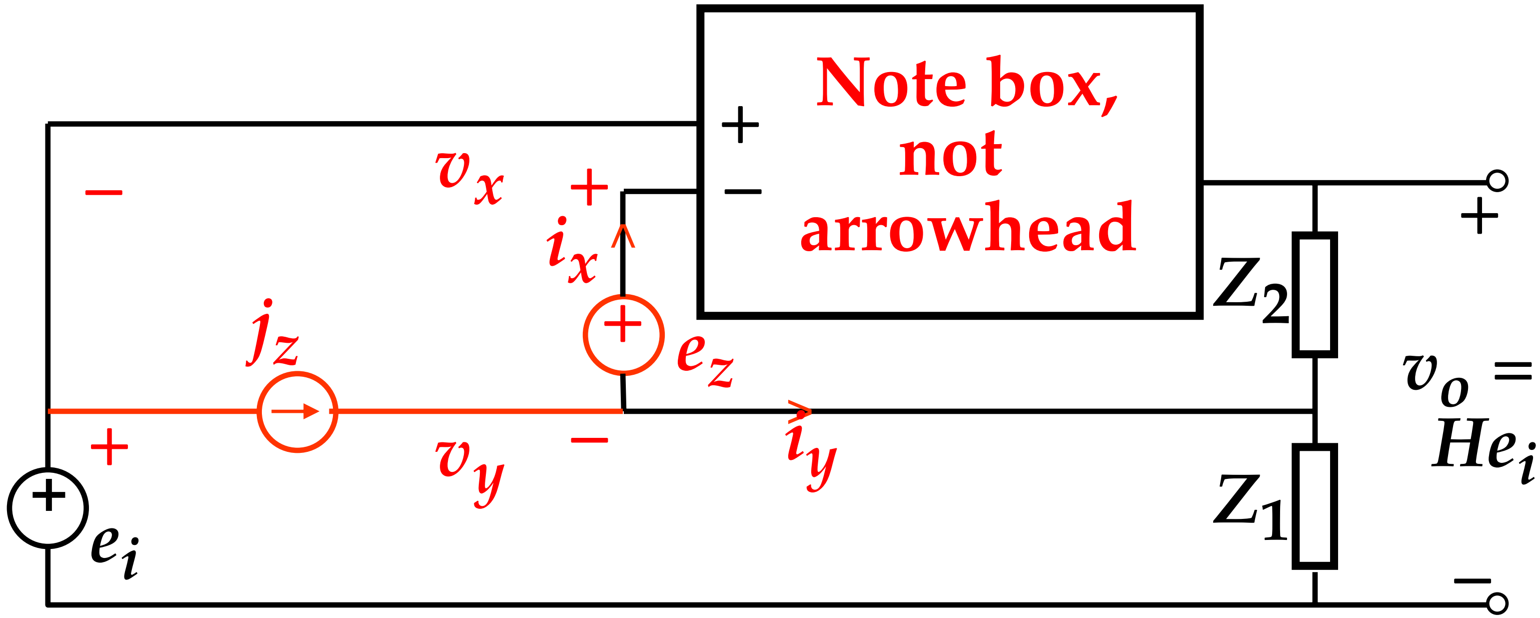
(Requirement 1)

Identify v_x drive to the forward path, and hence where e_z is to be injected (Requirement 2):

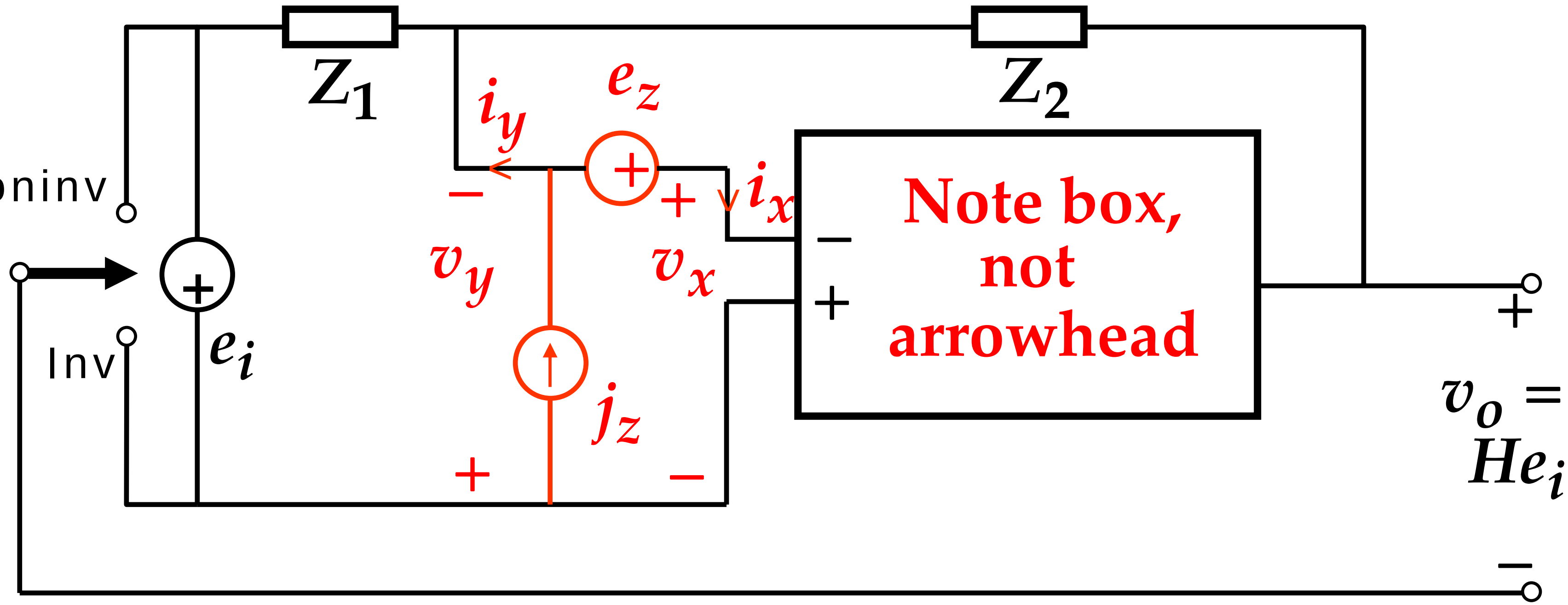


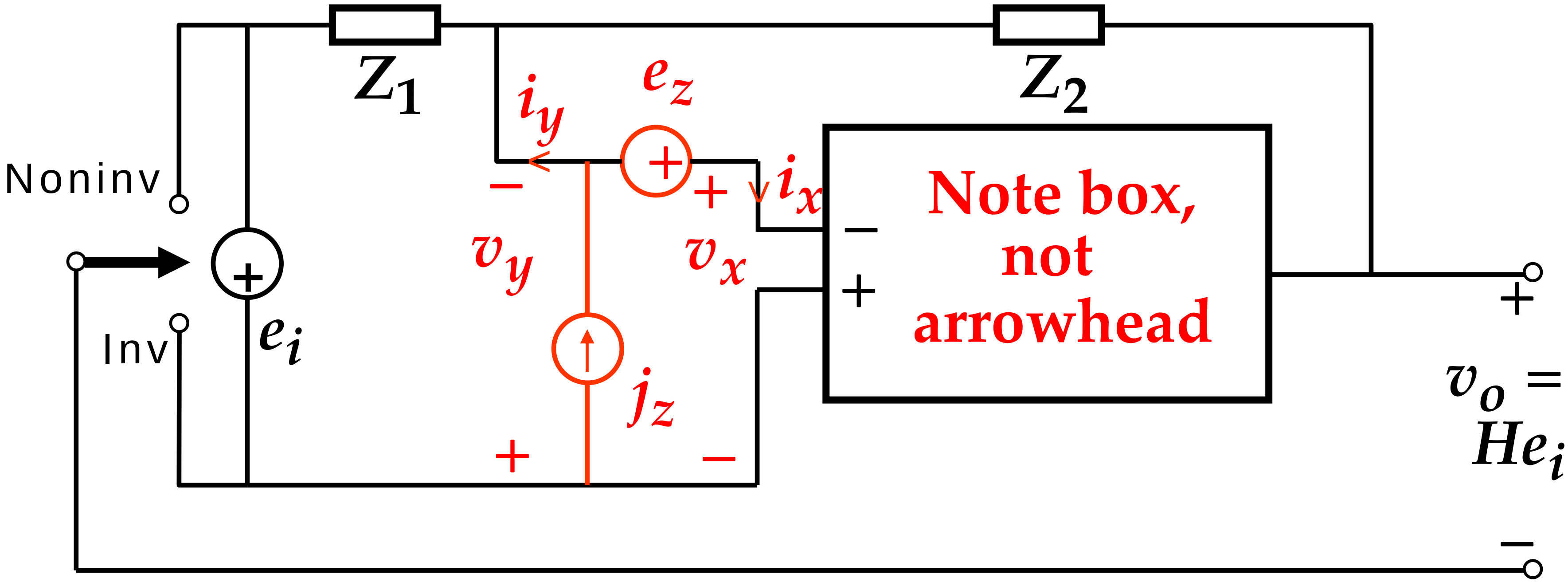
When v_y is nulled, is i_y automatically nulled?

If not, identify i_x drive to the forward path, and hence where j_z is to be injected:



Despite the apparently different layouts, both the Inv and Noninv configurations are the same, except that the reference node for the output voltage is different:





$$H_{\mathbf{Inv}} = H_{\infty\mathbf{Inv}} \frac{1+\frac{1}{T_{n\mathbf{Inv}}}}{1+\frac{1}{T}} \qquad H_{\mathbf{Noninv}} = H_{\infty\mathbf{Noninv}} \frac{1+\frac{1}{T_{n\mathbf{Noninv}}}}{1+\frac{1}{T}}$$

The principal loop gain T is the *same* for both;

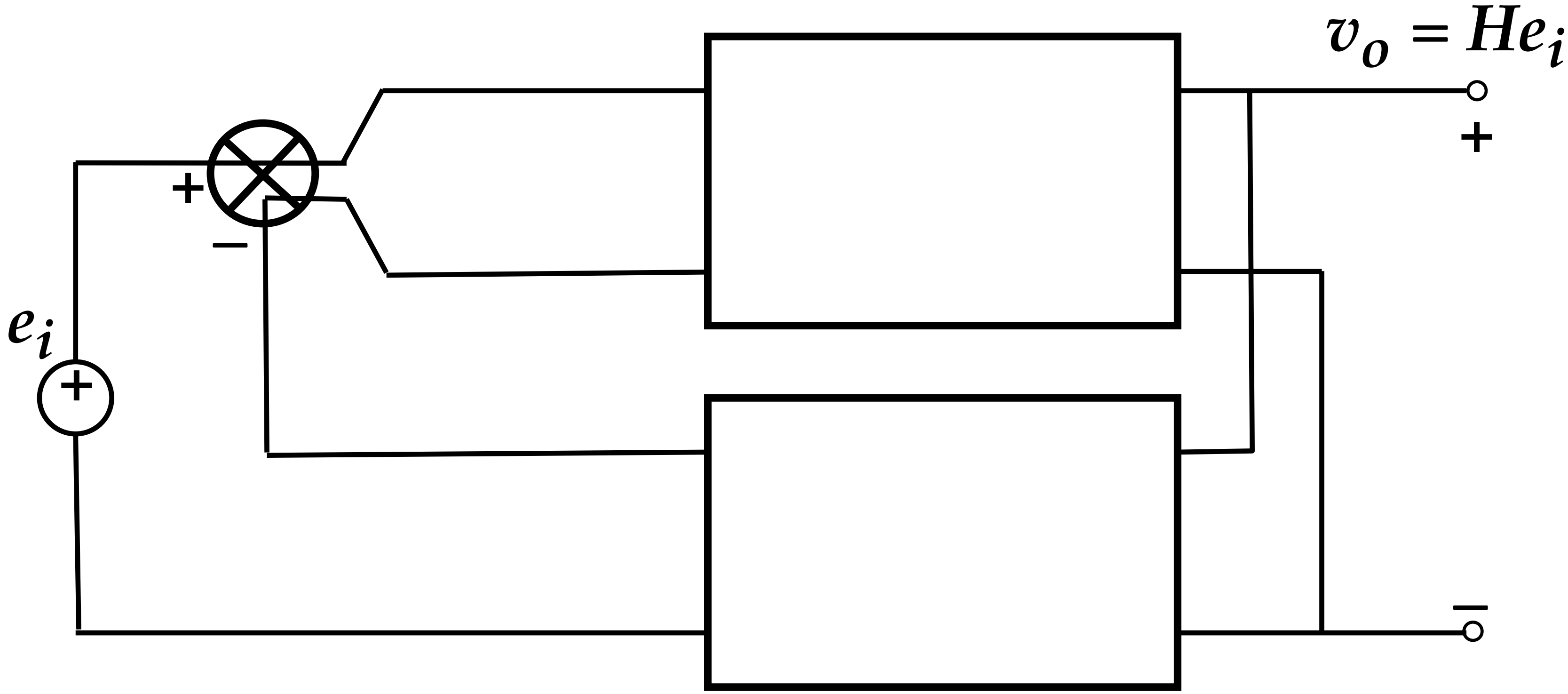
The amplifier CM gain affects $T_{n\mathbf{Noninv}}$, but does not affect $H_{\infty\mathbf{Noninv}}$, $H_{\infty\mathbf{Inv}}$, $T_{n\mathbf{Inv}}$, or T.

SUMMARY

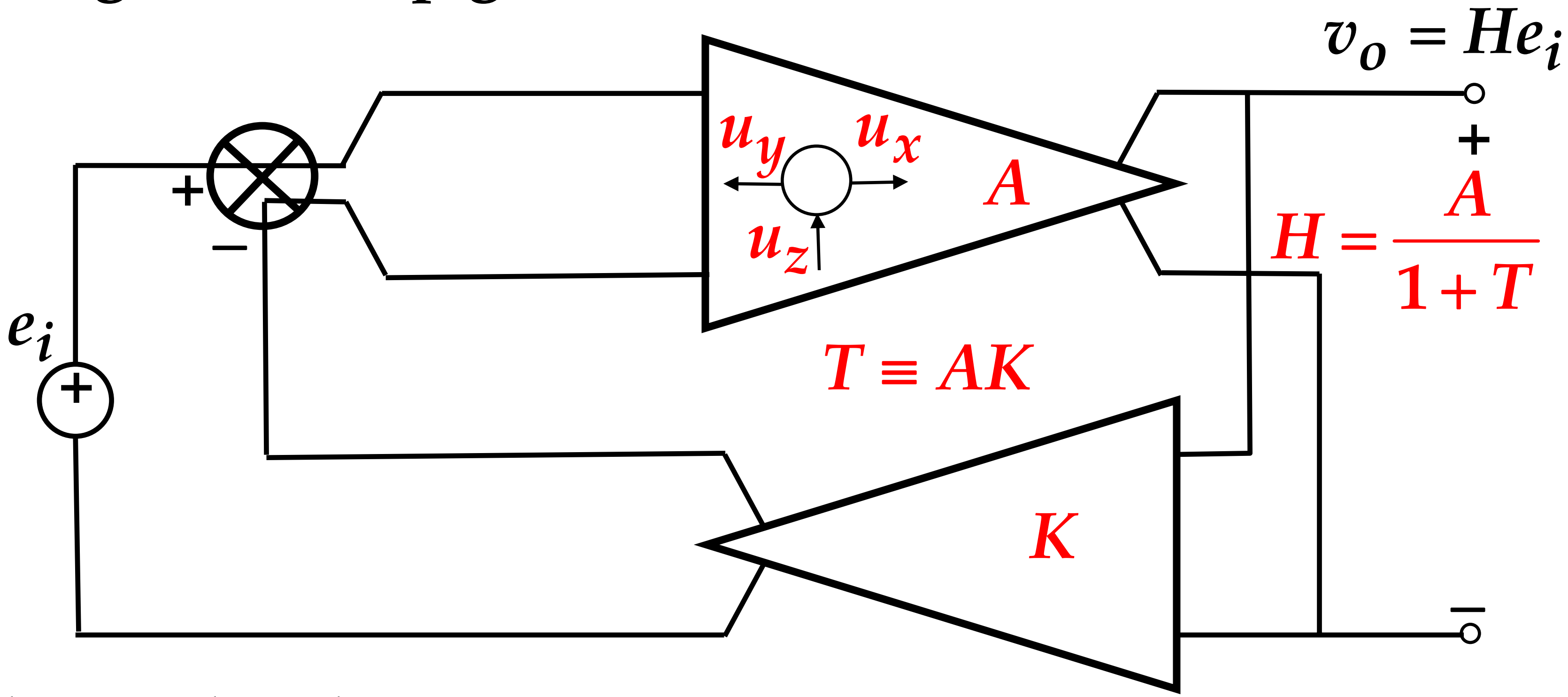
History of the conventional approach

The GFT: The Final Solution approach

Analysis History: Replace the boxes with arrowheads:



Spotlight the loop gain:



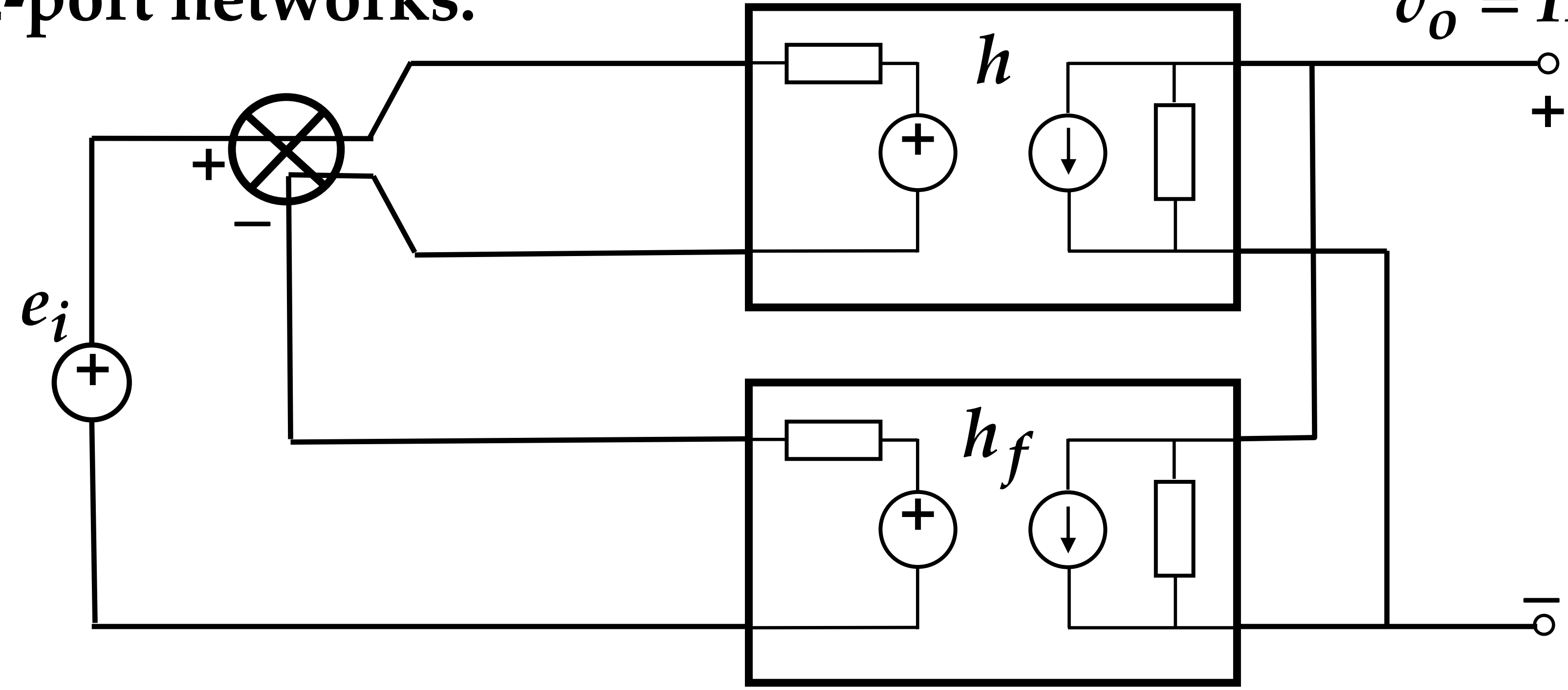
Development chronology:

1. Disconnect the feedback path, calculate A and K separately: ignores loading
2. Inject test signal where there is no loading (ideal injection point)
3. Inject coupled test signals (nonideal injection point)

Result: reverse transmission is ignored

Alternative: set up the forward and feedback paths as 2-port networks:

$v_o = He_i$



This requires four different sets of 2-port parameters for the four feedback connections, and it's still not clear how to account for reverse loop gain.

Disadvantage:

The model itself is inaccurate, in three of the four connections, because common-mode gain is ignored.

Greater Disadvantage:

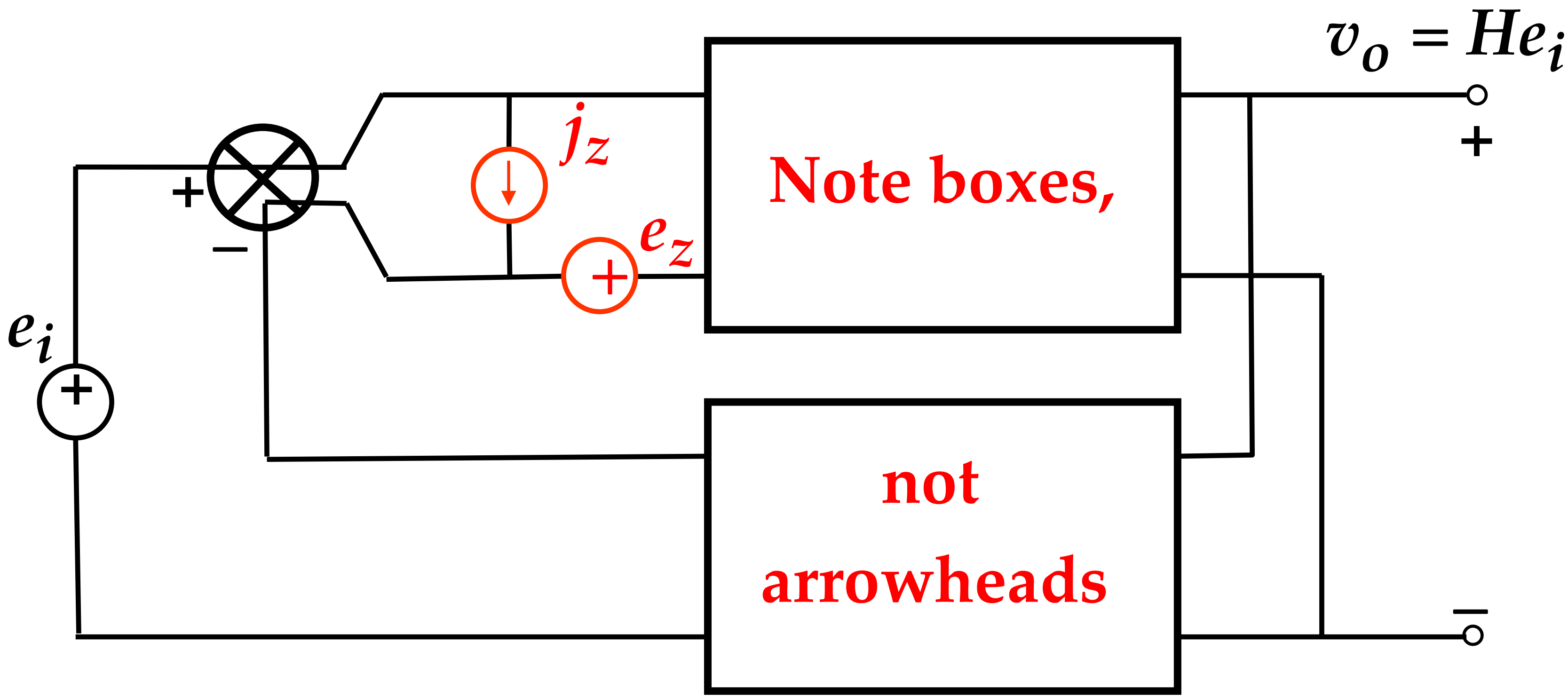
Essentially useless for design, because the circuit elements are buried inside the 2-port parameters, which are themselves buried in expressions for closed-loop gain and loop gain.

WHAT THE GFT DOES:

Defines and calculates the Principal Loop Gain

Identifies and calculates nonidealities

The GFT Approach: the Final Solution



Inject coupled test signals at error summing point.

The GFT gives all the second-level TFs directly

in terms of the circuit elements.

HOW THE GFT DOES IT:

$$H = H_\infty \frac{1 + \frac{1}{T_n}}{1 + \frac{1}{T}}$$

The Test Signal Injection Configuration is chosen to meet the following criteria:

1. The test signal(s) must be injected at the error summing point. This makes H_∞ equal to the Ideal Closed-Loop Gain $1/K$, the reciprocal of the feedback ratio.

2. The test signal(s) must be injected inside the major loop, but outside any minor loops. This makes T represent the Principal Loop Gain.

1 plus 2 makes the Null Loop Gain T_n represent all the nonidealities, including:

forward transmission through the feedback path

reverse transmission through the forward path

If $T_n \gg 1$, the nonidealities are negligible.

Injection of both voltage and current test signals at a nonideal point includes, as special cases, injection of only one test signal at an ideal point, so you don't have to know in advance whether or not the injection point is ideal.

Benefit:

Results include reverse transmission in both forward and feedback paths, and thus include reverse loop gain. Approximations are easily made, if desired.

Benefit:

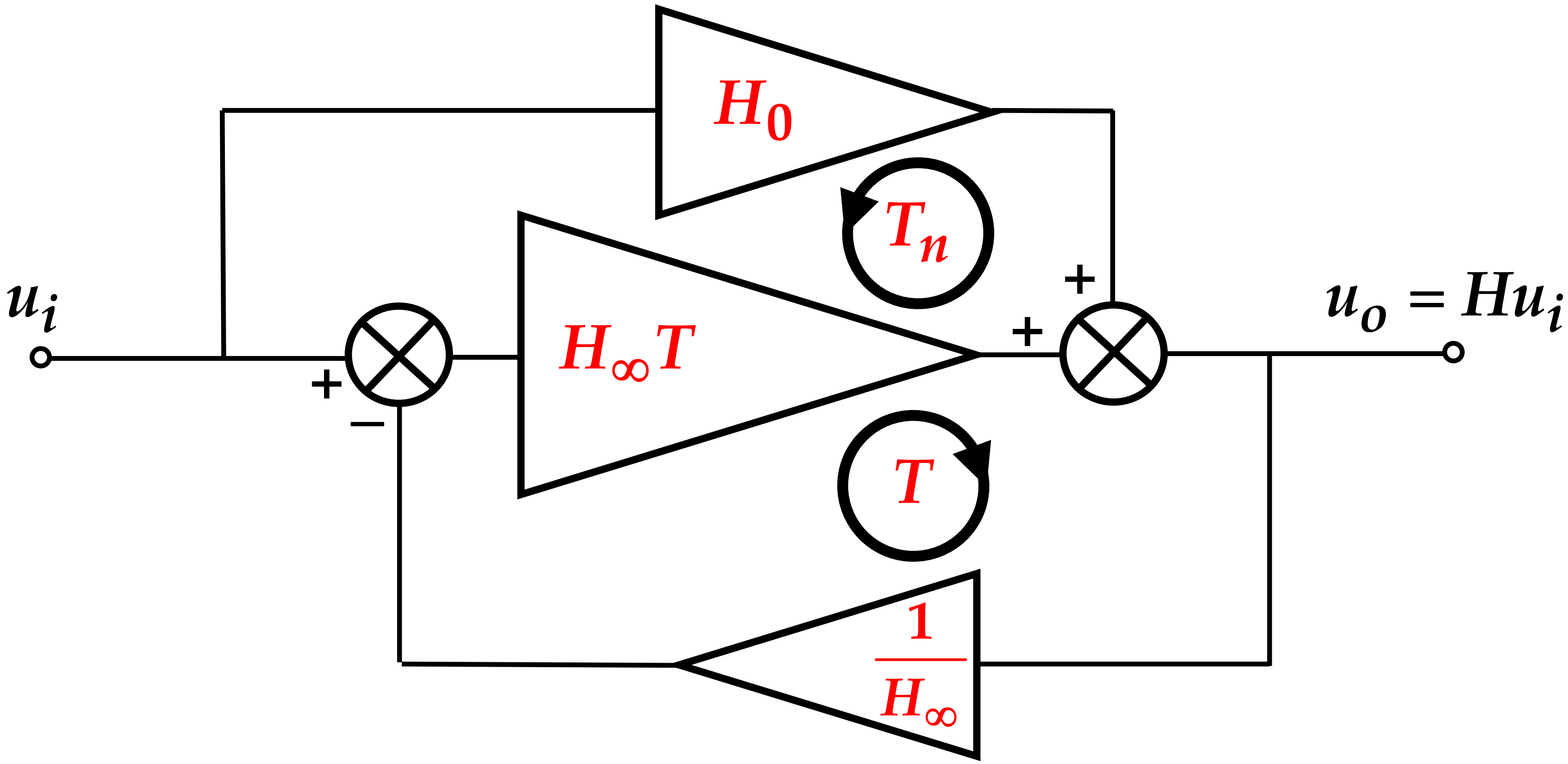
All four second level TFs H_∞, T, T_n, H_0 are produced, either numerically by Intusoft ICAP/4 GFT Template, or symbolically by ndi and dnti analysis.

Benefit:

The second-level TFs combine in the GFT to produce the first-level result, the closed-loop TF H:

$$H = H_\infty \frac{T}{1+T} + H_0 \frac{1}{1+T}$$

This result fits the block diagram:



Greater Benefit:

Thus, the block diagram, with unidirectional blocks, is a *result* of simple and straightforward *exact* calculation, *not an initial assumption*.

Greatest Benefit:

The second-level TFs H_∞, T, T_n, H_0 are obtained directly in terms of the circuit elements, which makes it possible for the results to be used backwards for design. This is the principal objective of

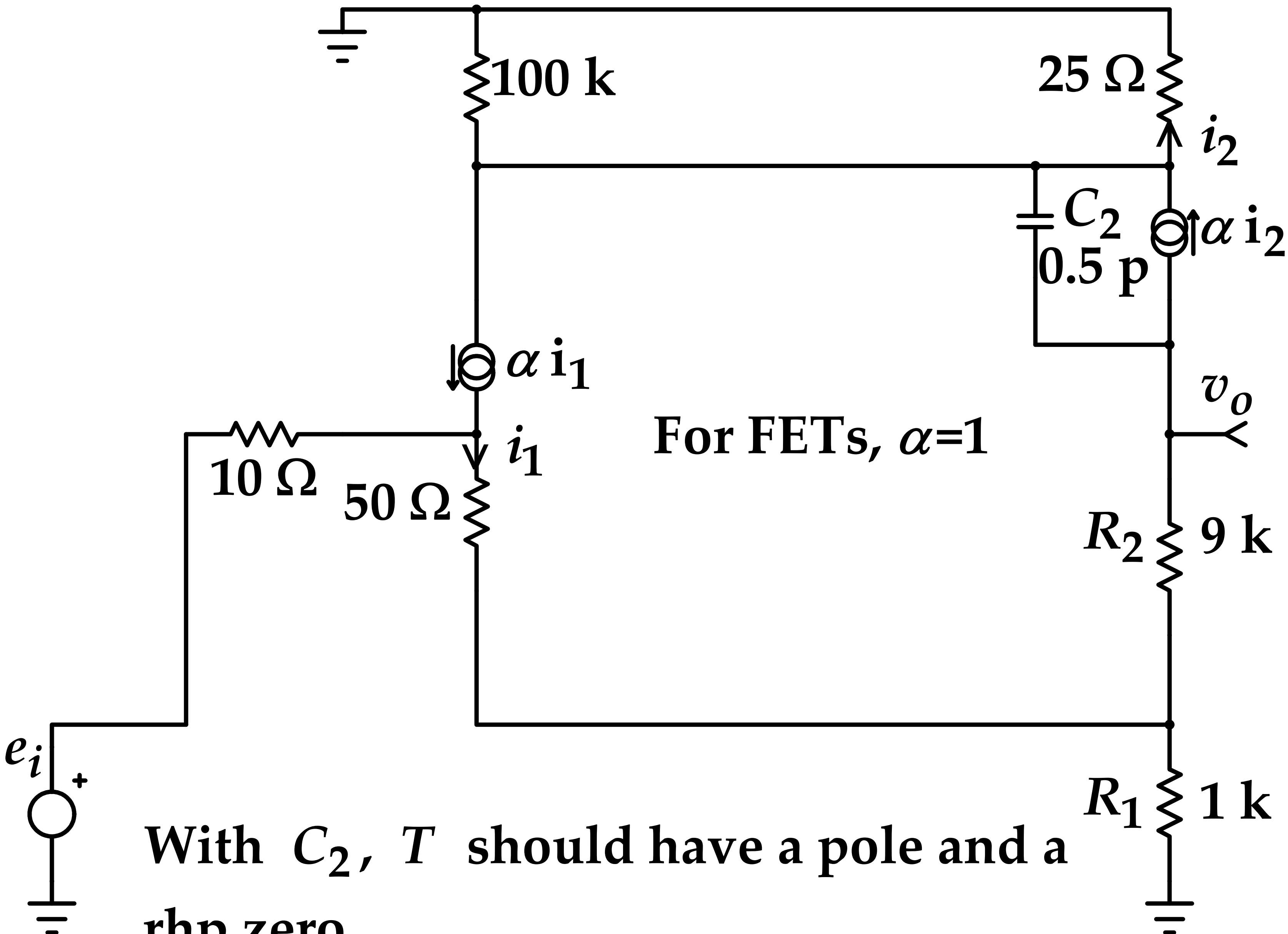
Design-Oriented Analysis:

the Only Kind of Analysis Worth Doing.

EXAMPLE

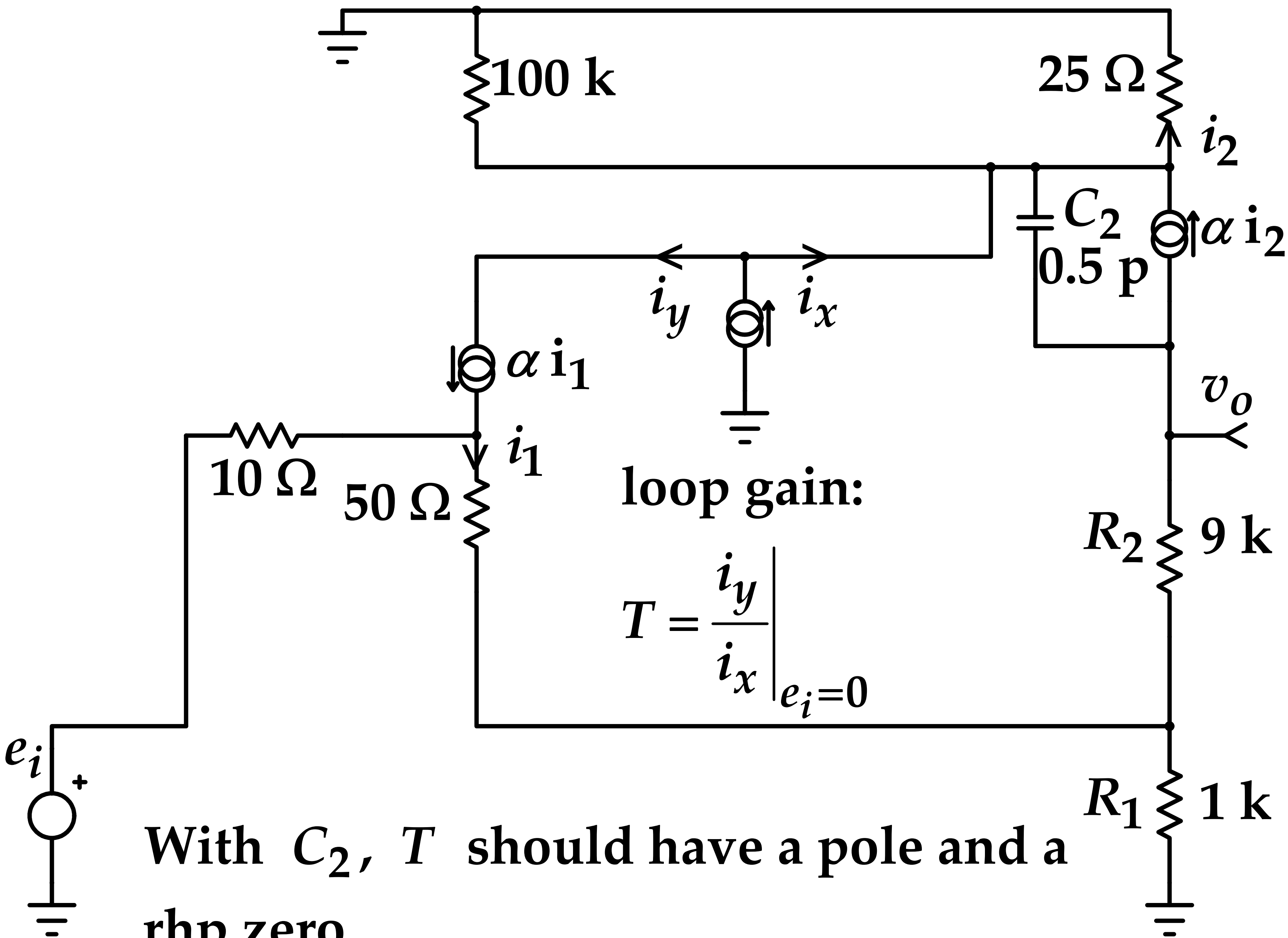
14. A 2CE/S FEEDBACK AMPLIFIER

A 2-stage Common-Emitter/-Source Amplifier



With C_2, T should have a pole and a rhp zero

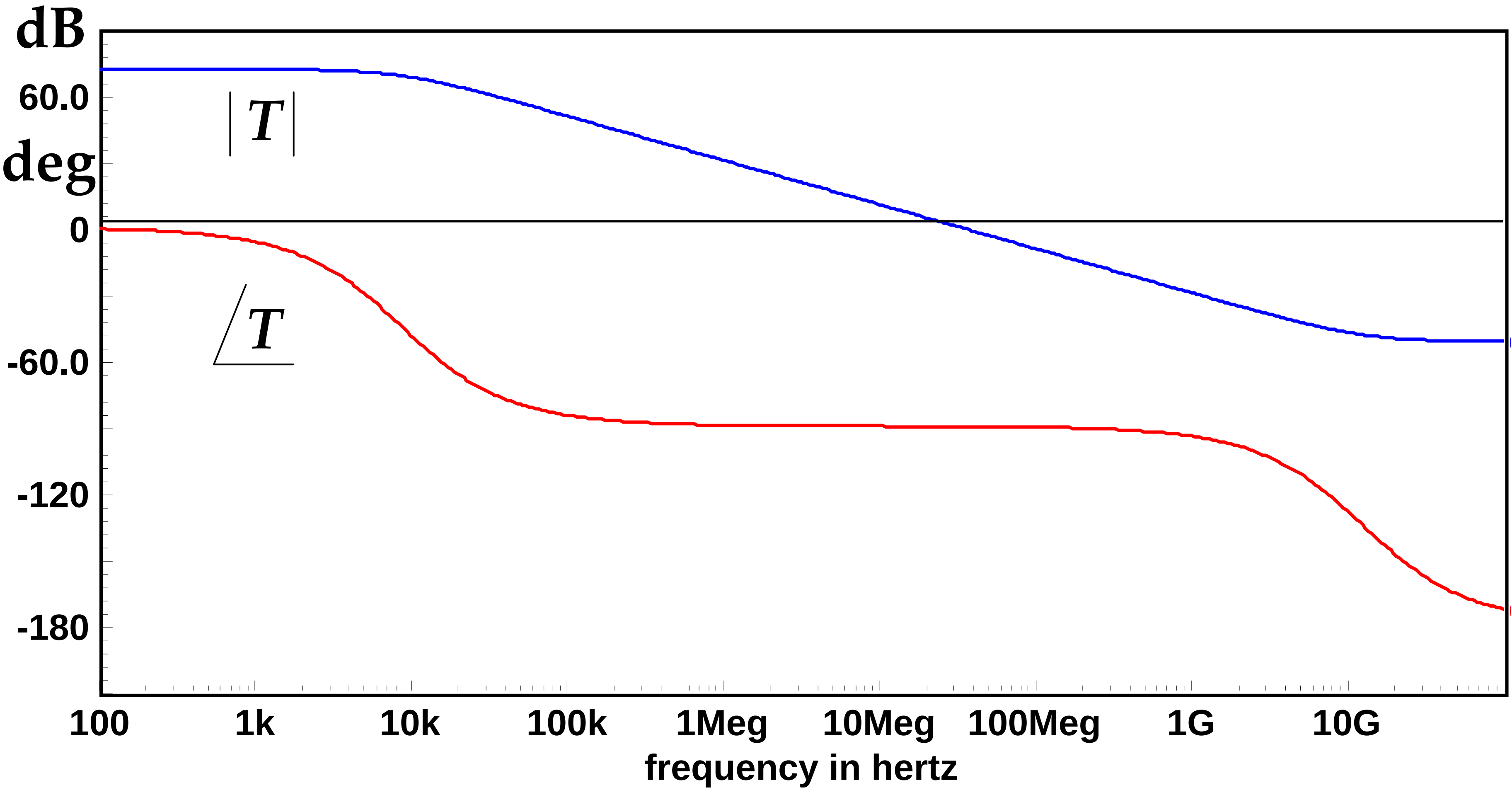
An ideal test current injection point is at the output of the first stage:



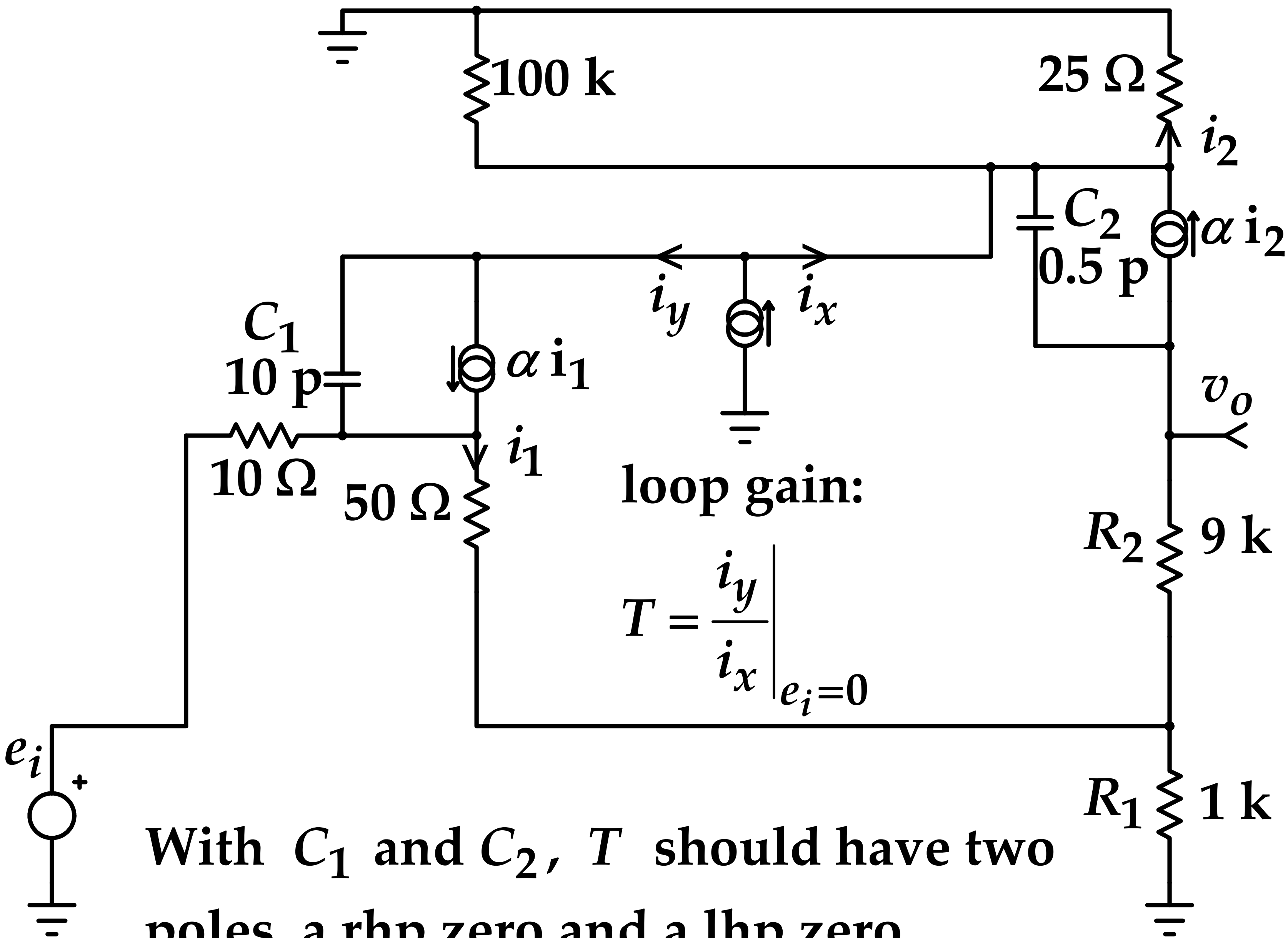
loop gain:

$$T = \left.\frac{i_y}{i_x}\right|_{e_i=0}$$

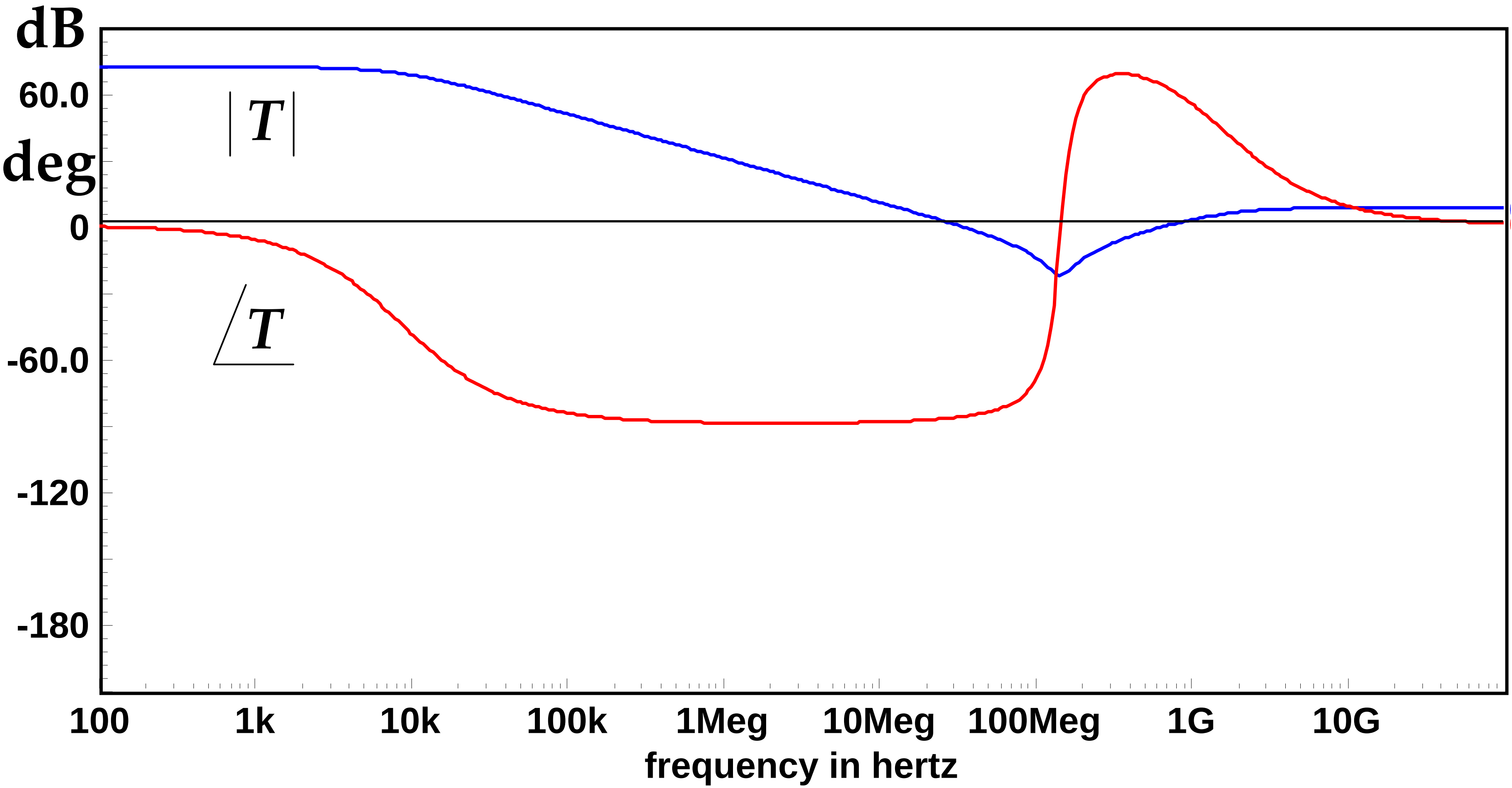
With C_2, T **should have a pole and a rhp zero**



With C_2: "right" answer



With C_1 and C_2, T should have two poles, a rhp zero and a lhp zero



With C_1 and C_2 : "wrong" answer

Why?

In the presence of C_2, the selected test current injection point is no longer "ideal", and in fact there is no ideal injection point for either a test current or a test voltage

A 1975[*] paper showed that *successive* injection, at a nonideal point, of current and voltage test signals to give T_i and T_v could be combined to give the correct result for T :

$$\frac{1}{1+T} = \frac{1}{1+T_i} + \frac{1}{1+T_v} \quad \text{or} \quad T = \frac{T_i T_v - 1}{2 + T_i + T_v}$$

[*] R.D.Middlebrook, "Measurement of loop gain in feedback systems," Int. J. Electronics, 1975, vol. 38, No. 4, pp. 485 – 512.

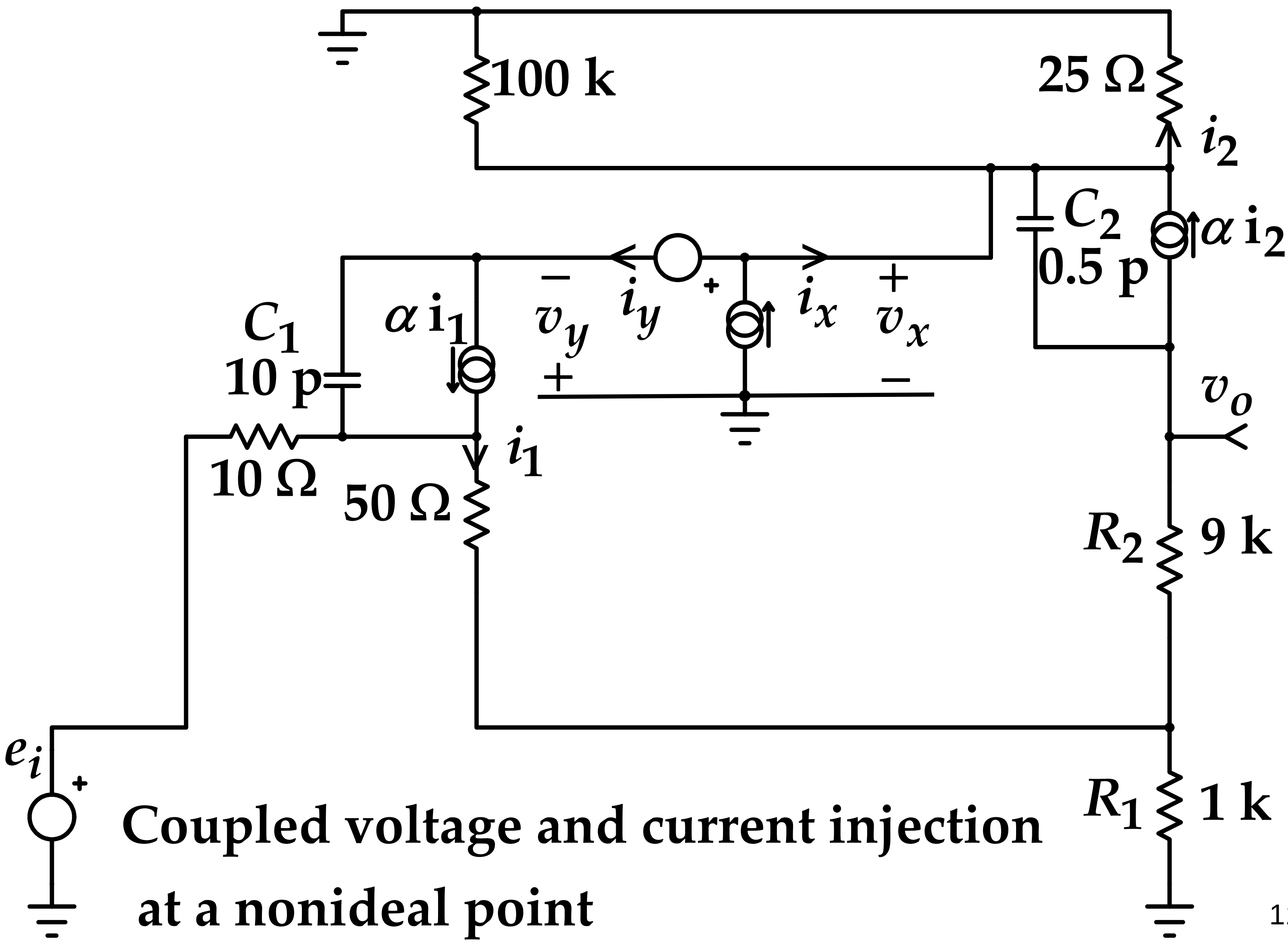
The same paper also showed that *simultaneous* injection of current and voltage test signals, adjusted to give the short-circuit current loop gain $T_i^{v_y}$ and open-circuit voltage loop gain $T_v^{i_y}$ could be combined to give the correct result

$$\frac{1}{T} = \frac{1}{T_i^{v_y}} + \frac{1}{T_v^{i_y}}$$

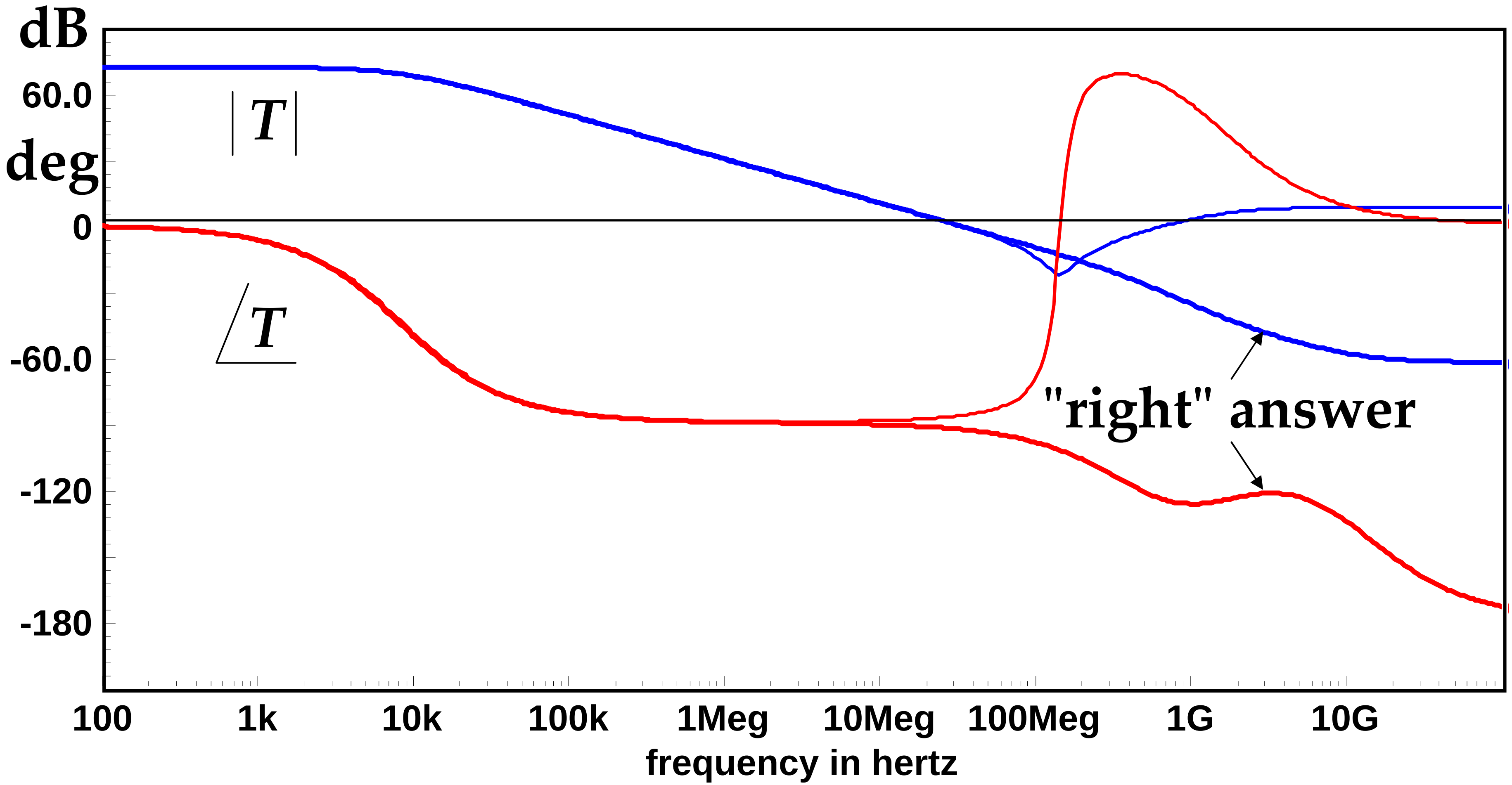
The advantage of this method is that the ndi calculations of $T_i^{v_y}$ and $T_v^{i_y}$ are symbolically simpler and easier than the si calculations of T_i and T_v.

Nevertheless, the conclusions of that paper were incomplete, because the nonidealities were still ignored.

The 2GFT includes the nonidealities, and gives the correct answer for T :

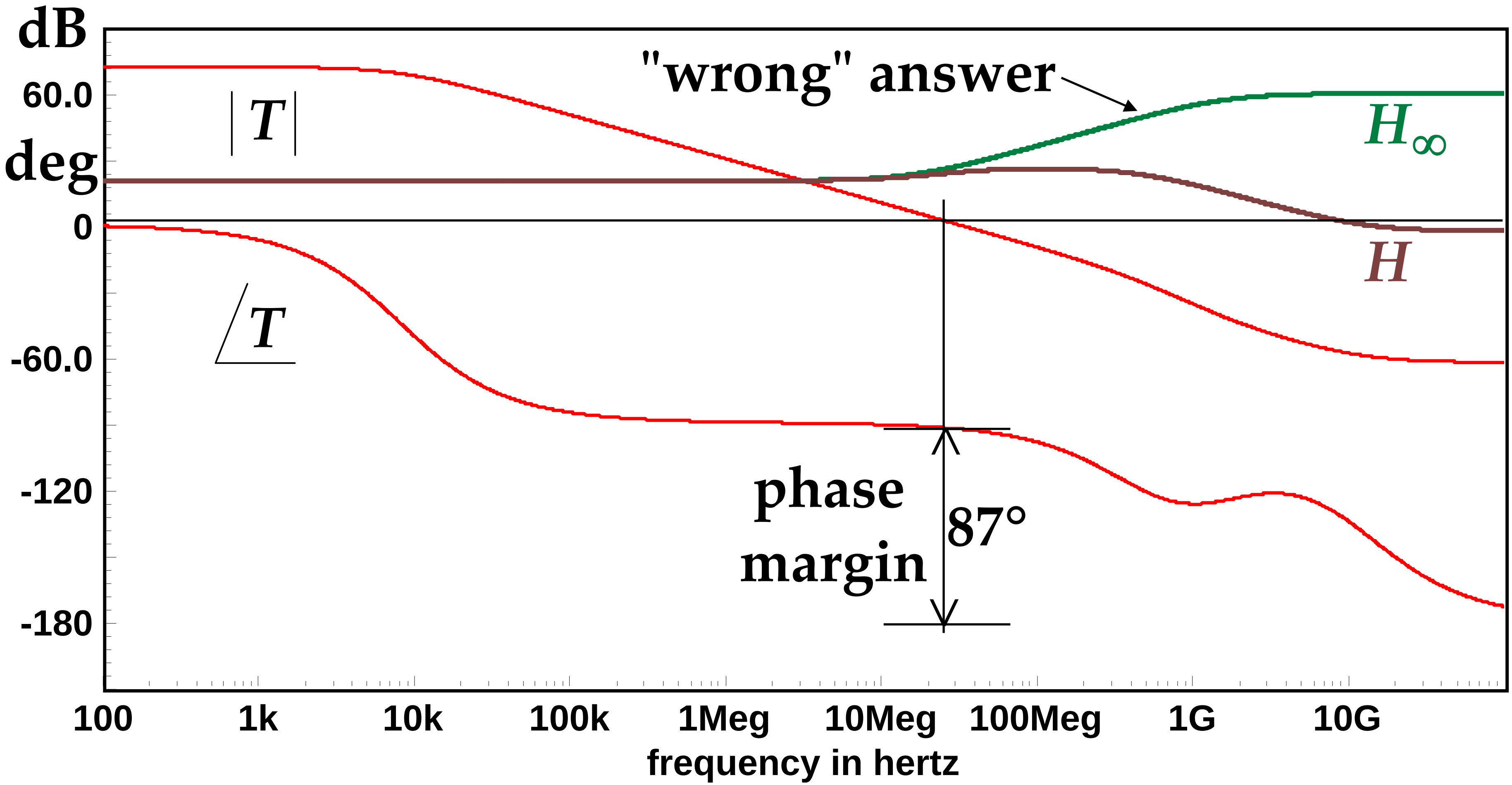


Coupled voltage and current injection at a nonideal point



The "right" answer (*T* has two poles, a lhp and a rhp zero) is obtained by coupled voltage and current injection (GFTv Template) at a nonideal point.

But, there is another problem:



H_∞ is not equal to $1/K$

Why?

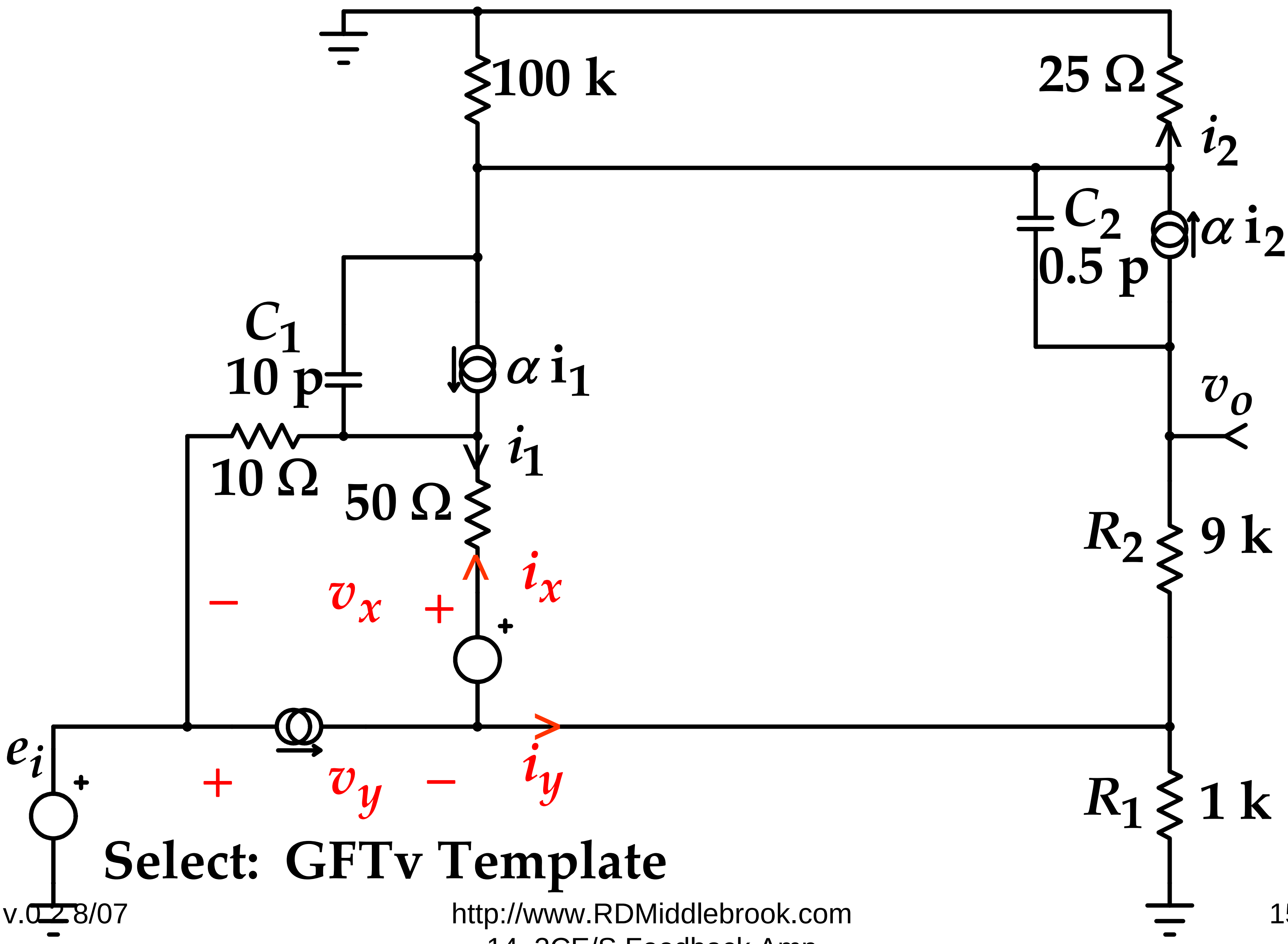
The test signal configuration meets the first criterion, that the test signal must be inside the loop, but it does not meet the second: the test signal must be able to null the error signal.

Therefore, choose a test signal configuration that meets the second criterion as well:

v_y **must be the error voltage, and**

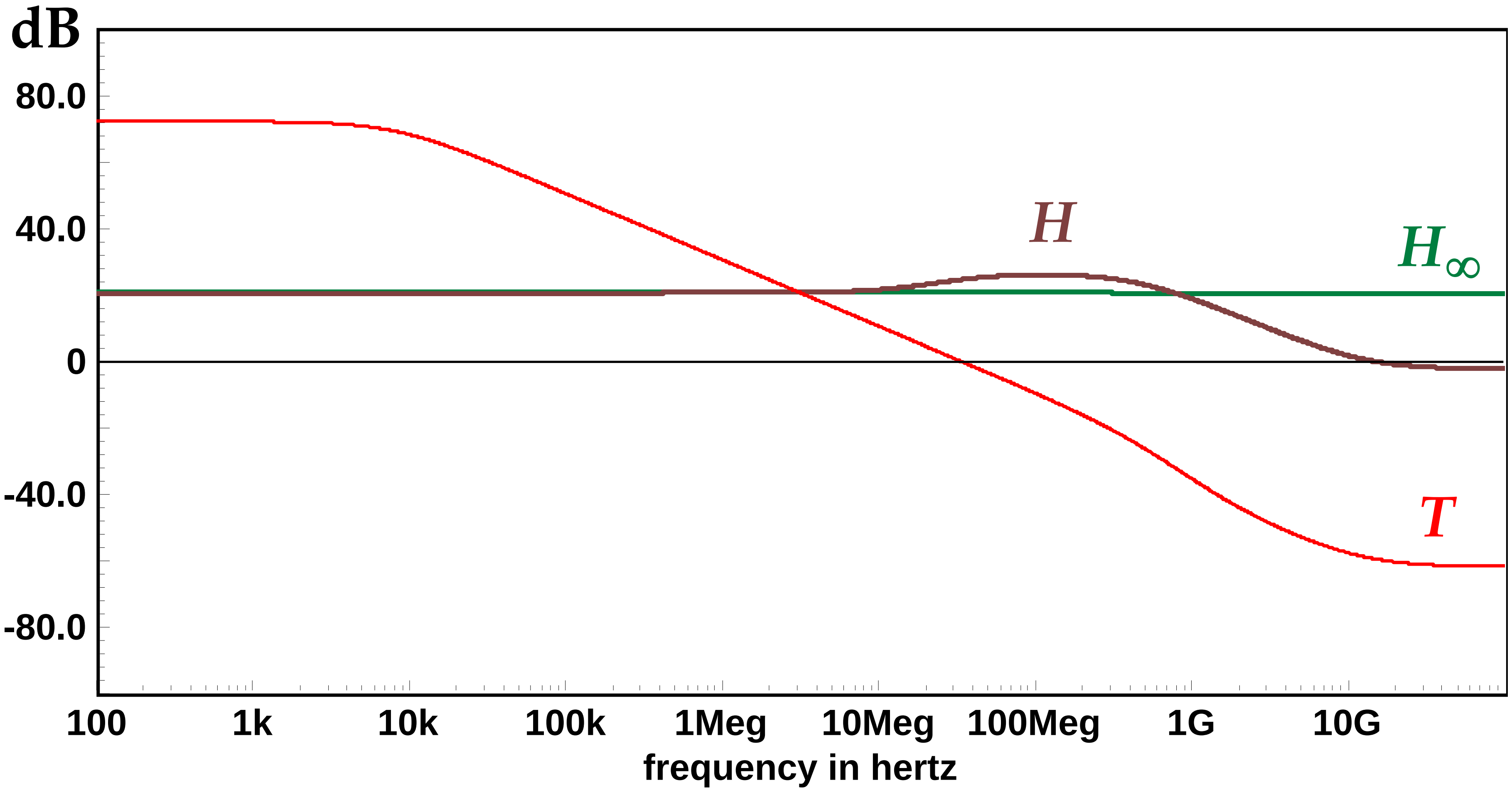
i_y **must be the error current.**

Coupled injection must be used, since v_y and i_y do not null simultaneously with only single injection.

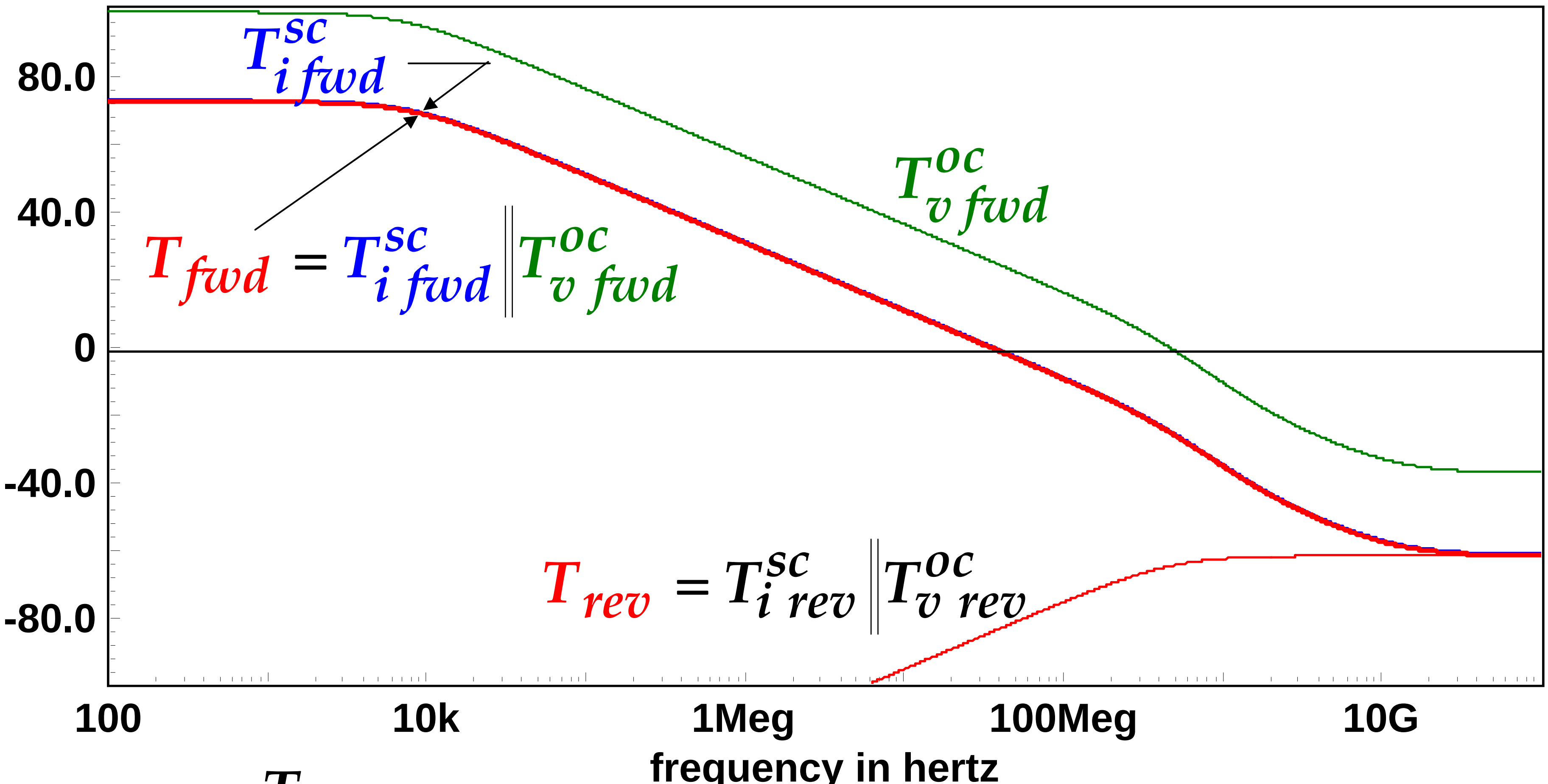


Select: GFTv Template

Check that H_∞ is the expected $1/K = 20$ dB:

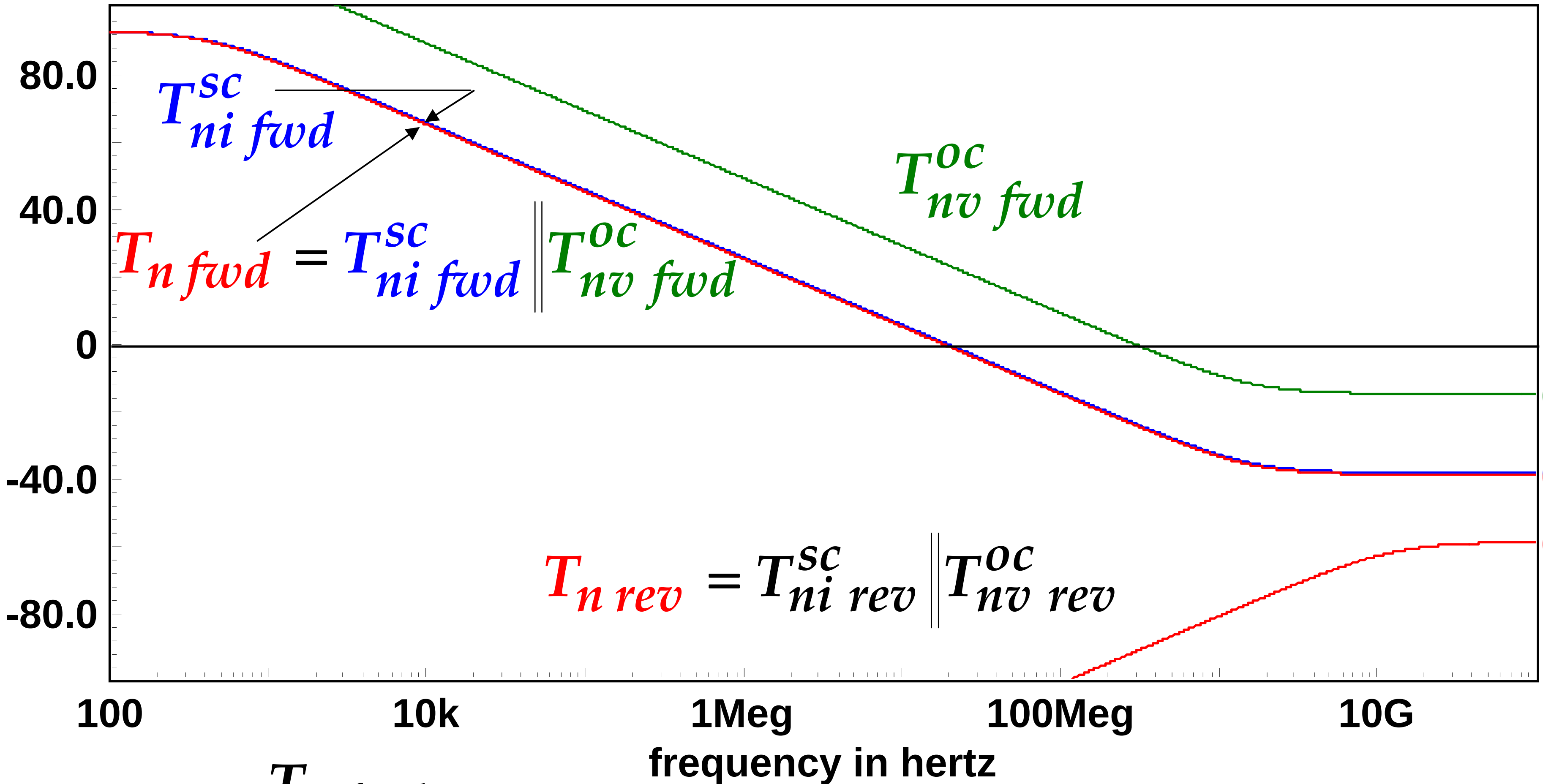


T and H are still the same



$T = \dfrac{T_{fwd}}{1+T_{rev}}$, **and since** $T^{oc}_{v\,fwd} >> 1$ **and** $T_{rev} << 1$,

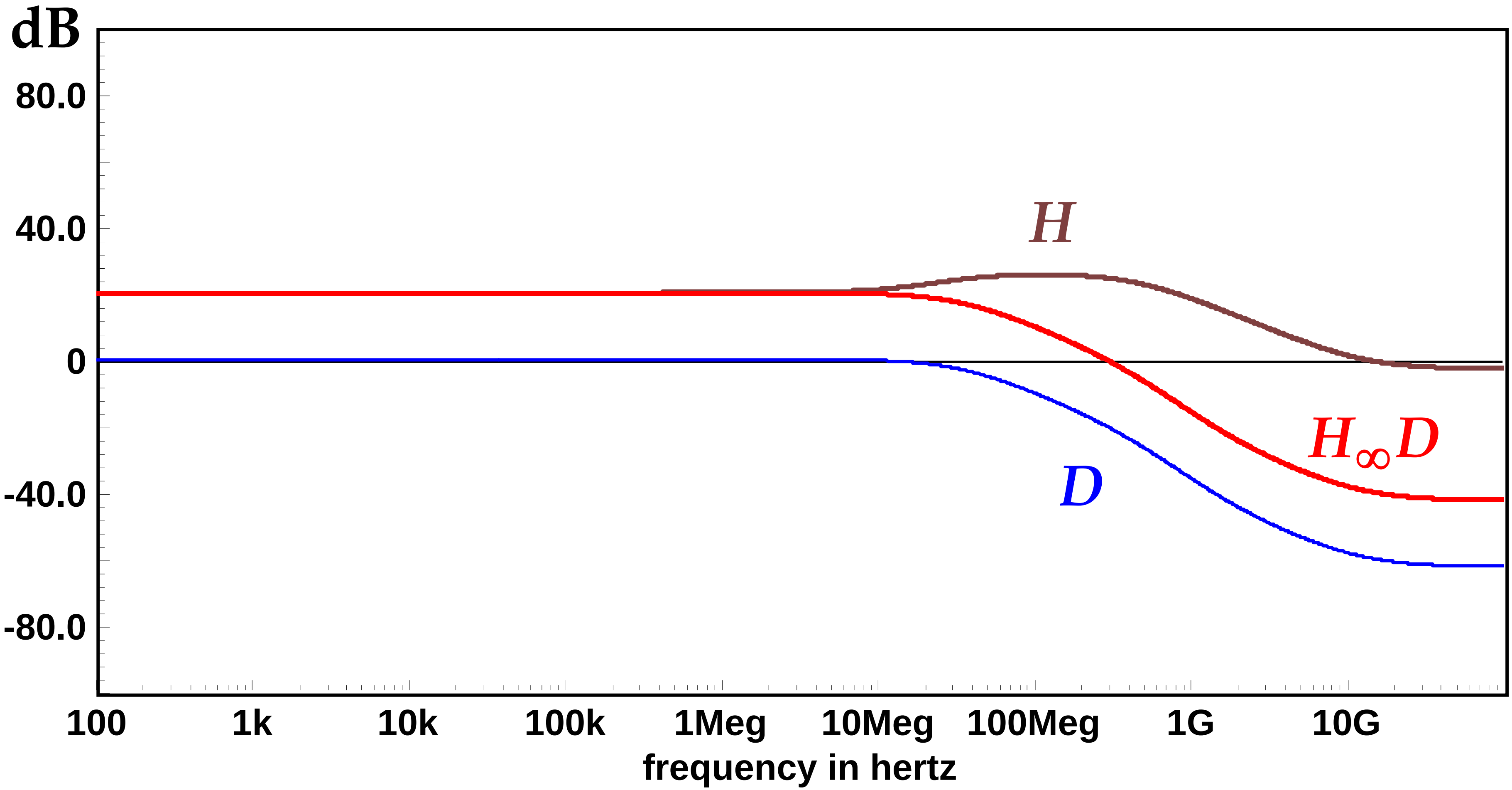
$T \approx T^{sc}_{i\,fwd}$



$T_n = \dfrac{T_{n\ fwd}}{1+T_{n\ rev}}$, and since $T^{oc}_{nv\ fwd} >> 1$ and $T_{n\ rev} << 1$,

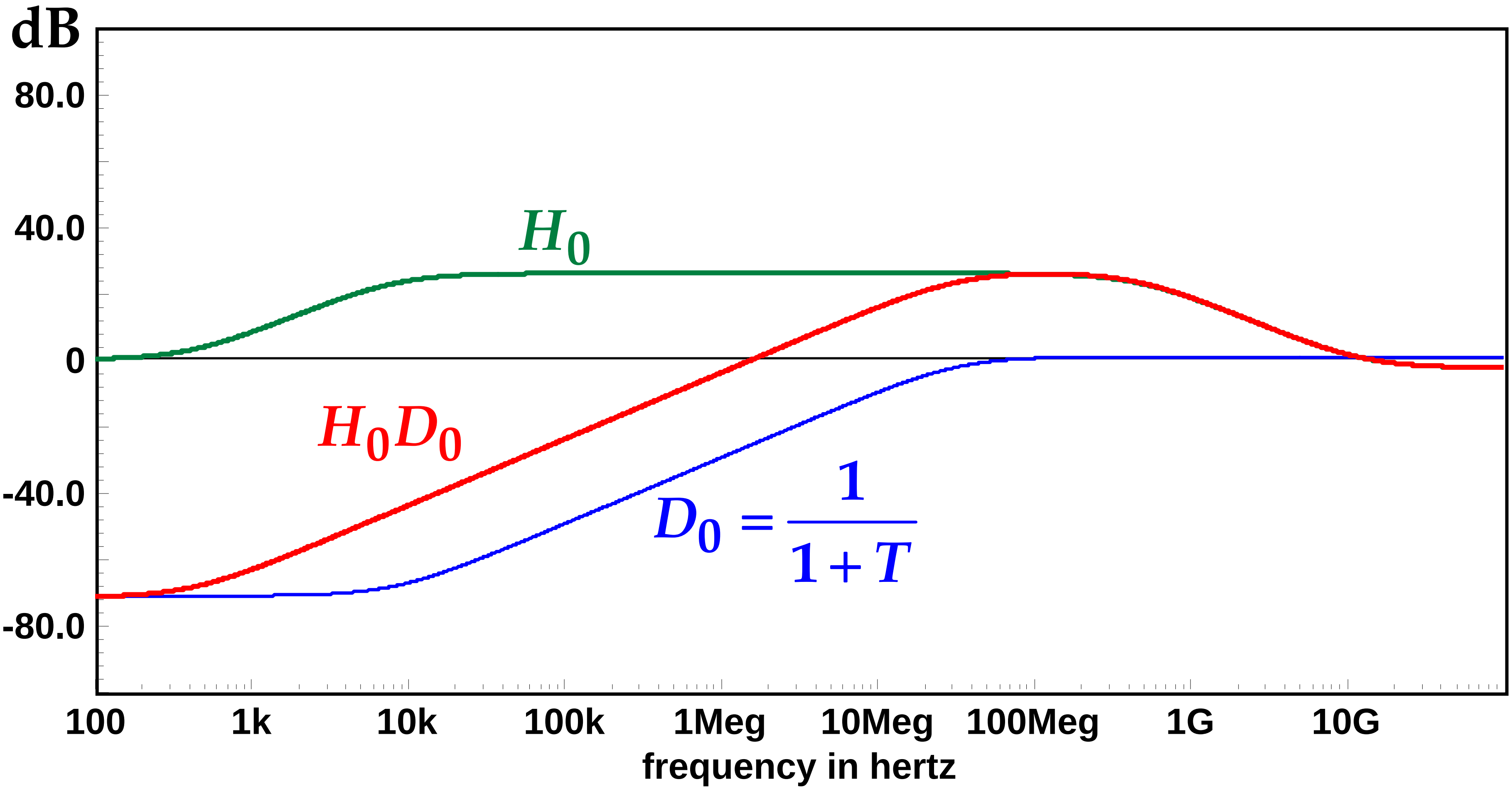
$T_n \approx T^{sc}_{ni\ fwd}$

The component $H_\infty D$ of $H = H_\infty D + H_0 D_0$:

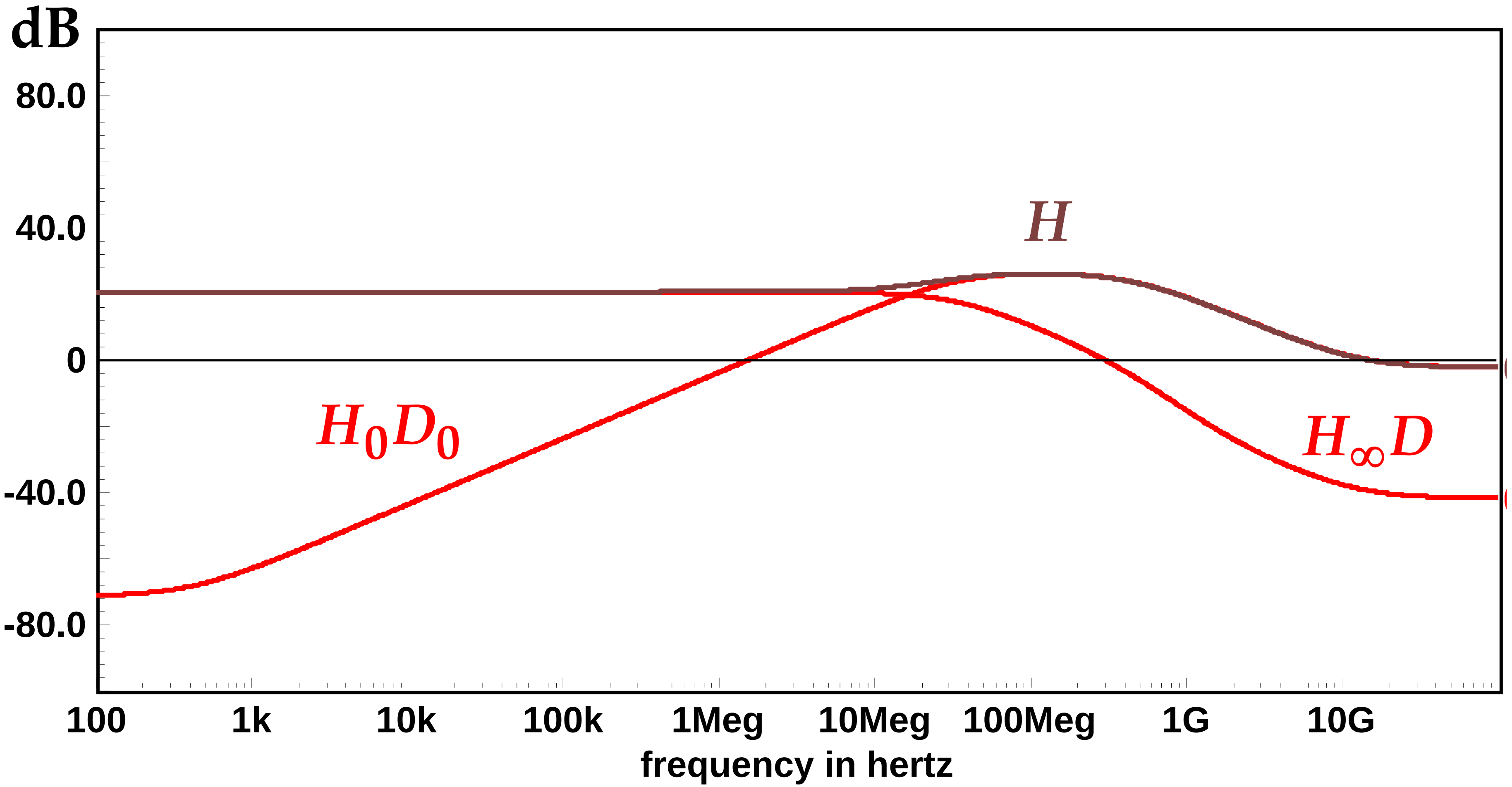


It may be thought that the peaking in H is due to insufficient phase margin, but in fact it comes from H_0 :

The component $H_0 D_0$ of $H = H_\infty D + H_0 D_0$:



Both components of $H = H_\infty D + H_0 D_0$:

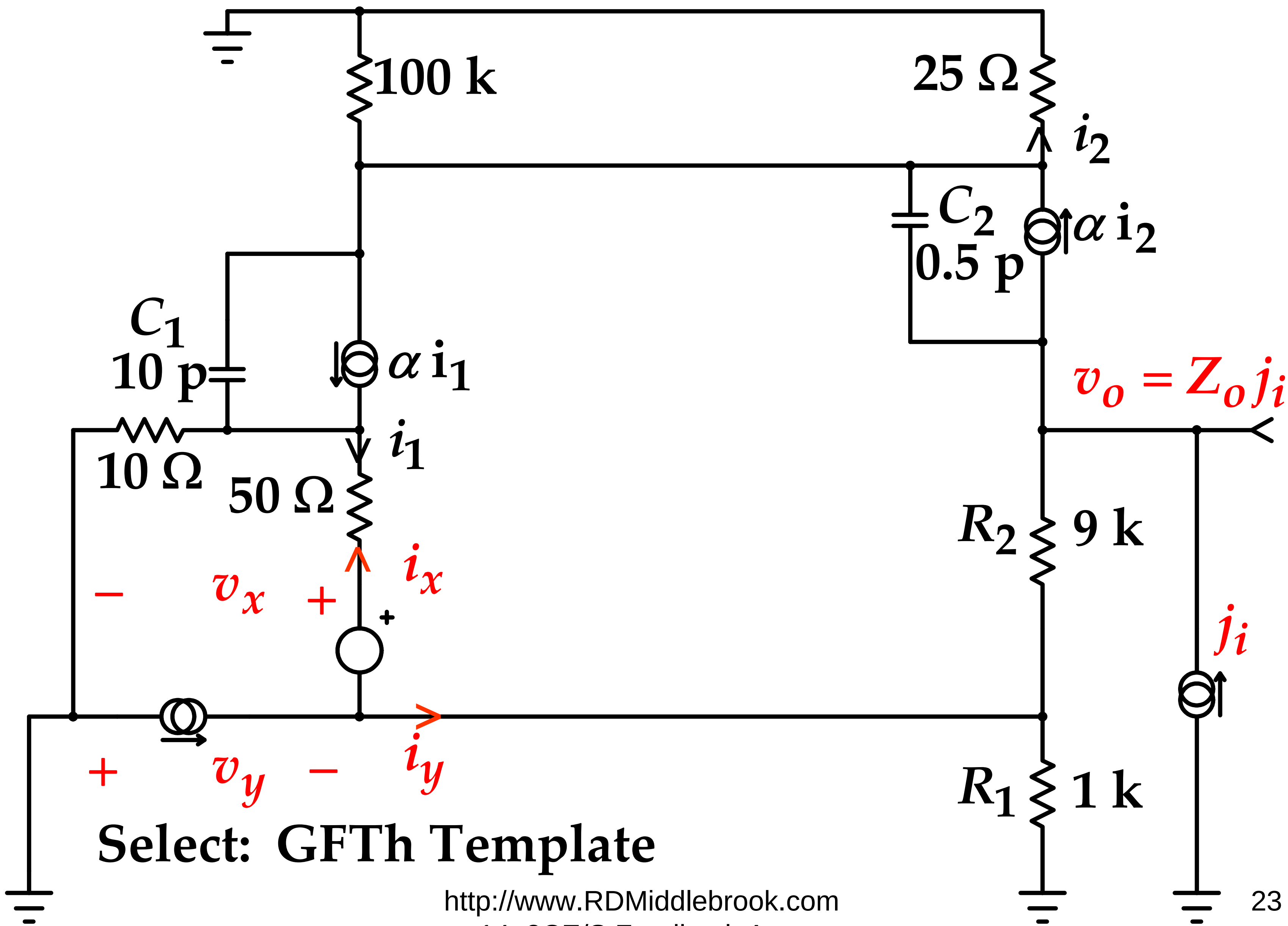


The first-level TF H can be a current gain, transadmittance, transimpedance, or a voltage gain.

As applied to an output impedance Z_o, which is a self-impedance, the GFT becomes

$$Z_{\mathbf{o}} = Z_{o\infty}\frac{T}{1+T} + Z_{o0}\frac{1}{1+T}$$

The "output" is v_o as for voltage gain, but the "input" is now a current j_i driven into the output.

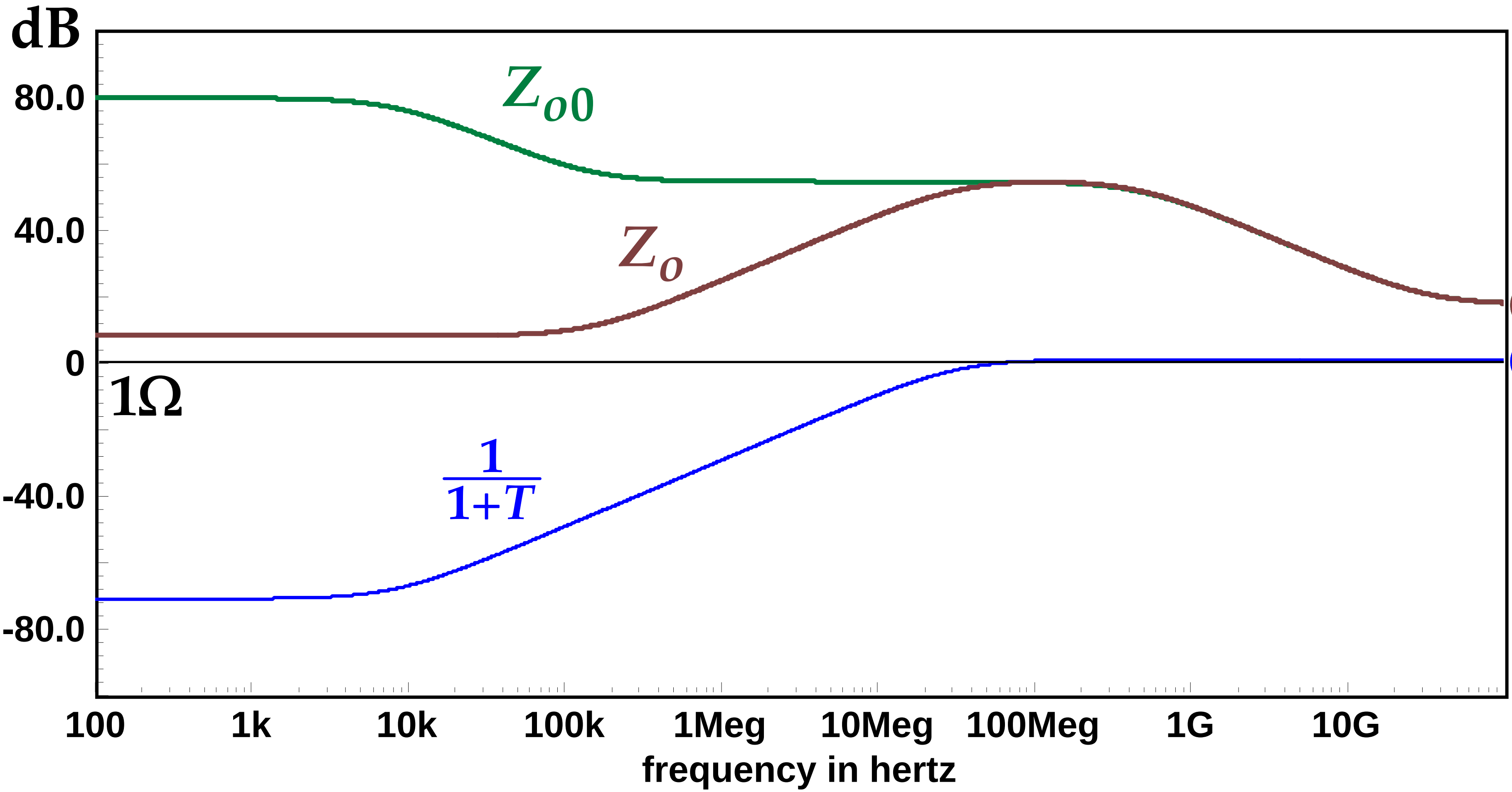


Select: GFTh Template

For voltage feedback, as in this example, $Z_{o\infty}$ is always zero, so

$$Z_{\mathbf{o}} = Z_{o0} \frac{1}{1+T}$$

which is the familiar result that the closed-loop output impedance is the open-loop value divided by the feedback factor.

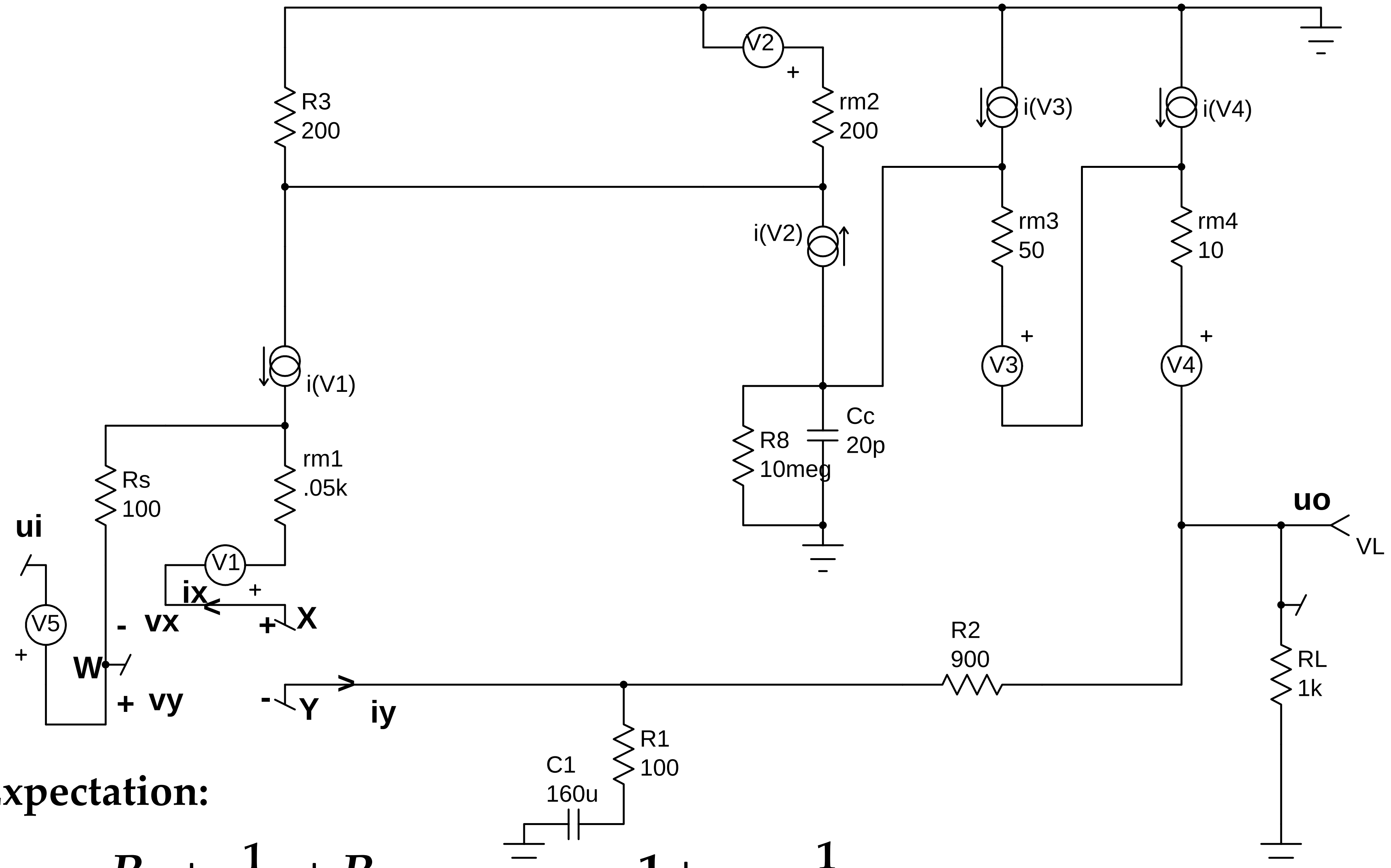


The closed-loop output impedance Z_o rises to the open-loop value Z_{o0} as loop gain crossover is approached.

EXAMPLE

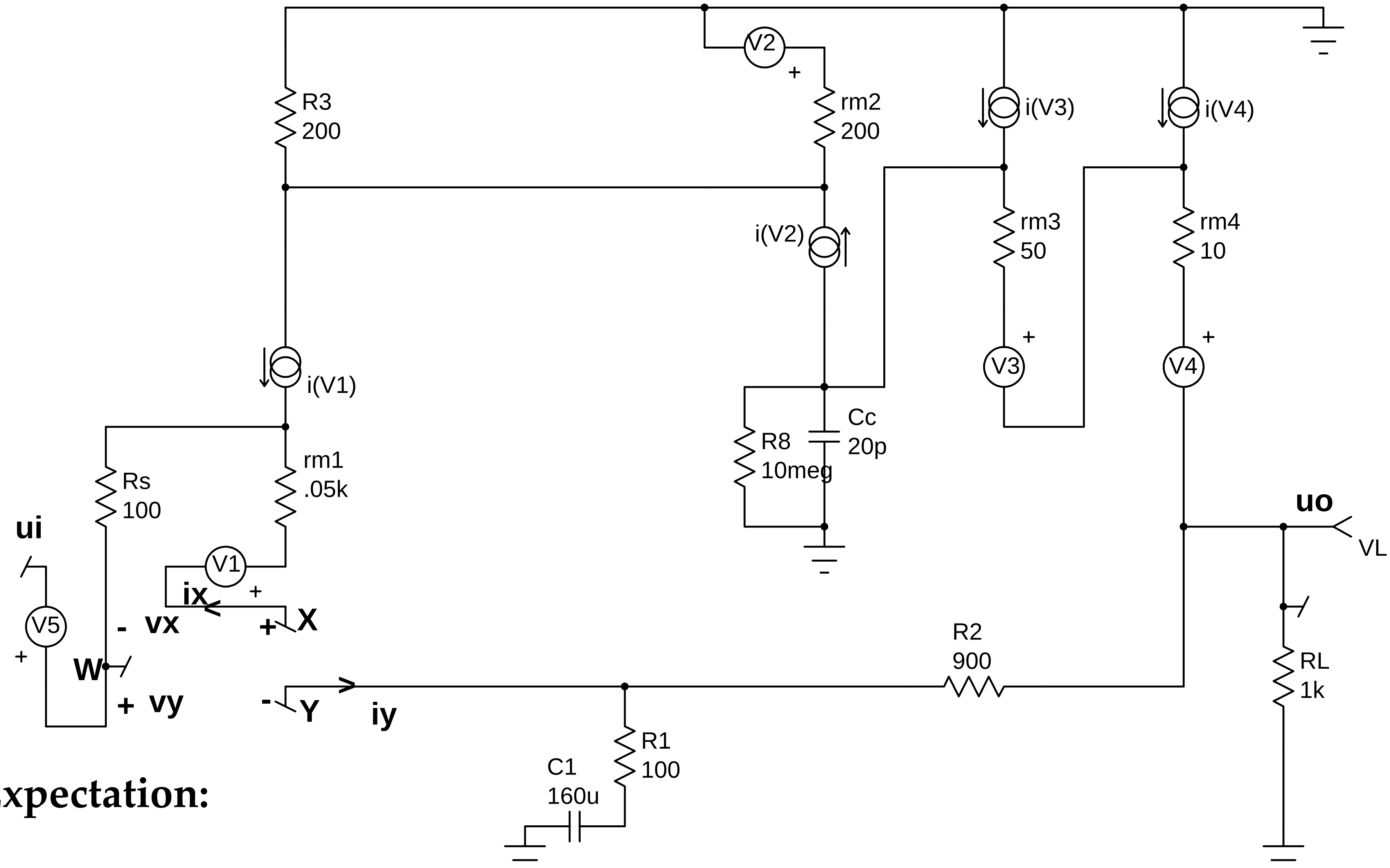
15. A REALISTIC IC FEEDBACK AMPLIFIER

Example 1: Noninverting Amplifier with nonflat H_∞



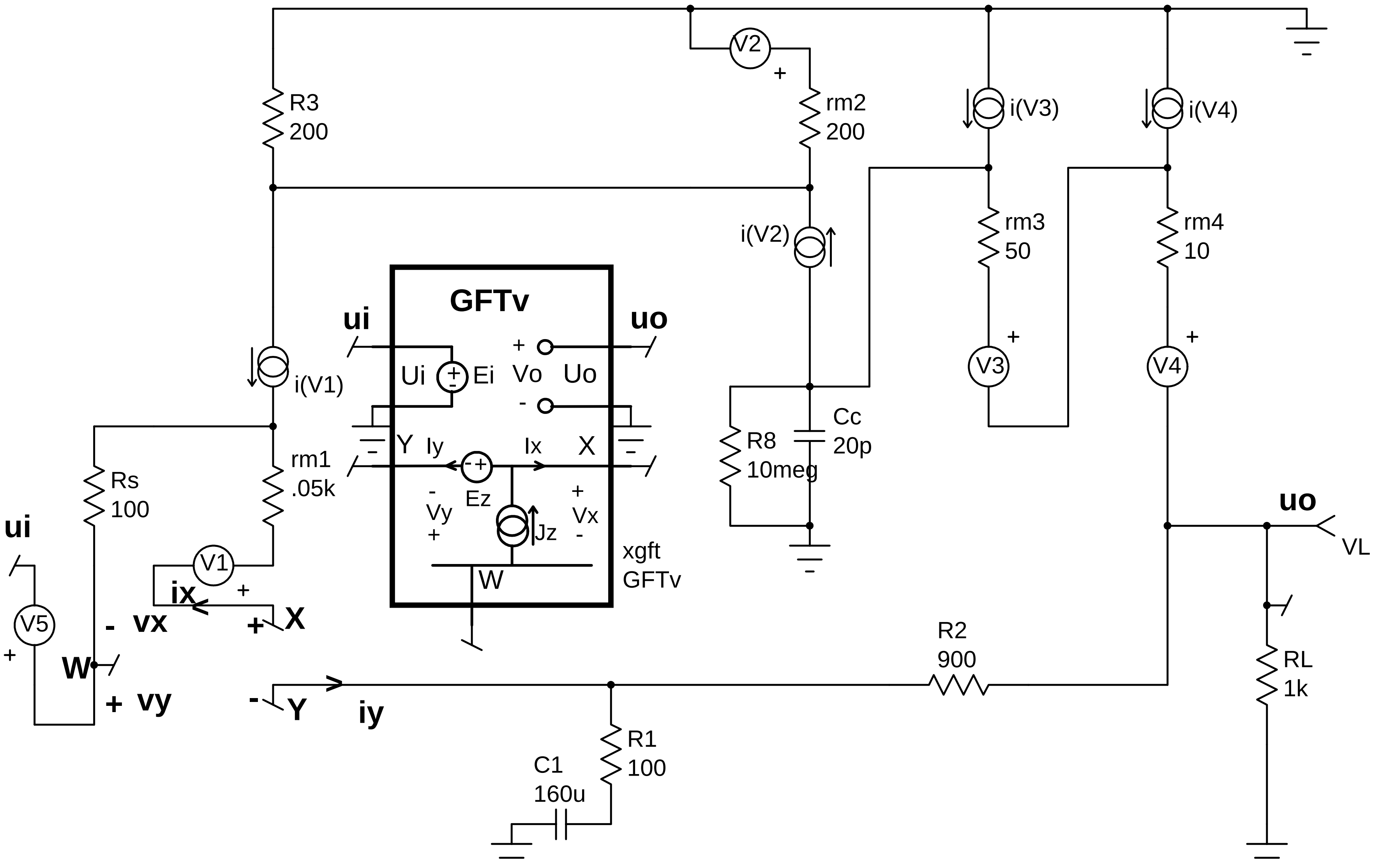
Expectation:

$$H_\infty = \frac{R_1 + \frac{1}{sC_1} + R_2}{R_1 + \frac{1}{sC_1}} = \frac{R_1 + R_2}{R_1} \frac{1 + \frac{1}{sC_1(R_1+R_2)}}{1 + \frac{1}{sC_1R_1}}$$



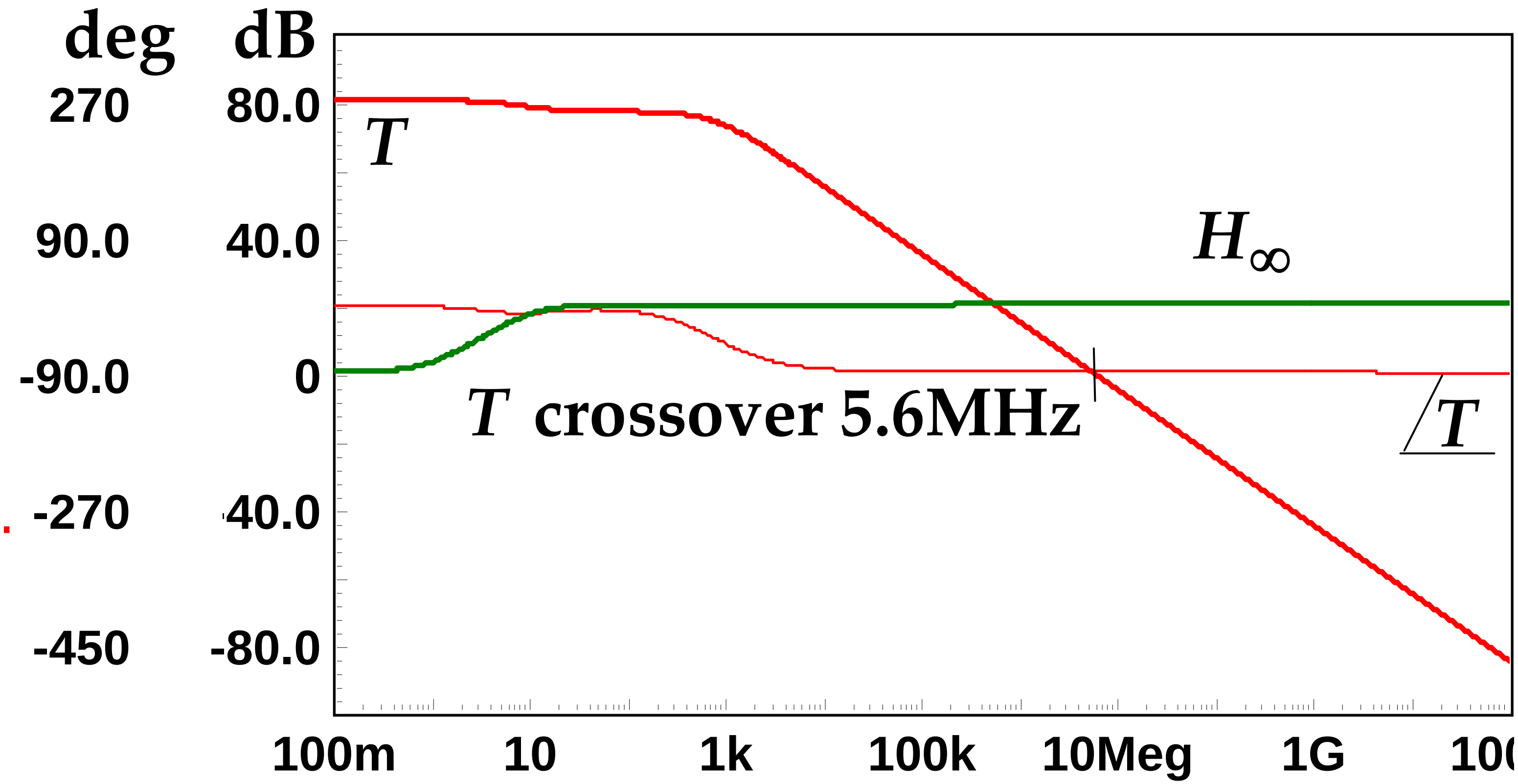
Expectation:

T should be flat at low frequencies, with a small shelf due to C_1, plus a dominant pole due to C_c.

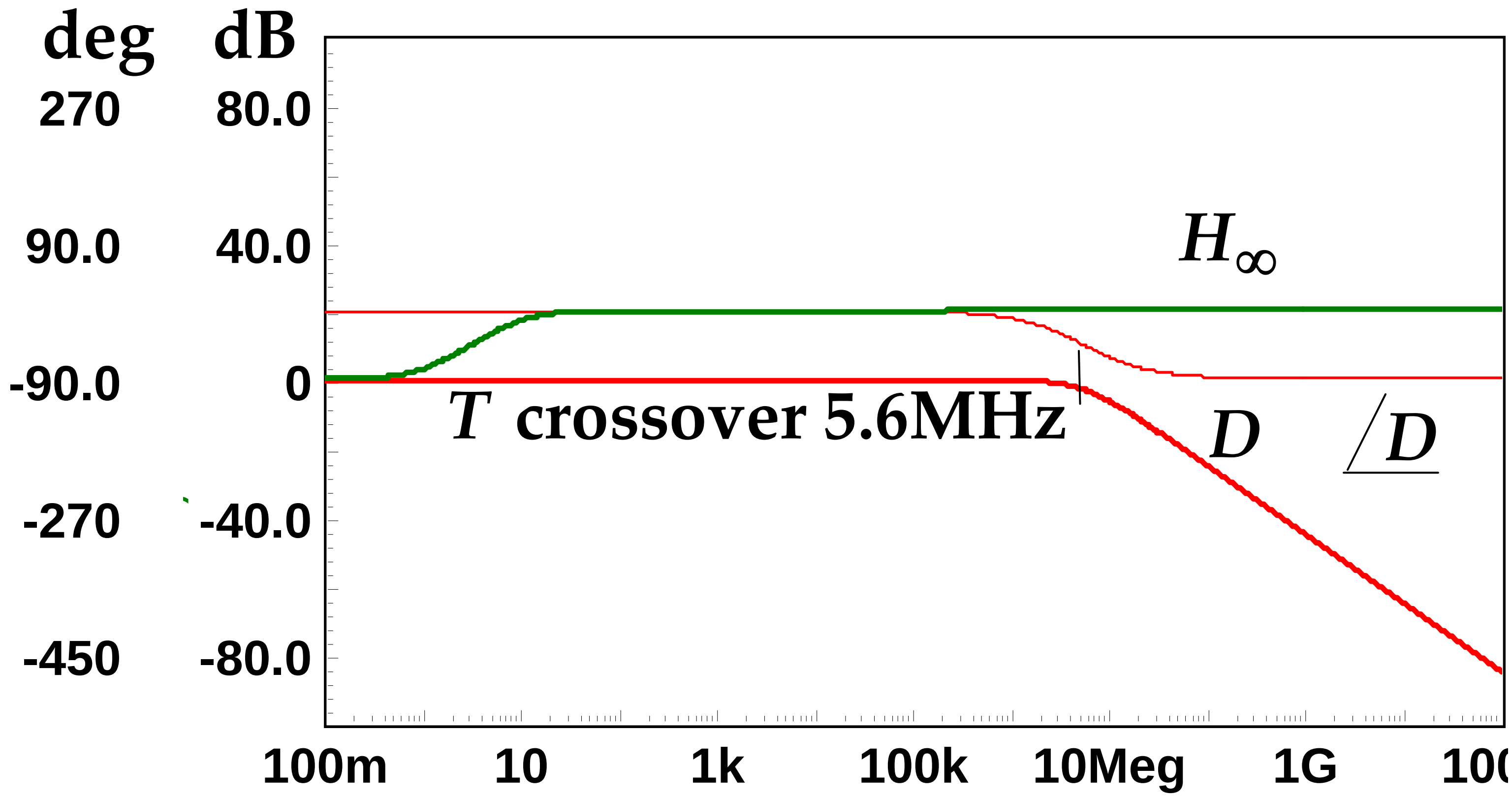


This Test Signal Injection Configuration meets the two requirements

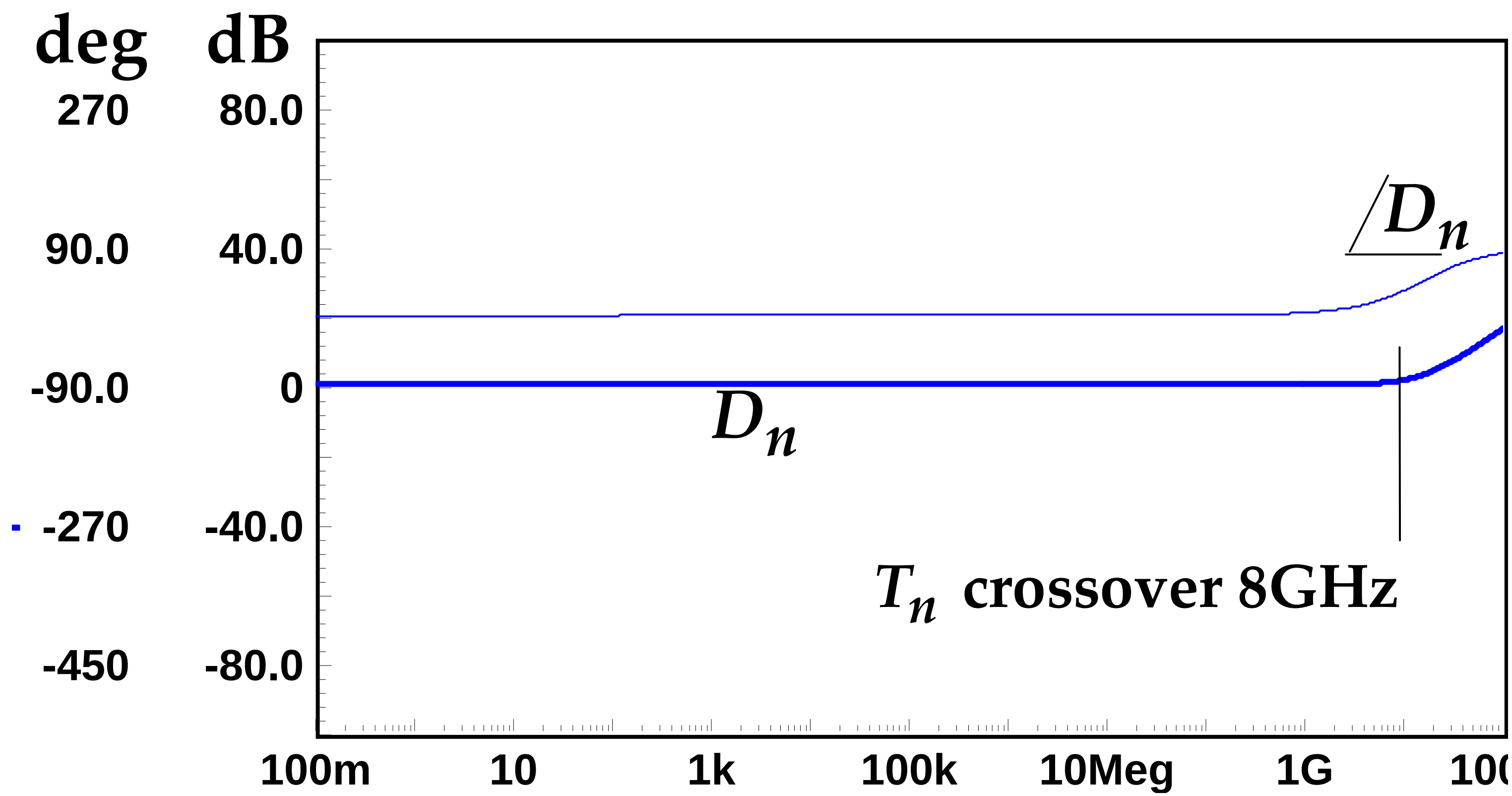
The expectations are borne out:



The discrepancy factor D should be flat at 0dB at low frequencies, with a dominant pole at the T crossover frequency, beyond which $T = D$:



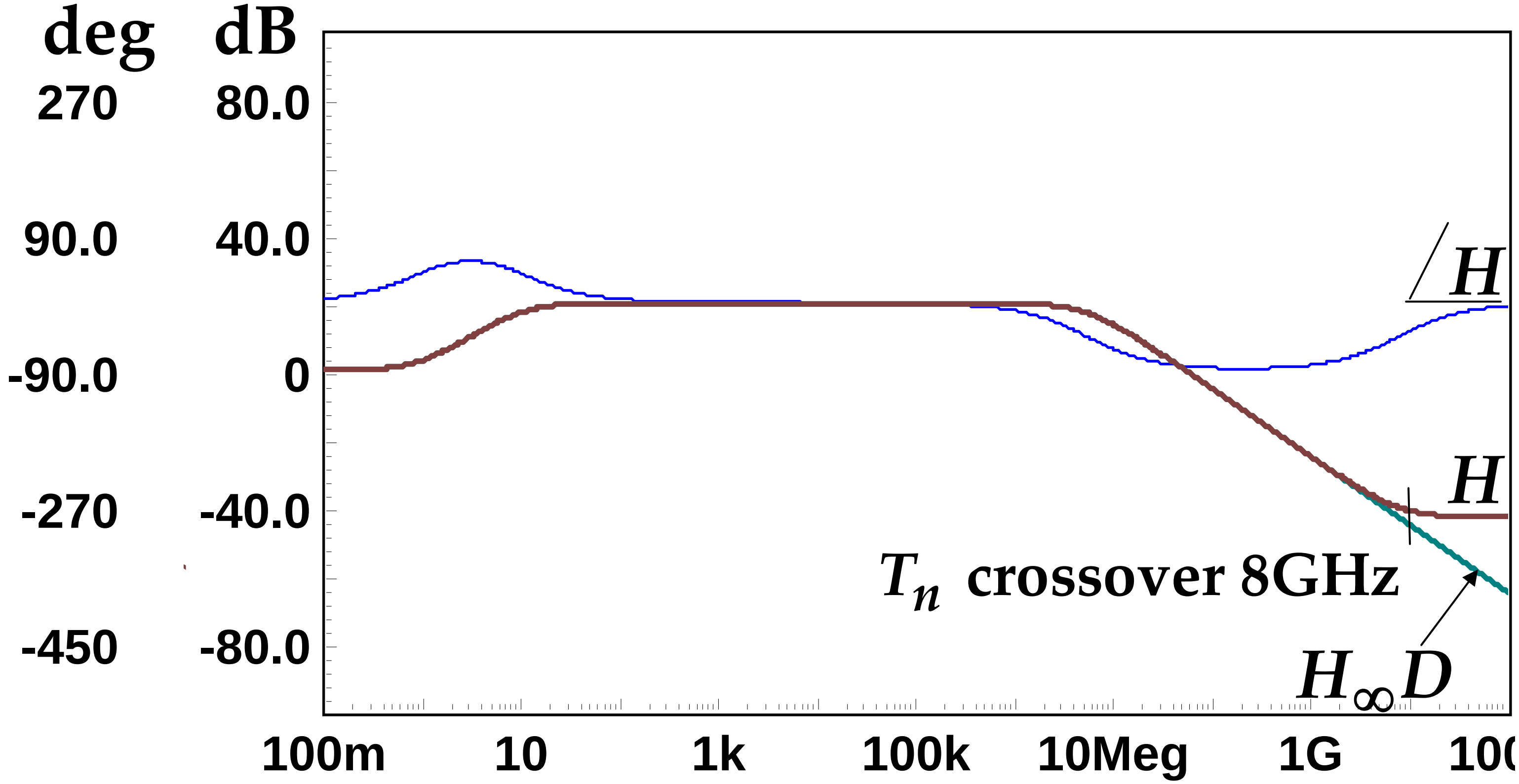
The null discrepancy factor D_n would be 0dB at all frequencies,



but the nonideality direct forward transmission through the feedback path causes D_n, above the T_n crossover frequency, to rise.

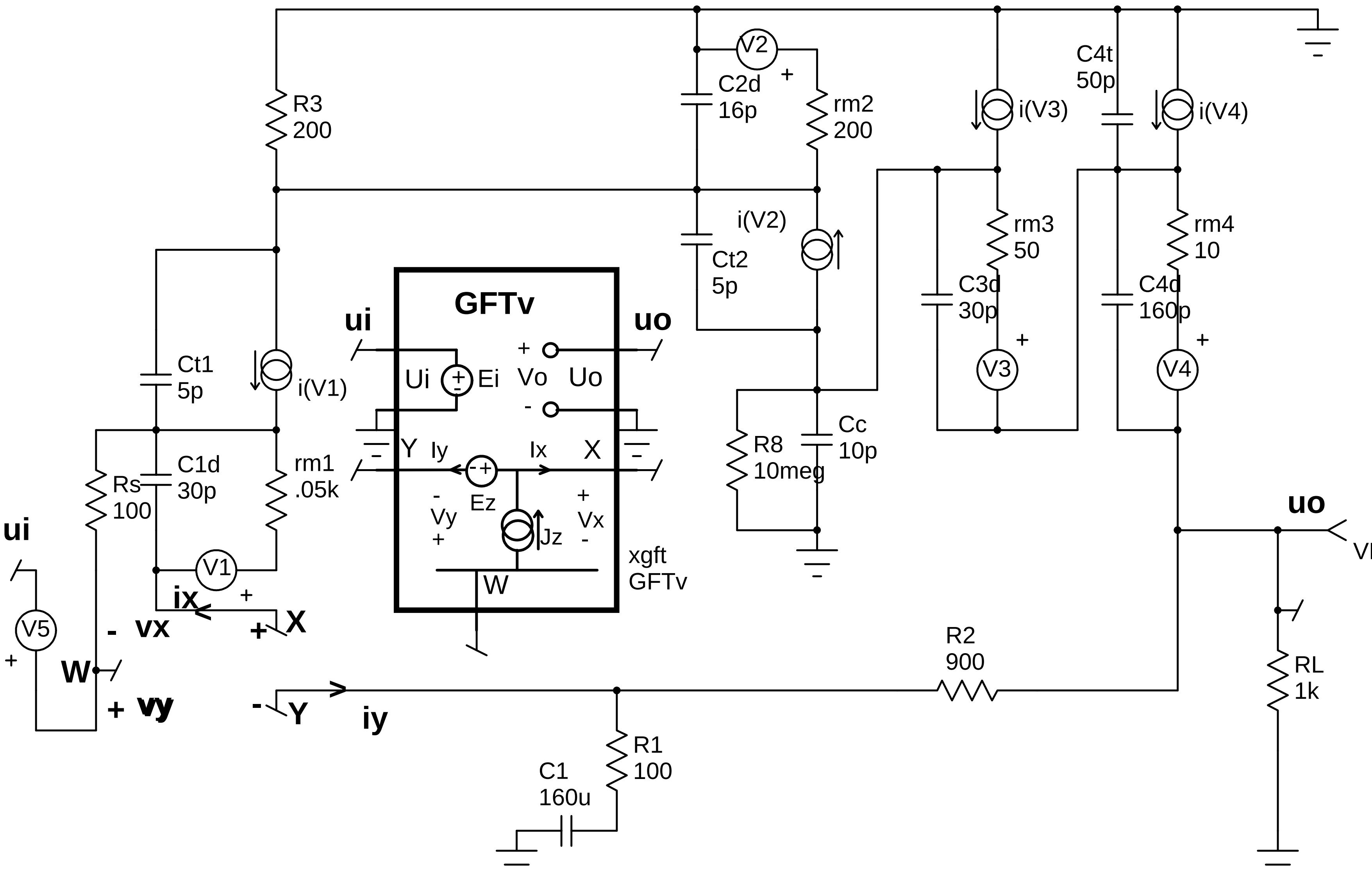
Assembled results:

The closed-loop gain H follows H_∞ up to T crossover, then falls with D. However, H levels off above T_n crossover:

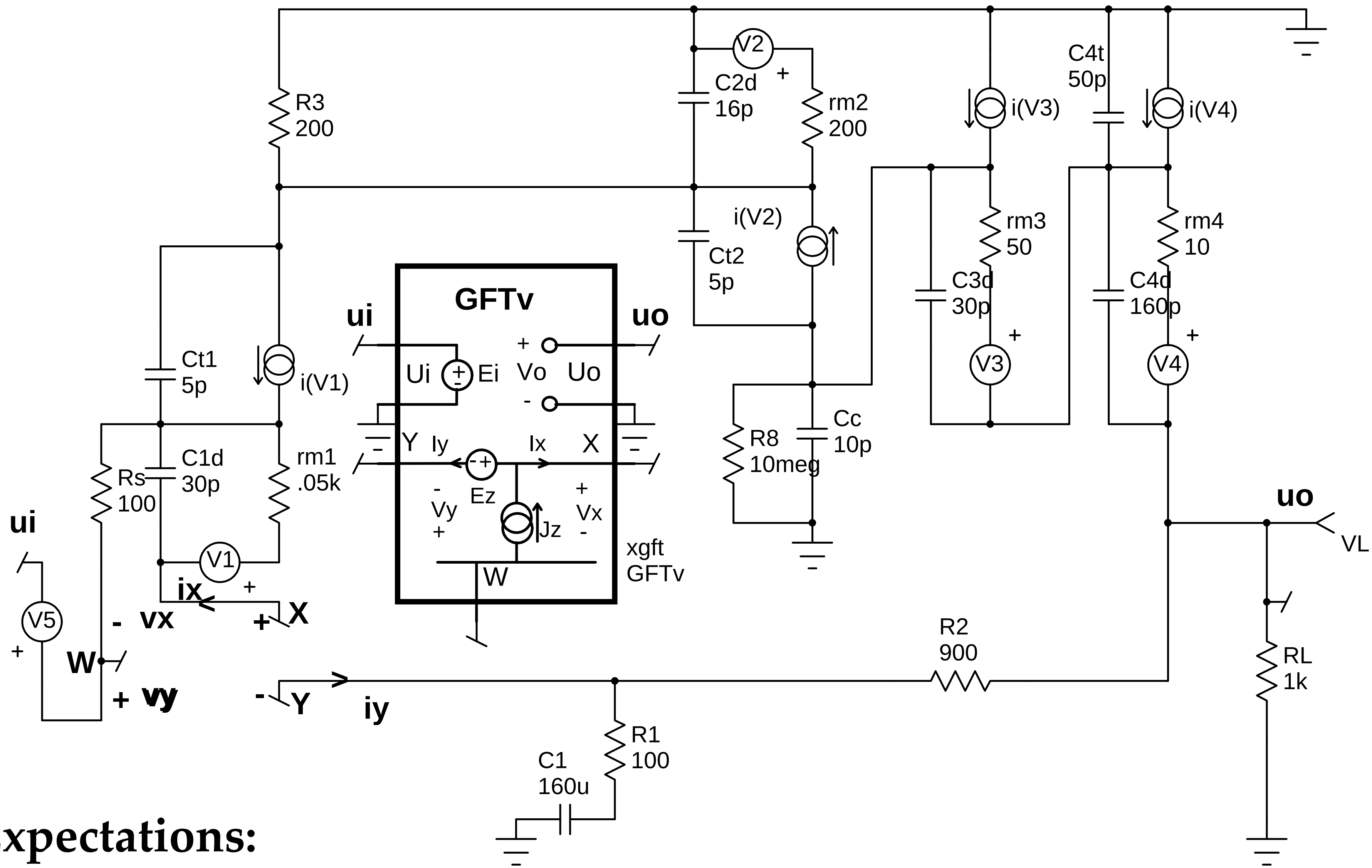


The nonideality is negligible.

Same basic circuit, but many device capacitances included:

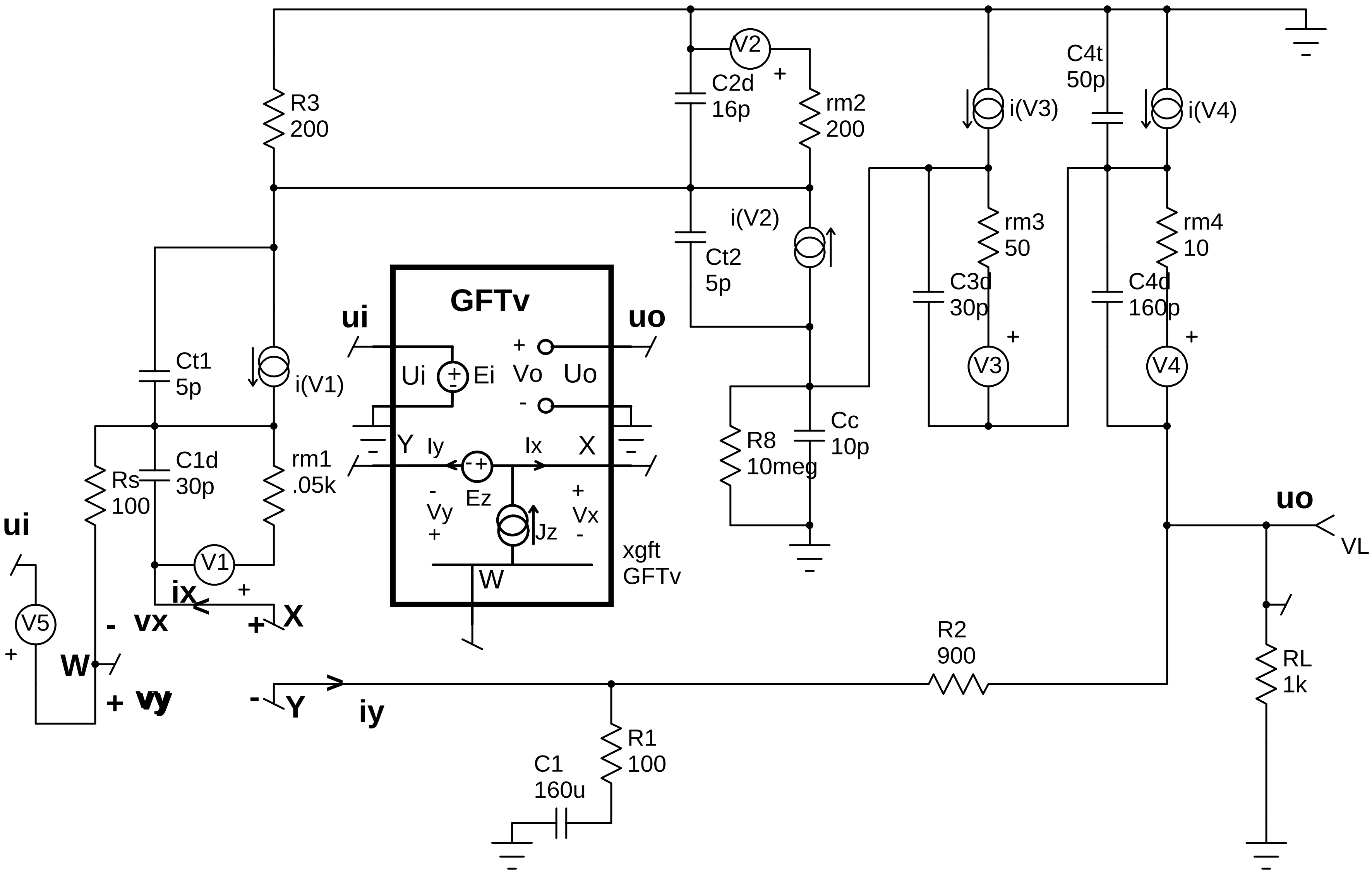


These cause additional nonidealities: reverse transmission in the forward path, and nonzero reverse loop gain.

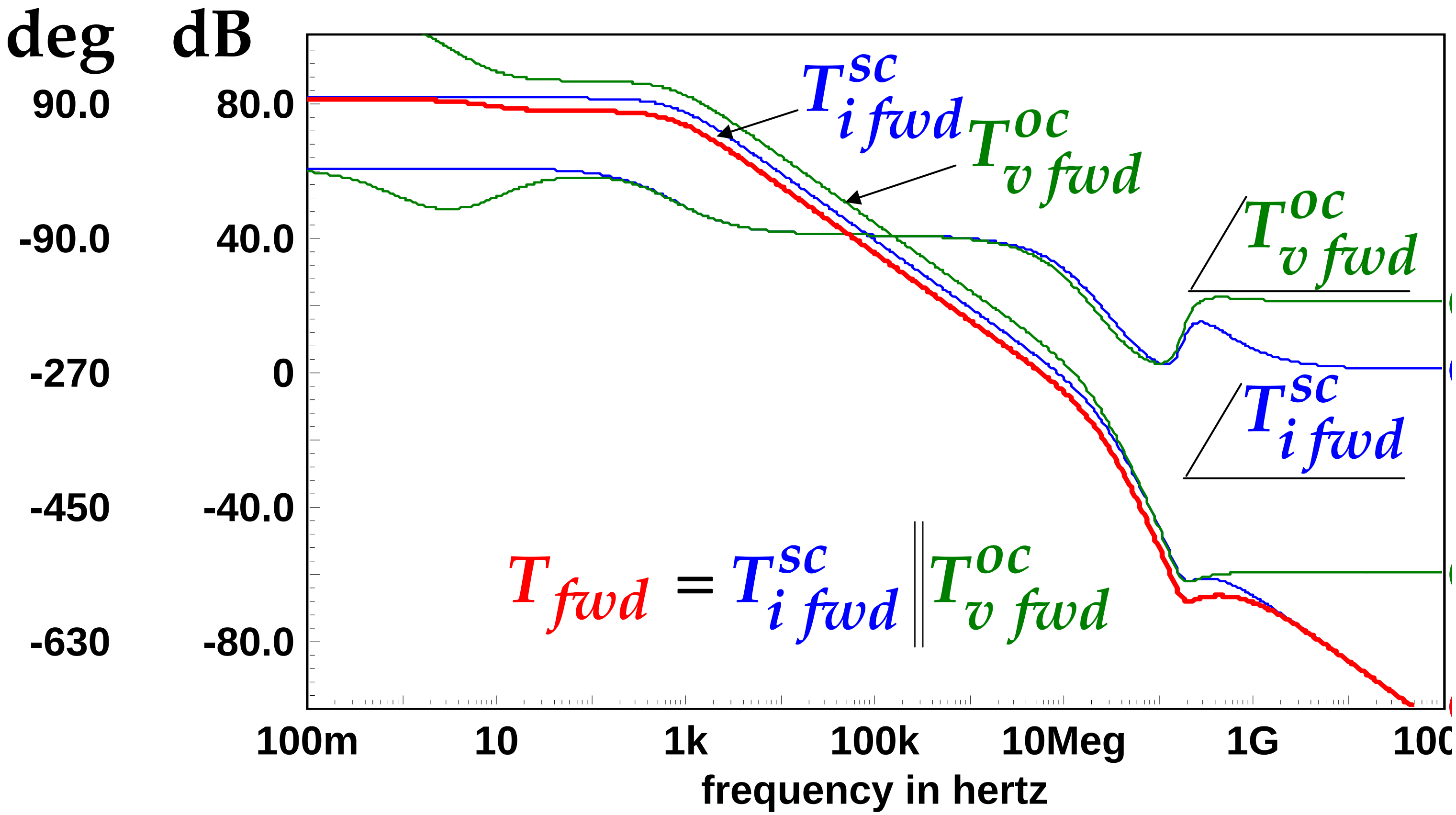


Expectations:

***T* crossover is lowered ; high-frequency *T* is more complicated; phase margin is lowered**

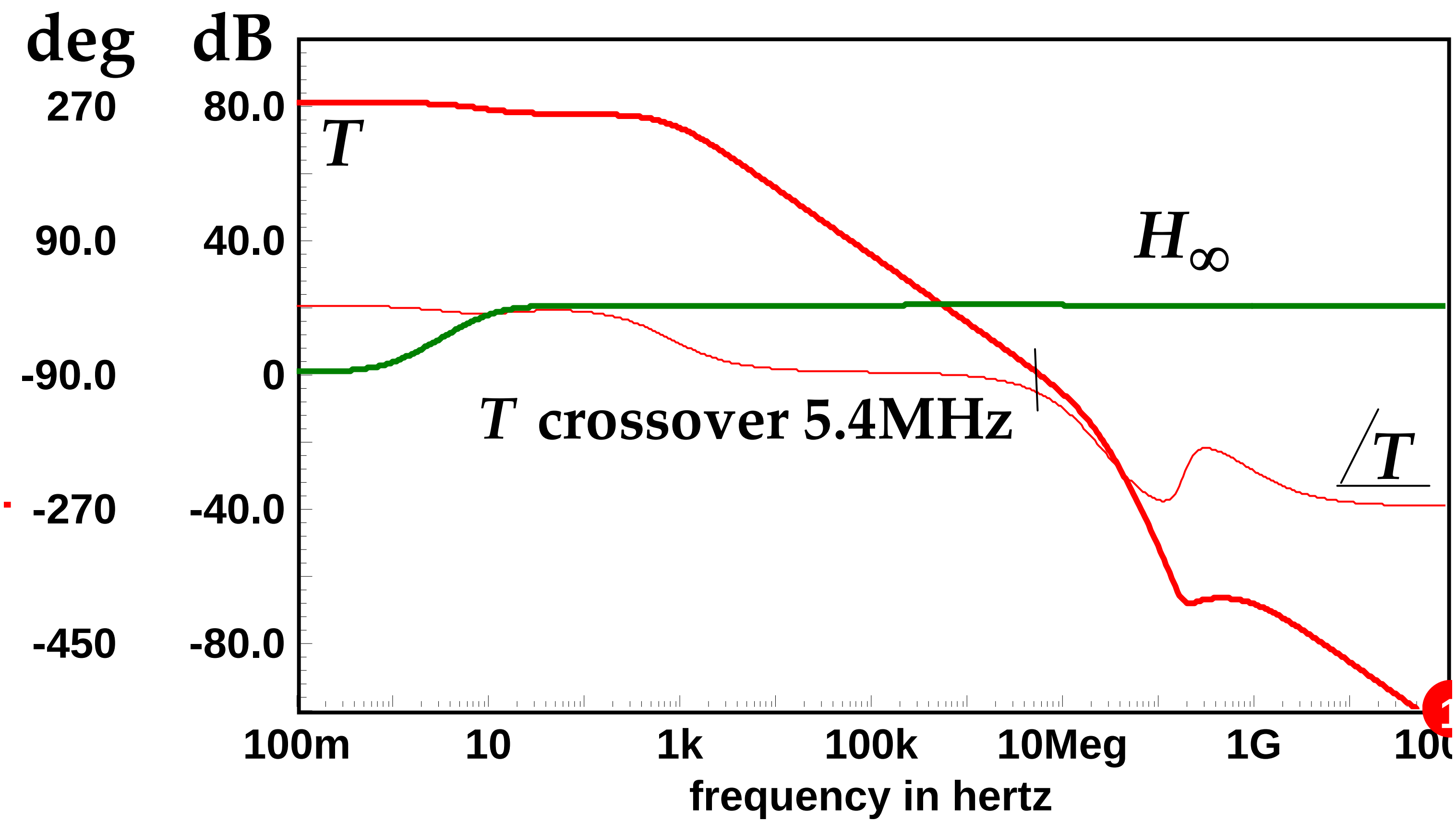


To retain about the same crossover frequency of 5.6MHz, C_c has been lowered from 20pF to 10pF.

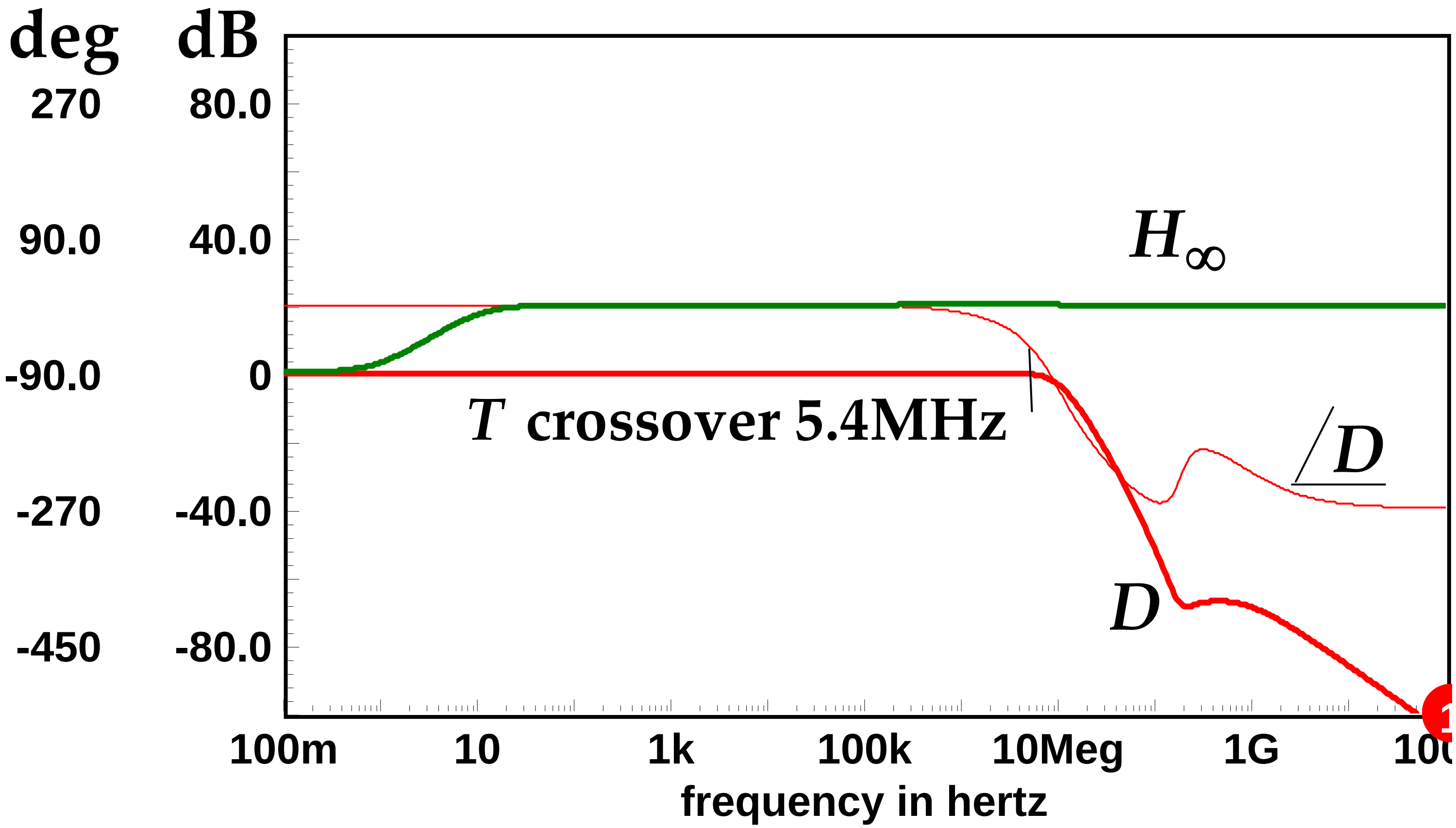


deg
dB
90.0
80.0
-90.0
40.0
-270
0
-450
-40.0
-630
-80.0
100m
10
1k
100k
10Meg
1G
frequency in hertz
$T^{sc}_{i\,fwd}$
$T^{oc}_{v\,fwd}$
$T^{oc}_{v\,fwd}$
$T^{sc}_{i\,fwd}$
$T_{fwd} = T^{sc}_{i\,fwd} \| T^{oc}_{v\,fwd}$

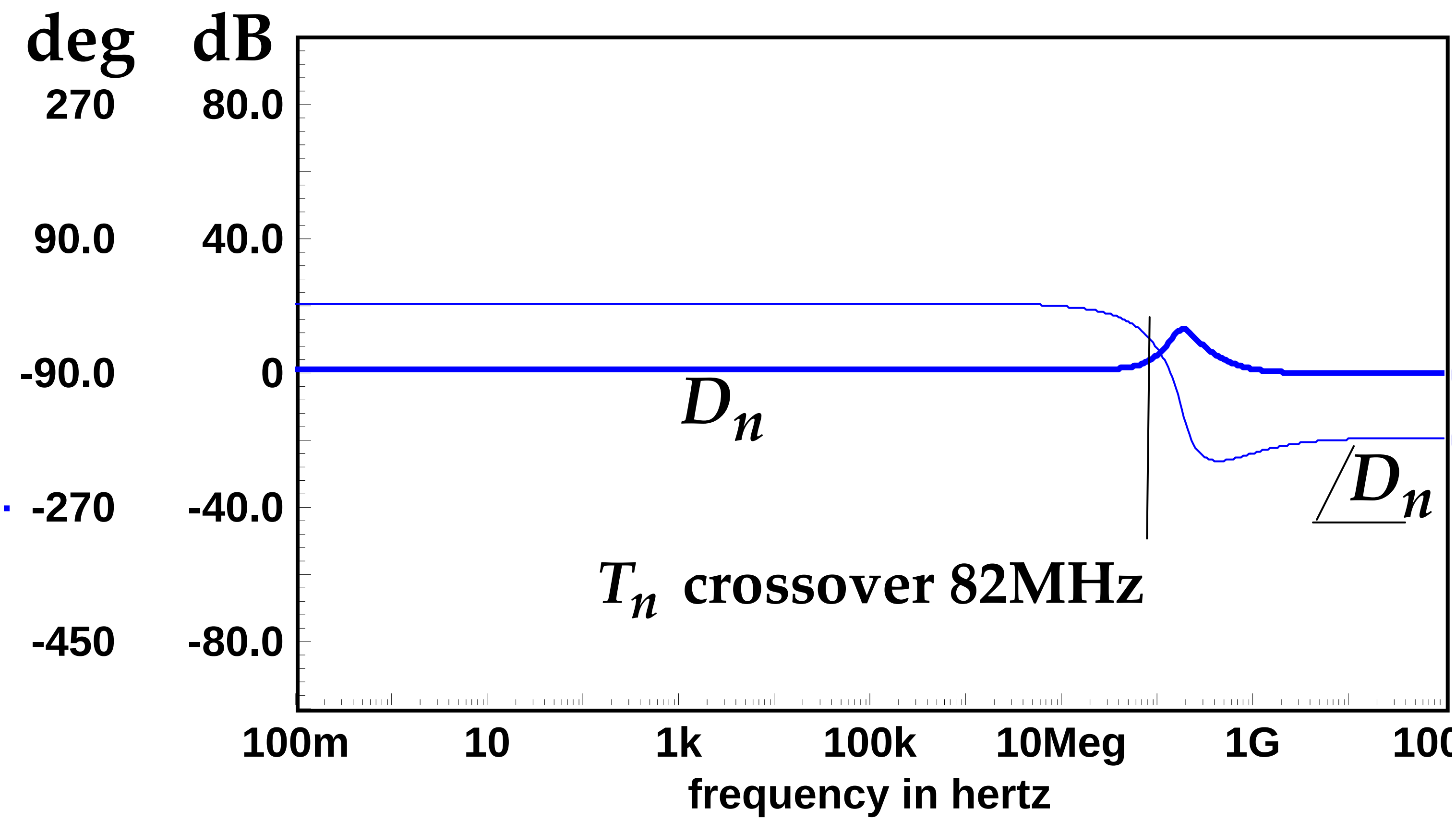
The expectations are borne out:



Beyond T crossover, $D = T$ and therefore is also more complicated:

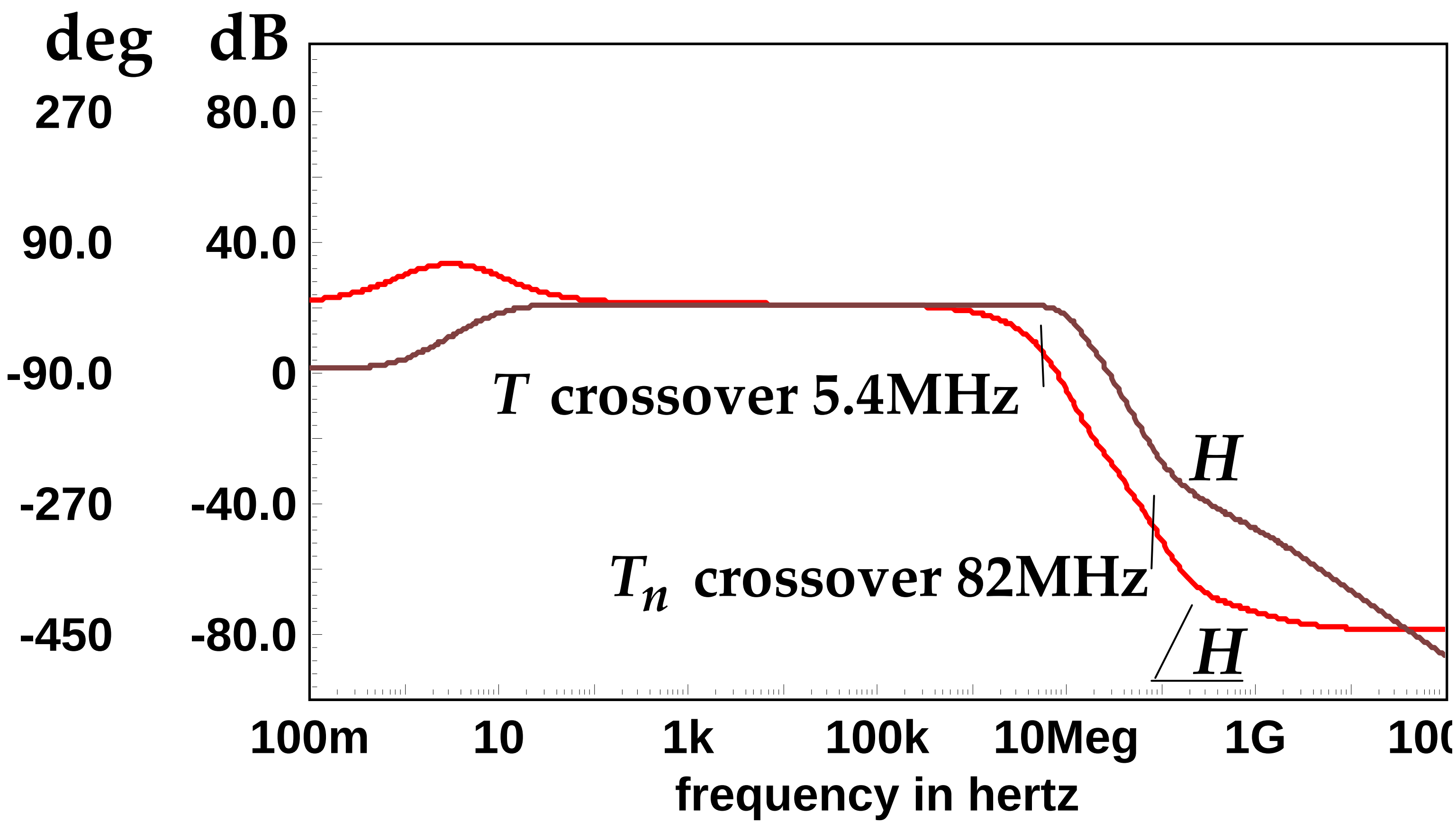


The T_n crossover is drastically lowered from 16GHz to 82MHz:



Even though D_n is approximately 0dB at both low and high frequencies, it undergoes a complete phase reversal

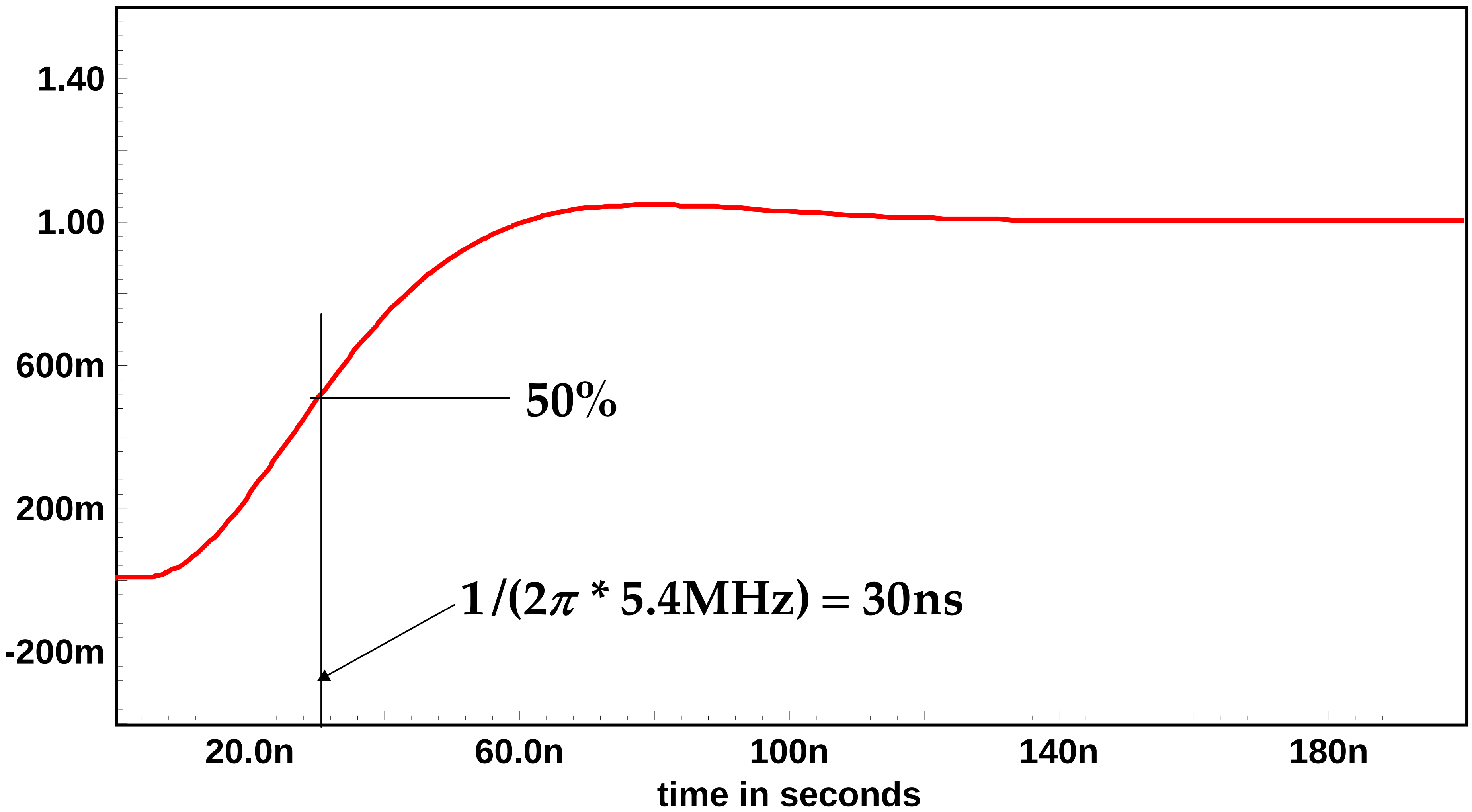
The major effect of the additional nonidealities is to cause $\angle H$ to fall off much more rapidly,



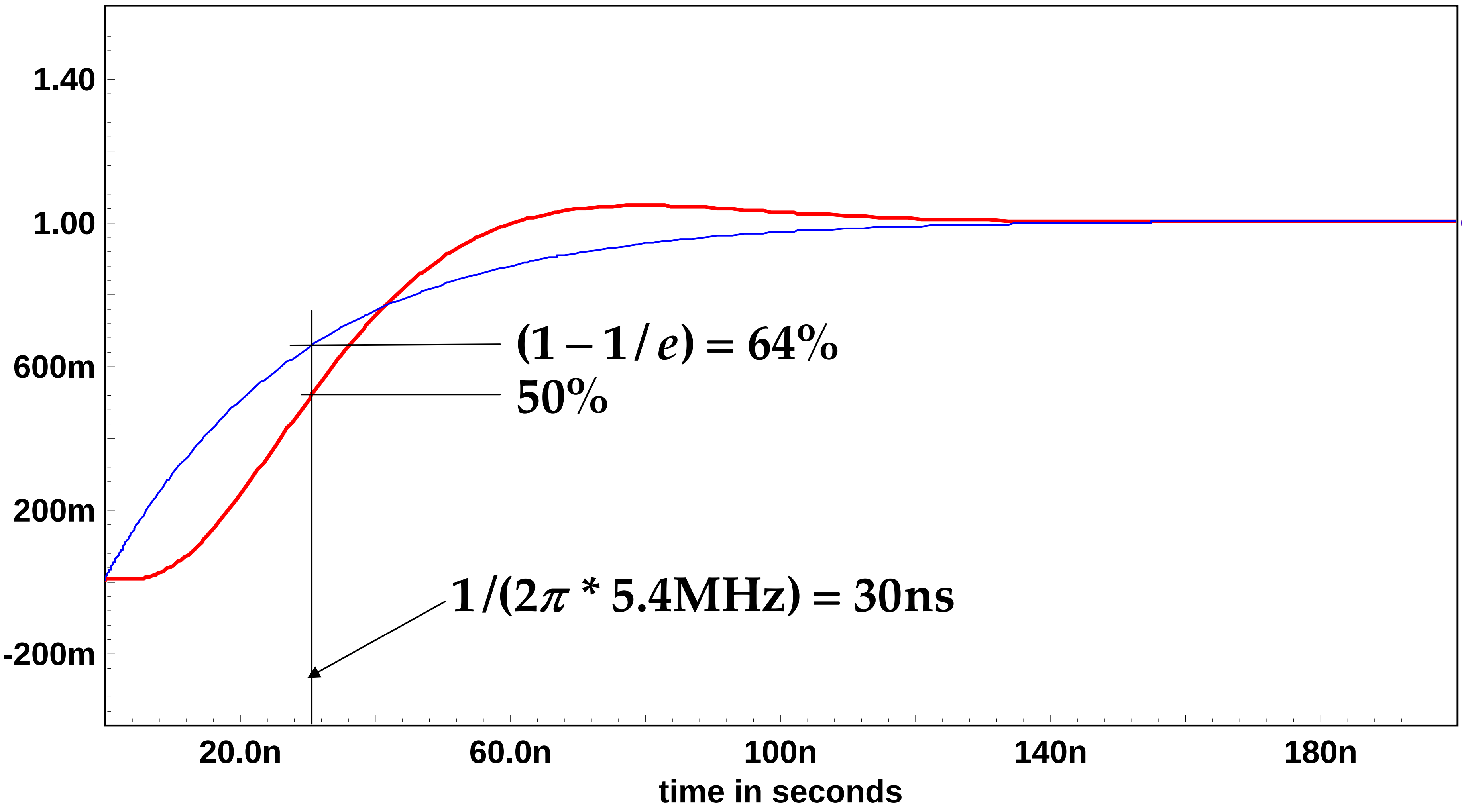
and it is now asymptotic to -450° instead of to 0°!

The transient response is therefore strongly degraded.

Step response:



Add step response with C_c only:



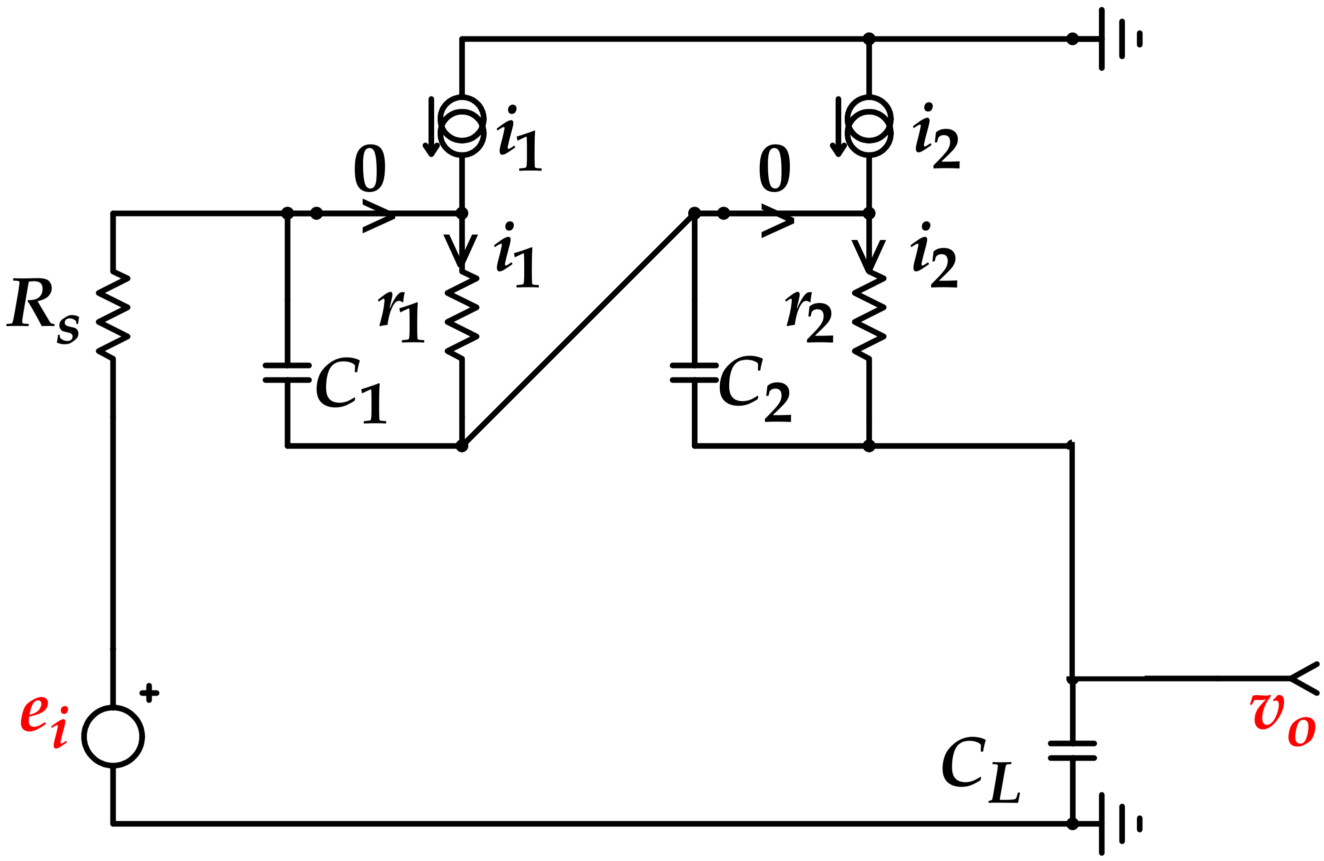
EXAMPLE

16. A DESIGN SOLUTION TO DARLINGTON FOLLOWER INSTABILITY

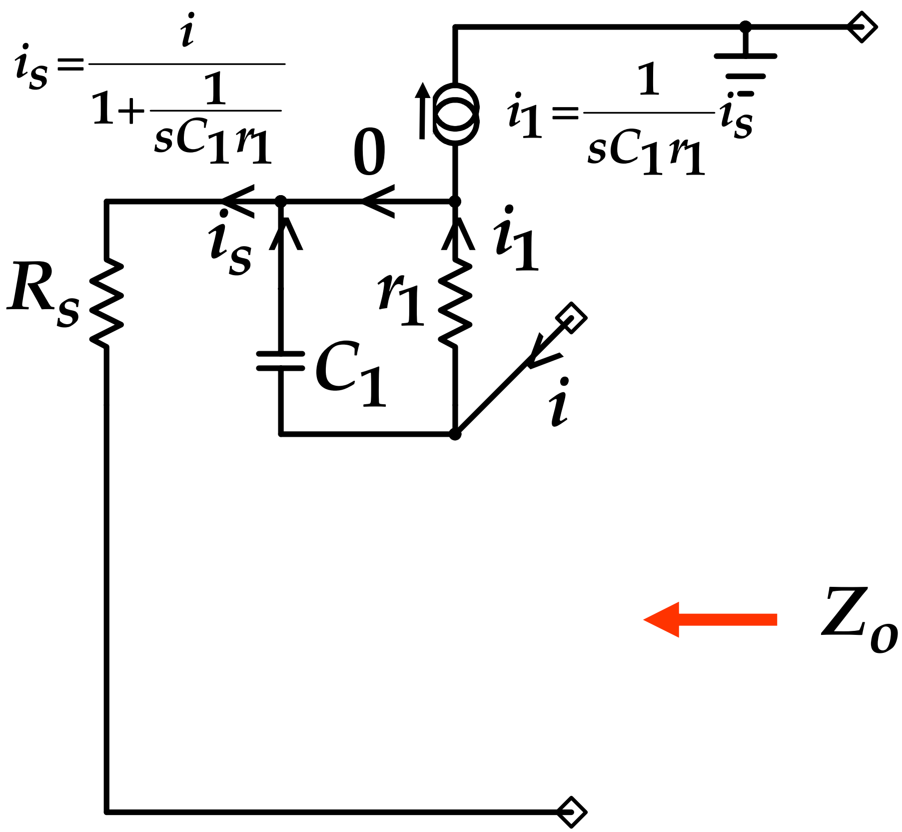
Example 3: Darlington Emitter/Source Follower

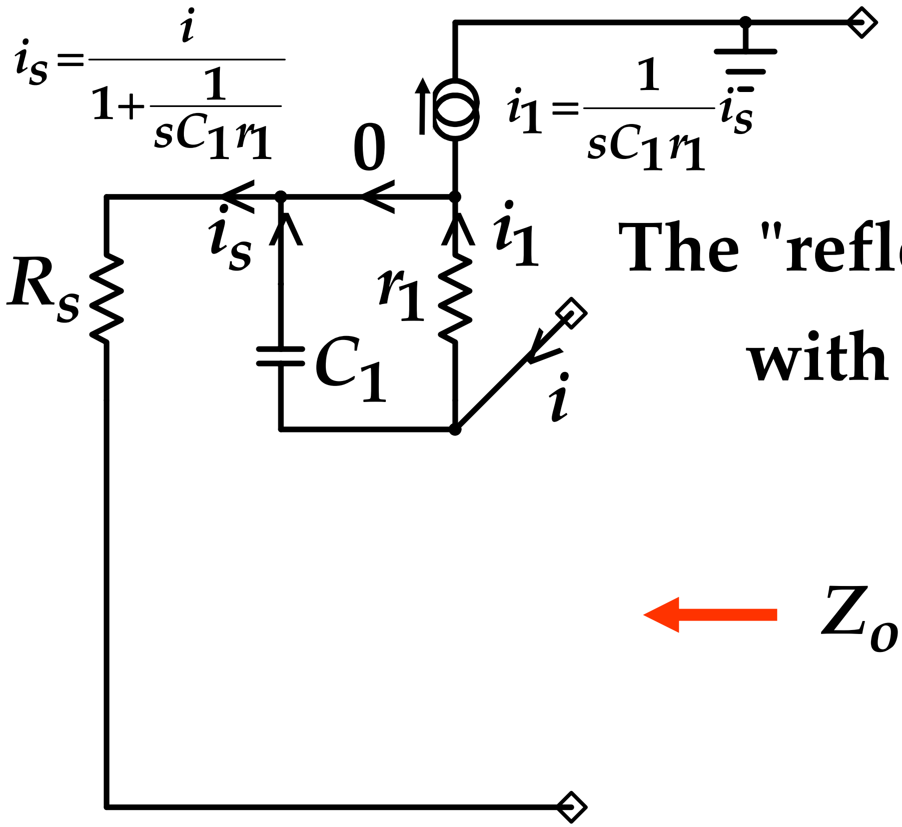
The Darlington Follower is known to be unstable for certain values of load capacitance.

The Design Problem is to select a damping resistance so that the Follower is not only stable, but has a maximum peaking , regardless of the load capacitance.



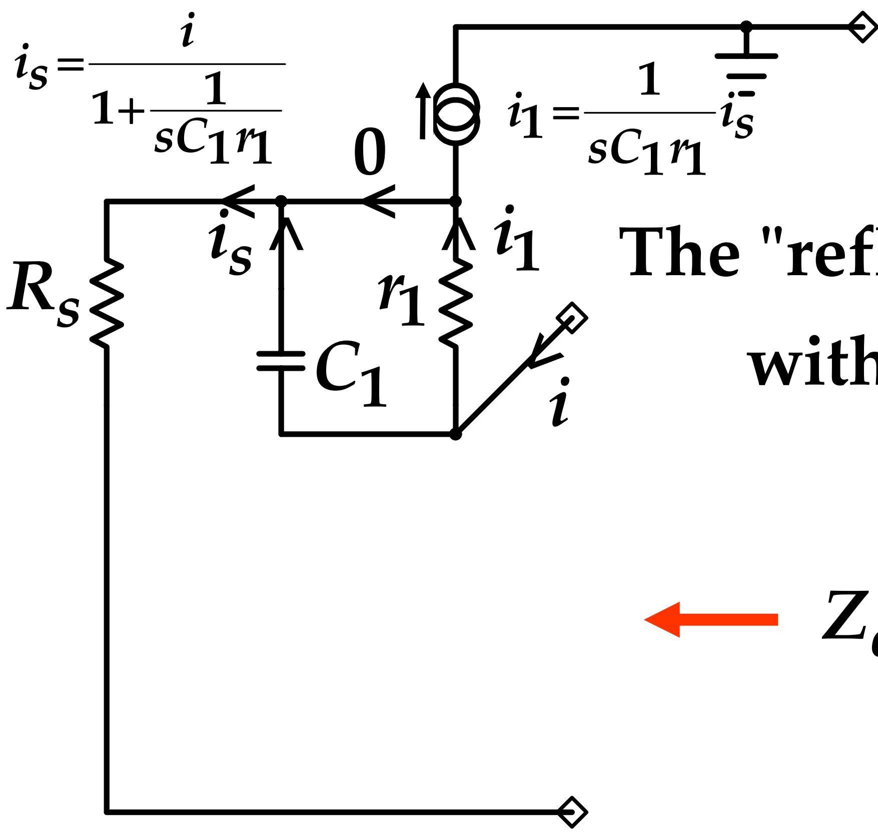
0
0
i_1
i_1
i_2
i_2
R_s
r_1
r_2
C_1
C_2
e_i
+
C_L
v_o


$i_s = \frac{i}{1+\frac{1}{sC_1 r_1}}$
$i_1 = \frac{1}{sC_1 r_1} i_s$
0
i_s
i_1
R_s
r_1
C_1
i
Z_o



The "reflection" process works with effective $\beta_1 = \frac{1}{sC_1 r_1}$:

$$Z_o = \frac{R_s}{1+\beta_1} + r_1 \left\| \frac{1}{sC_1} \right.$$

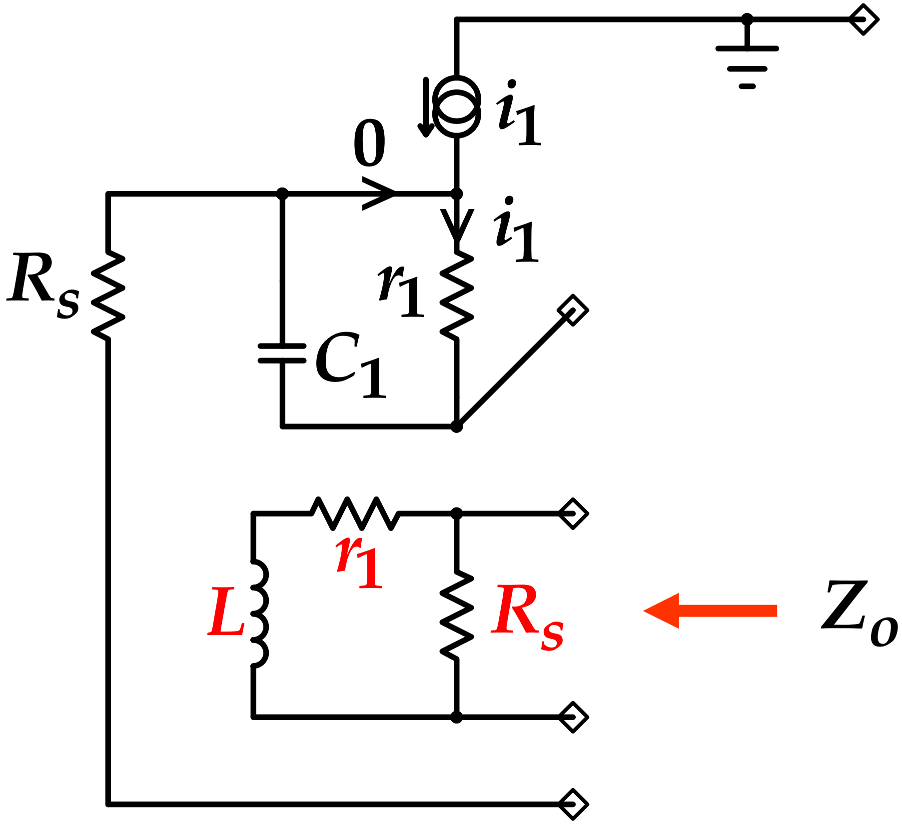


The "reflection" process works with effective $\beta_1 = \frac{1}{sC_1 r_1}$:

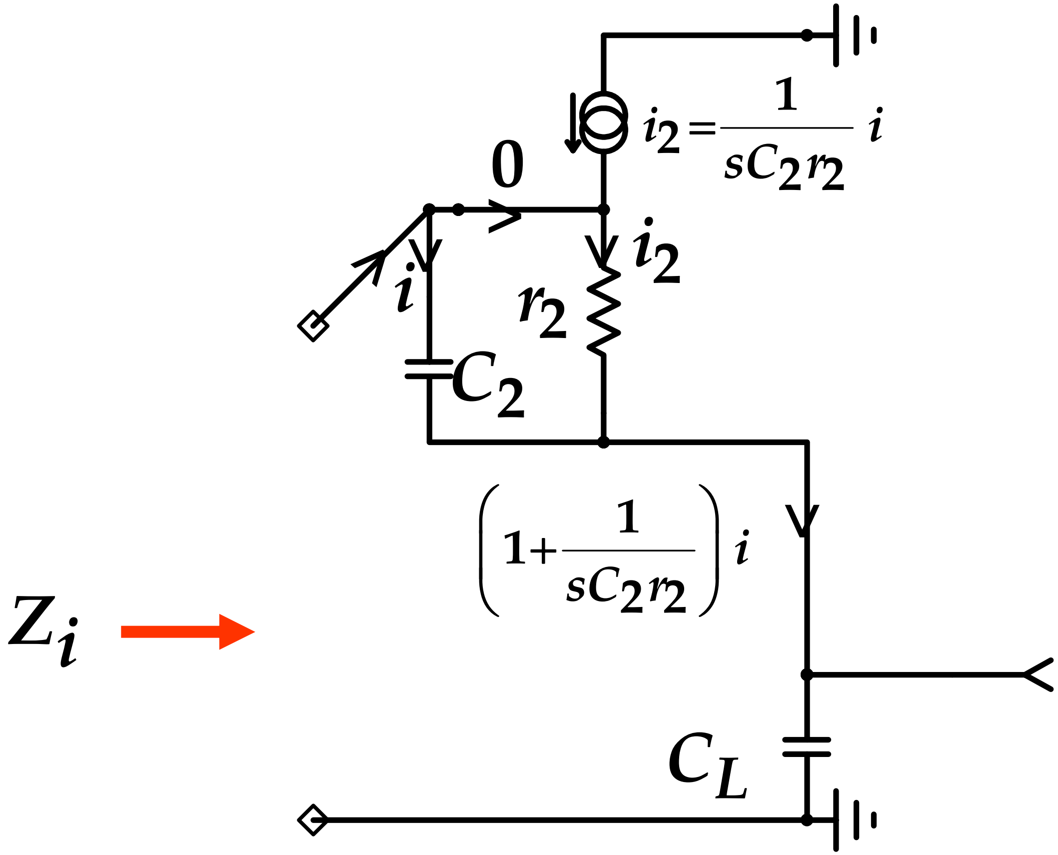
$$Z_o = \frac{R_s}{1+\beta_1} + r_1 \left\| \frac{1}{sC_1} \right.$$

$$Z_o = \frac{R_s + \frac{1}{sC_1}}{1 + \frac{1}{sC_1 r_1}} = \frac{R_s\left(r_1 + sC_1 R_s r_1\right)}{R_s + sC_1 R_s r_1} = \frac{R_s\left(r_1 + sL\right)}{R_s + sL}$$

where $L \equiv C_1 R_s r_1$. **If** $R_s >> r_1$, $Z_o \approx R_s \| \left(r_1 + sL\right)$



$$Z_o \approx R_s \| (r_1 + sL)$$

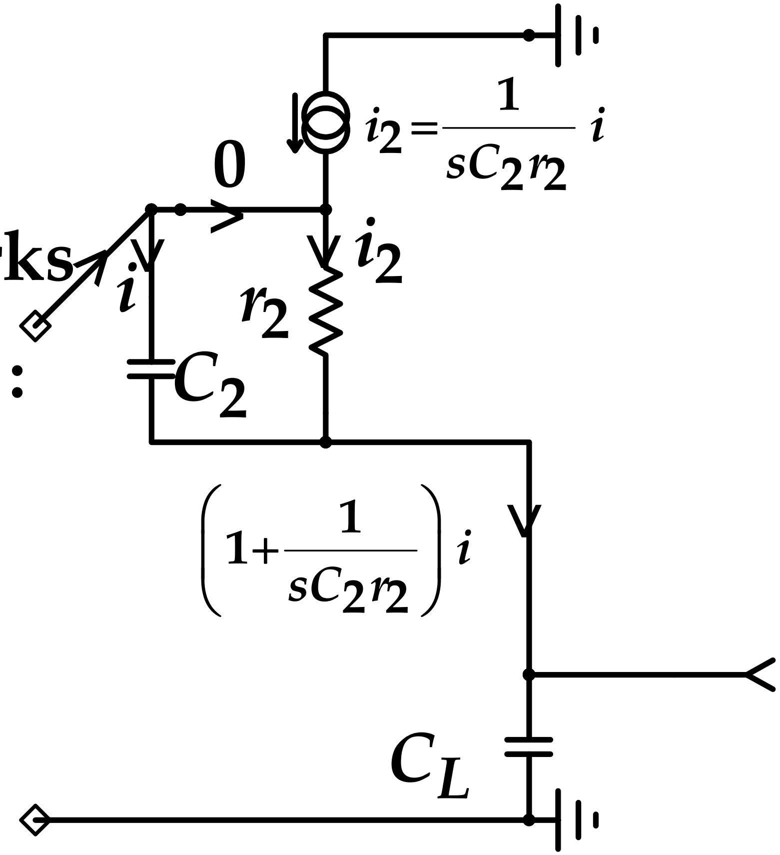


Z_i
$i_2 = \frac{1}{sC_2 r_2} i$
0
i_2
i
r_2
C_2
$\left(1+\frac{1}{sC_2 r_2}\right) i$
C_L

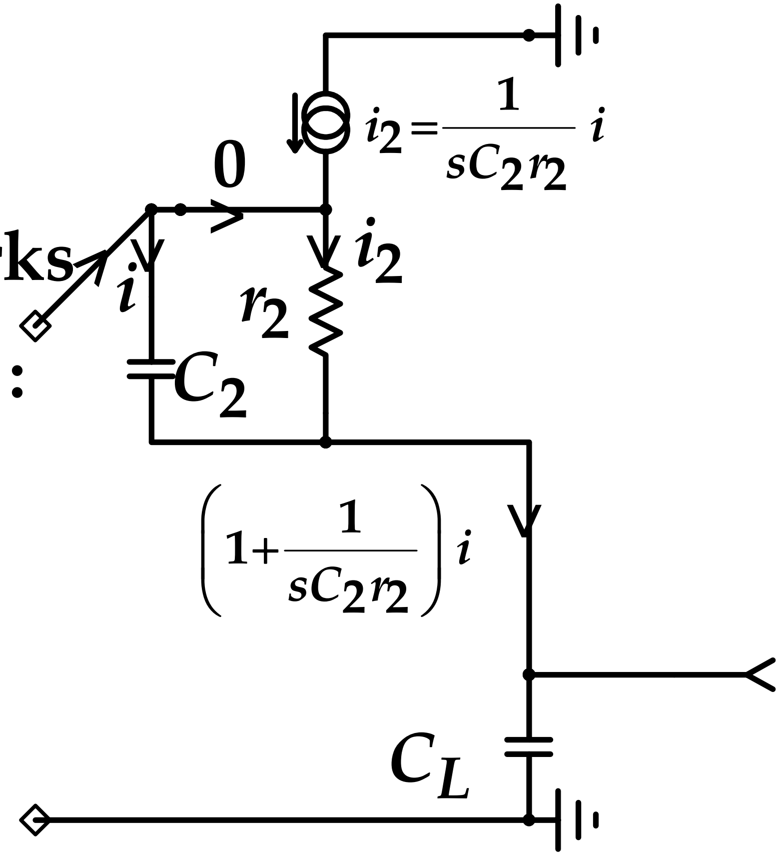
The "reflection" process works

with effective $\beta_2 = \frac{1}{sC_2 r_2}$:

$$Z_i = \frac{1}{sC_2} + \frac{1+\beta_2}{sC_L}$$



The "reflection" process works with effective $\beta_2 = \frac{1}{sC_2 r_2}$:

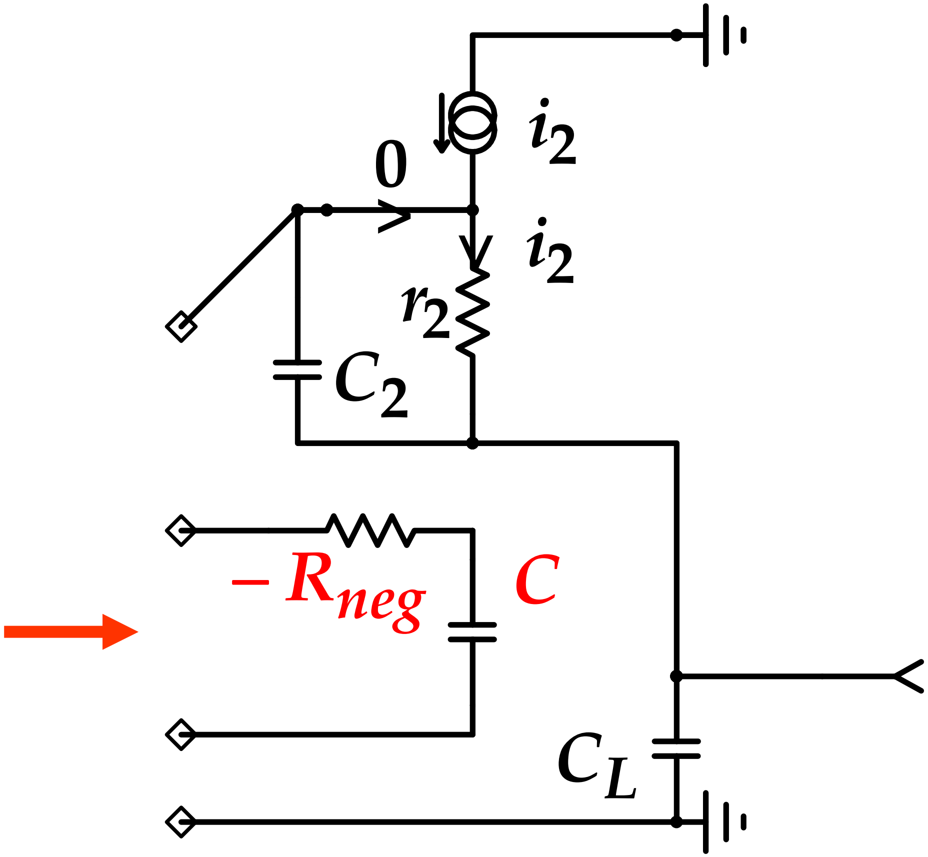


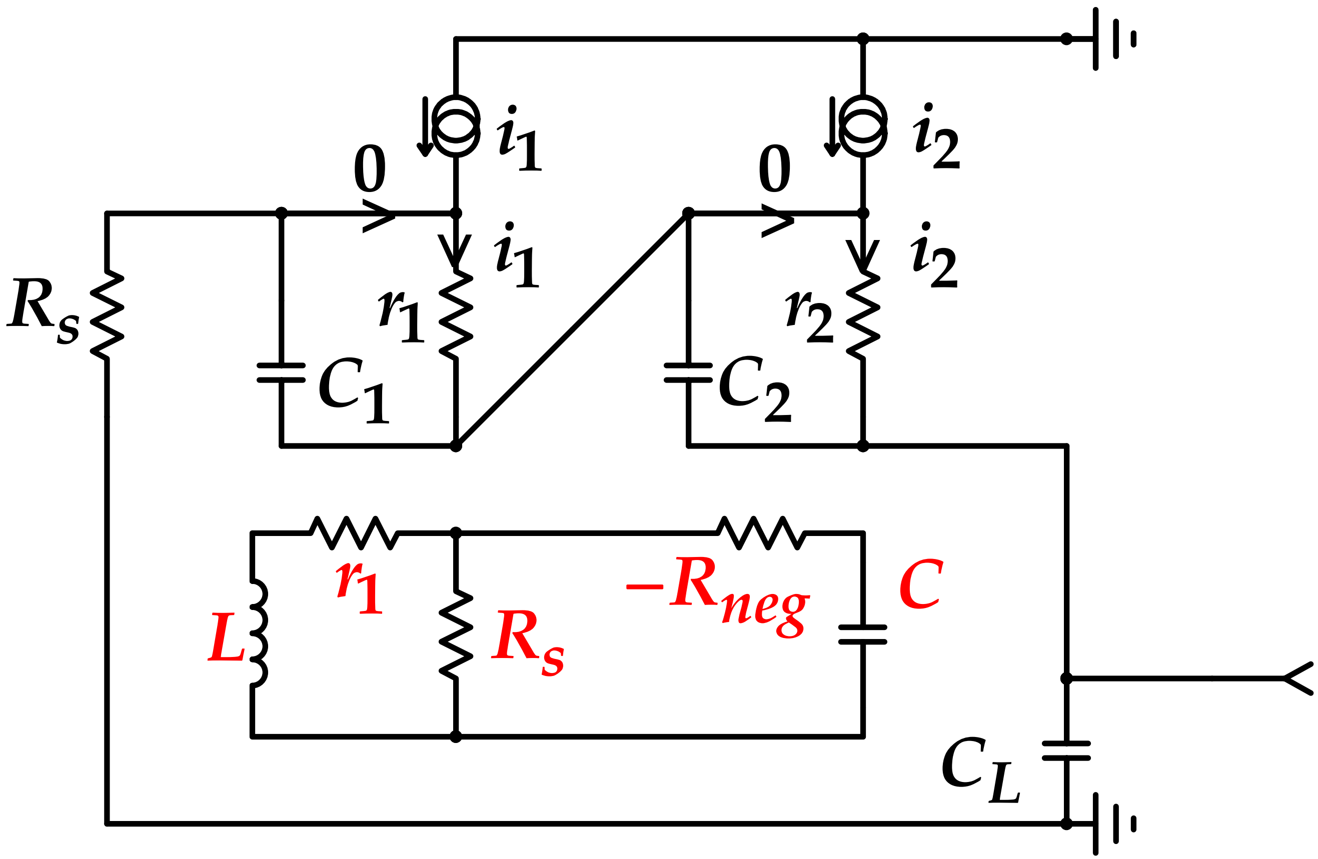
$$Z_i = \frac{1}{sC_2} + \frac{1+\beta_2}{sC_L}$$

$$Z_i = \frac{1}{sC_2} + \frac{1}{sC_L}\left(1 + \frac{1}{sC_2 r_2}\right) = \frac{1}{sC} - R_{neg}$$

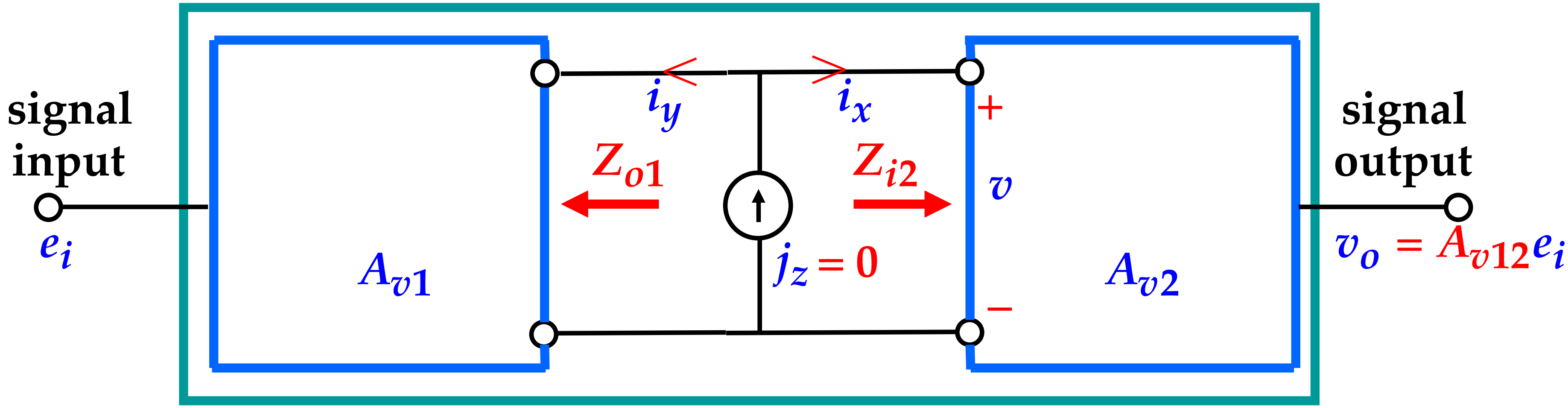
where $\frac{1}{C} \equiv \frac{1}{C_2} + \frac{1}{C_L}$ **and** $R_{neg} \equiv \frac{1}{\omega^2 C_2 r_2 C_L}$

$$Z_i = \frac{1}{sC} - R_{neg}$$




0
0
i_1
i_1
i_2
i_2
R_s
r_1
r_2
C_1
C_2
r_1
L
R_s
$-R_{neg}$
C
C_L

A detailed analysis can be done with the DT/CT

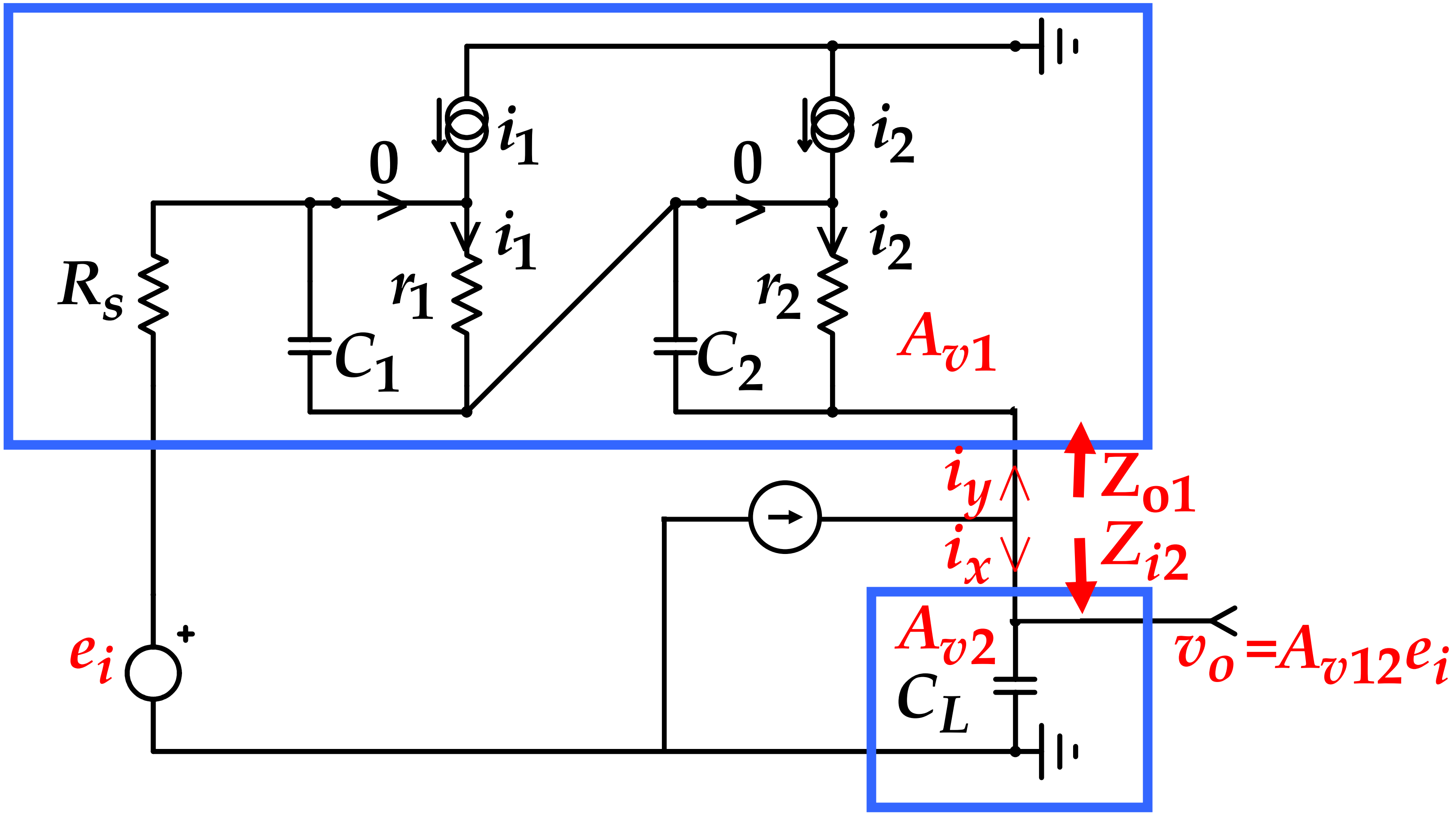


The gain $A_{v12} \equiv \frac{v_o}{e_i}$ is given by the DT:

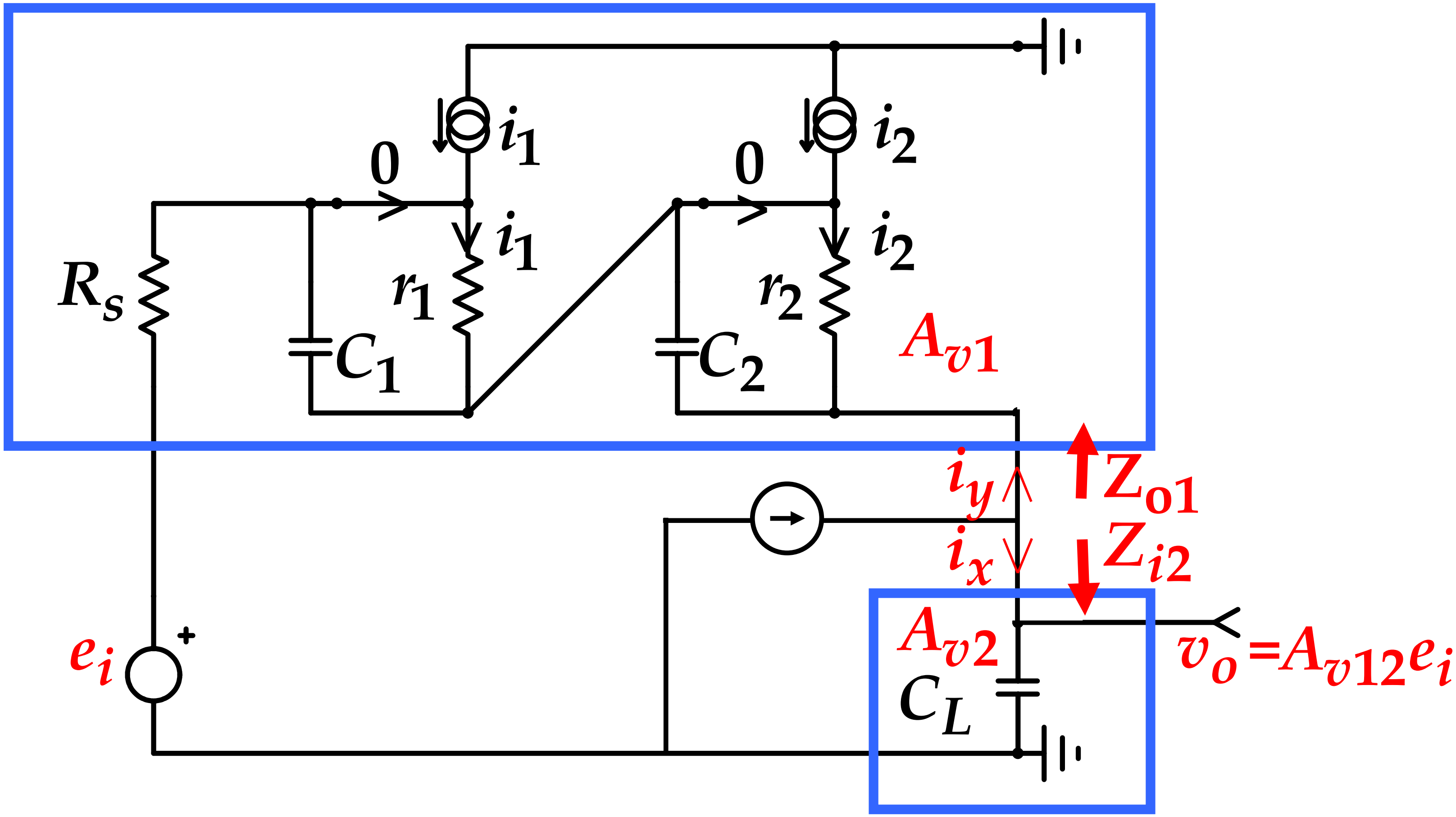
$$A_{v12} = A_{v12}^{i_y} \frac{1 + \frac{1}{T_{ni}}}{1 + \frac{1}{T_i}}$$

The null return ratio $T_{ni} \equiv i_y / i_x \big|_{v_o = 0}$ is an ndi calculation with the output v_o nulled. If v_o is nulled, so is i_x, so $T_{ni} = \infty$.

This implies that T_{ni} is infinite unless the signal can bypass the injection point.



0
0
i_1
i_1
i_2
i_2
R_s
r_1
r_2
C_1
C_2
A_{v1}
i_y
i_x
Z_{o1}
Z_{i2}
A_{v2}
C_L
e_i
+
$v_o = A_{v12} e_i$

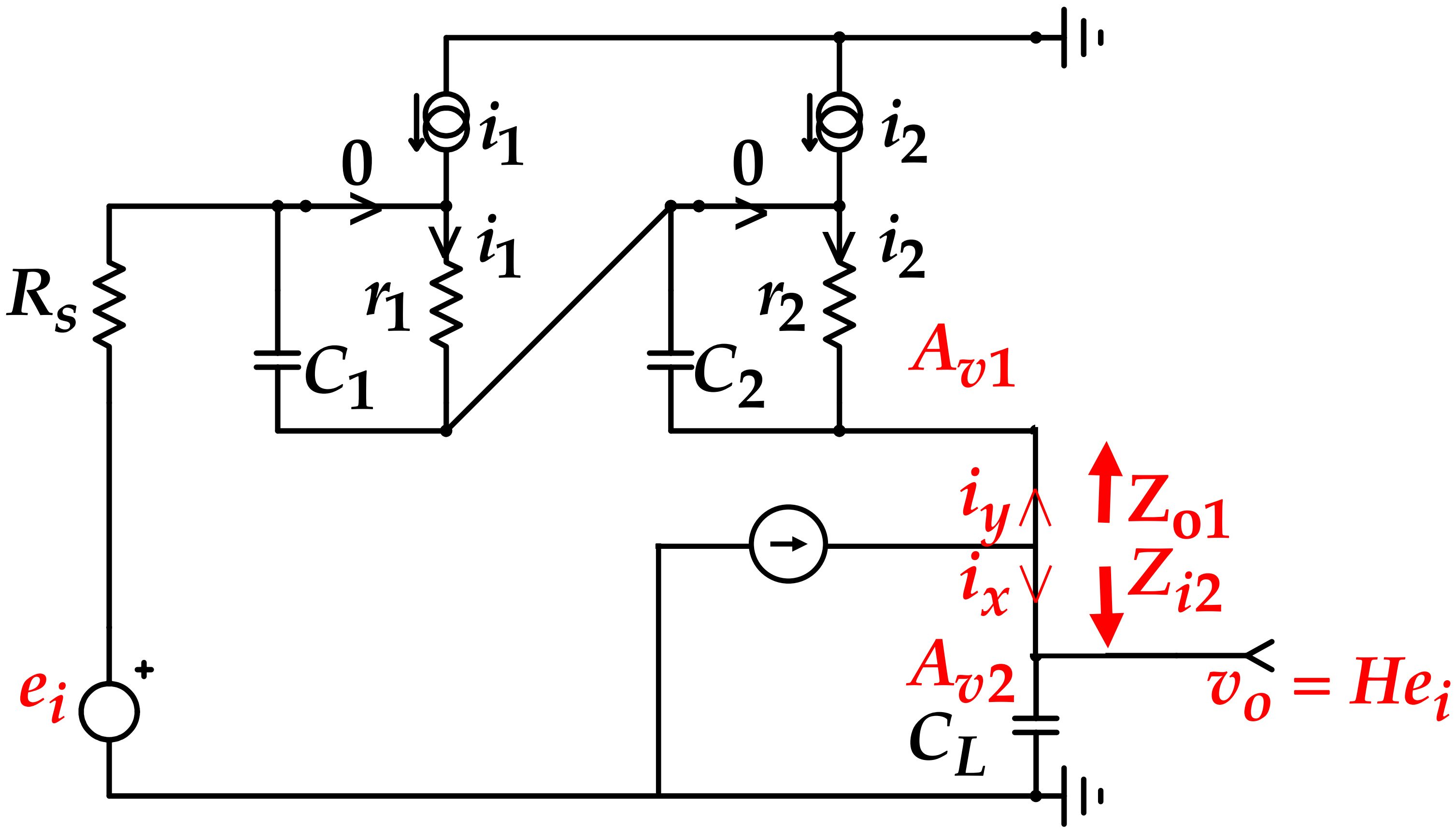


Nulled i_y means that the A_{v1} box is unloaded, so the input voltage to the A_{v2} box is the open-circuit (oc) output voltage of the A_{v1} box. Thus

$$A_{v12}^{i_y} = A_{v1}^{oc} A_{v2}$$

Also, $T_i = Z_{i2} / Z_{o1}$,

so the DT becomes $$A_{v12} = A_{v1}^{oc} A_{v2} \frac{1}{1 + \frac{Z_{o1}}{Z_{i2}}}$$



$$A_{v1}^{oc} = 1, \quad A_{v2} = 1, \quad \text{so}$$

$$H = A_{v12} = \frac{1}{1+\frac{1}{T}} = D \quad \text{where} \;\; T = T_i = \frac{Z_{i2}}{Z_{o1}}$$

We can figure out a lot about T before doing a simulation.

Z_{o1} has two poles, because it has two capacitances.

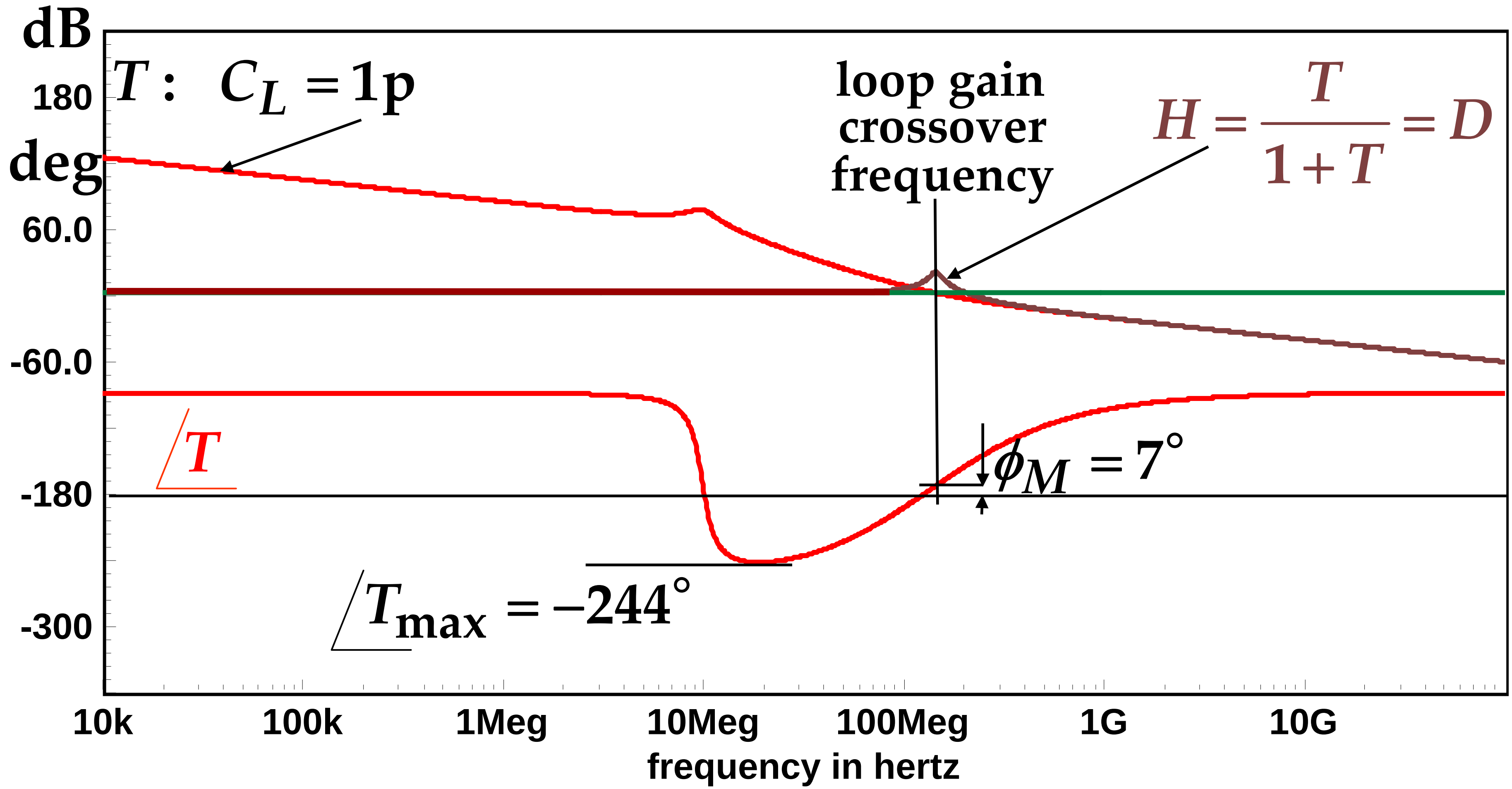
By "mental frequency sweep,"

$$Z_{o1}(0) = r_2, \qquad Z_{o1}(\infty) = R_s$$

Since Z_{o1} is flat at zero and infinite frequency, Z_{o1} must have two zeros as well as two poles.

Also, $Z_{i2} = \dfrac{1}{sC_L}$

Therefore, $T = \dfrac{Z_{i2}}{Z_{o1}}$ **has three poles and two zeros.**



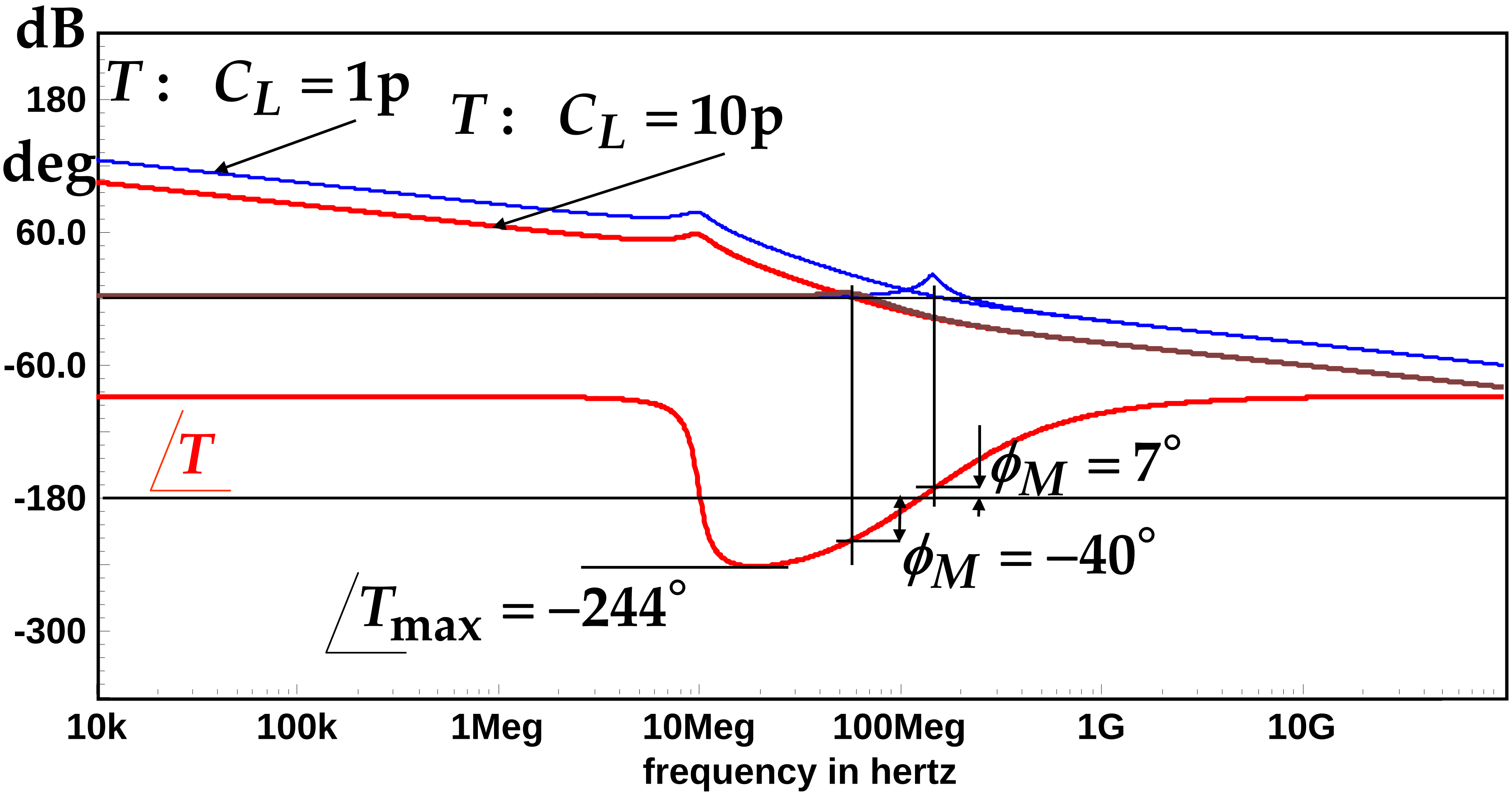
The phase margin ϕ_M is positive, so the Follower is stable. However, ϕ_M is small, so H has a large peak.

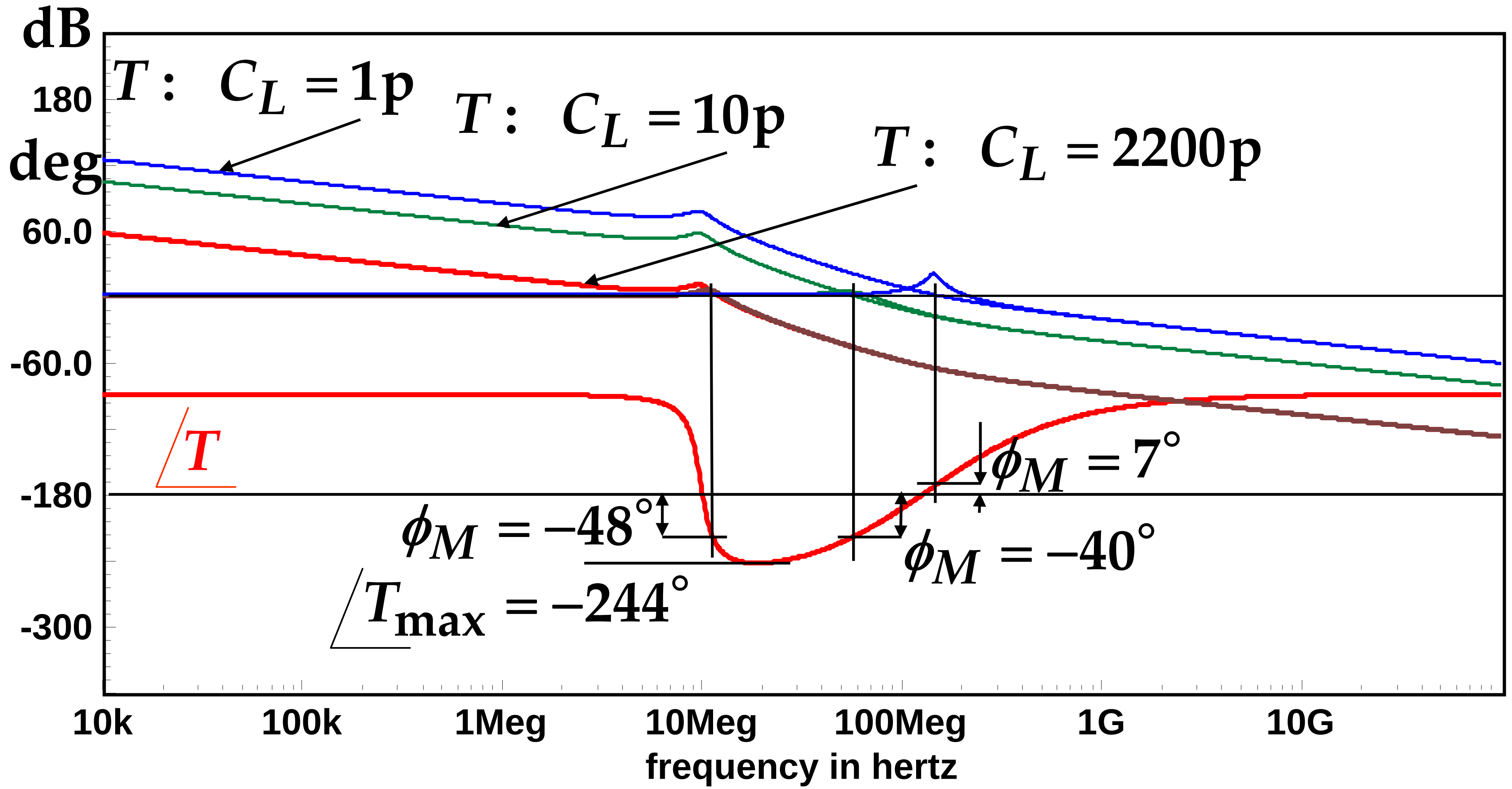
For design, C_L is to be considered a variable.

Examine the effect of C_L upon $T = 1/sC_LZ_{o1}$.

Since C_L is not inside Z_{o1}, increasing C_L does not change the shape of either $|T|$ or $\angle T$; $|T|$ decreases in inverse proportion to C_L, but $\angle T$ does not change at all.

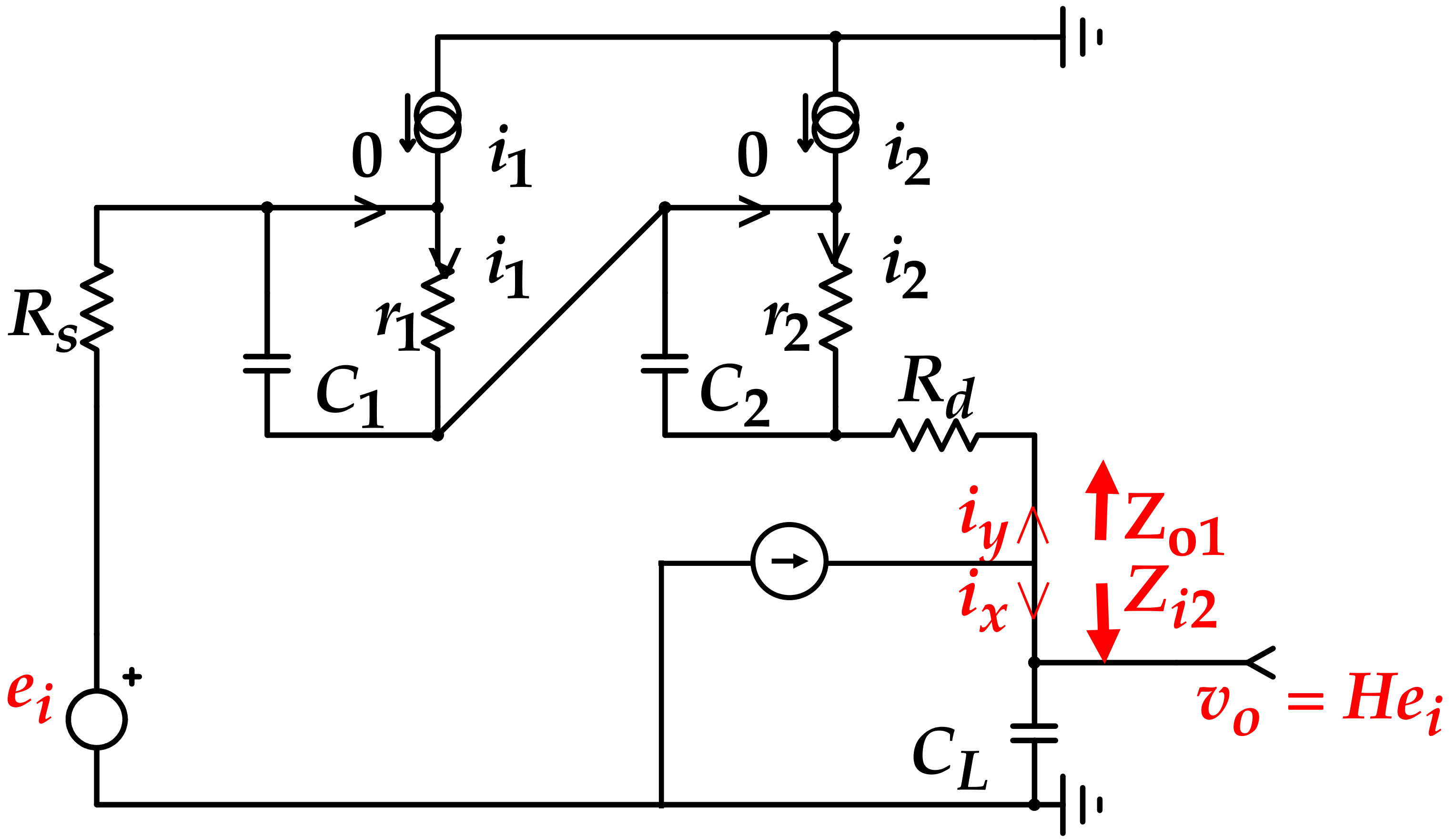
The decrease of $|T|$ results in lowered loop gain crossover frequency:


dB
180
deg
60.0
-60.0
-180
-300
$T: \;\; C_L = 1\text{p}$
$T: \;\; C_L = 10\text{p}$
$\angle T$
$\phi_M = 7^\circ$
$\phi_M = -40^\circ$
$\angle T_{\max} = -244^\circ$
10k
100k
1Meg
10Meg
100Meg
1G
10G
frequency in hertz

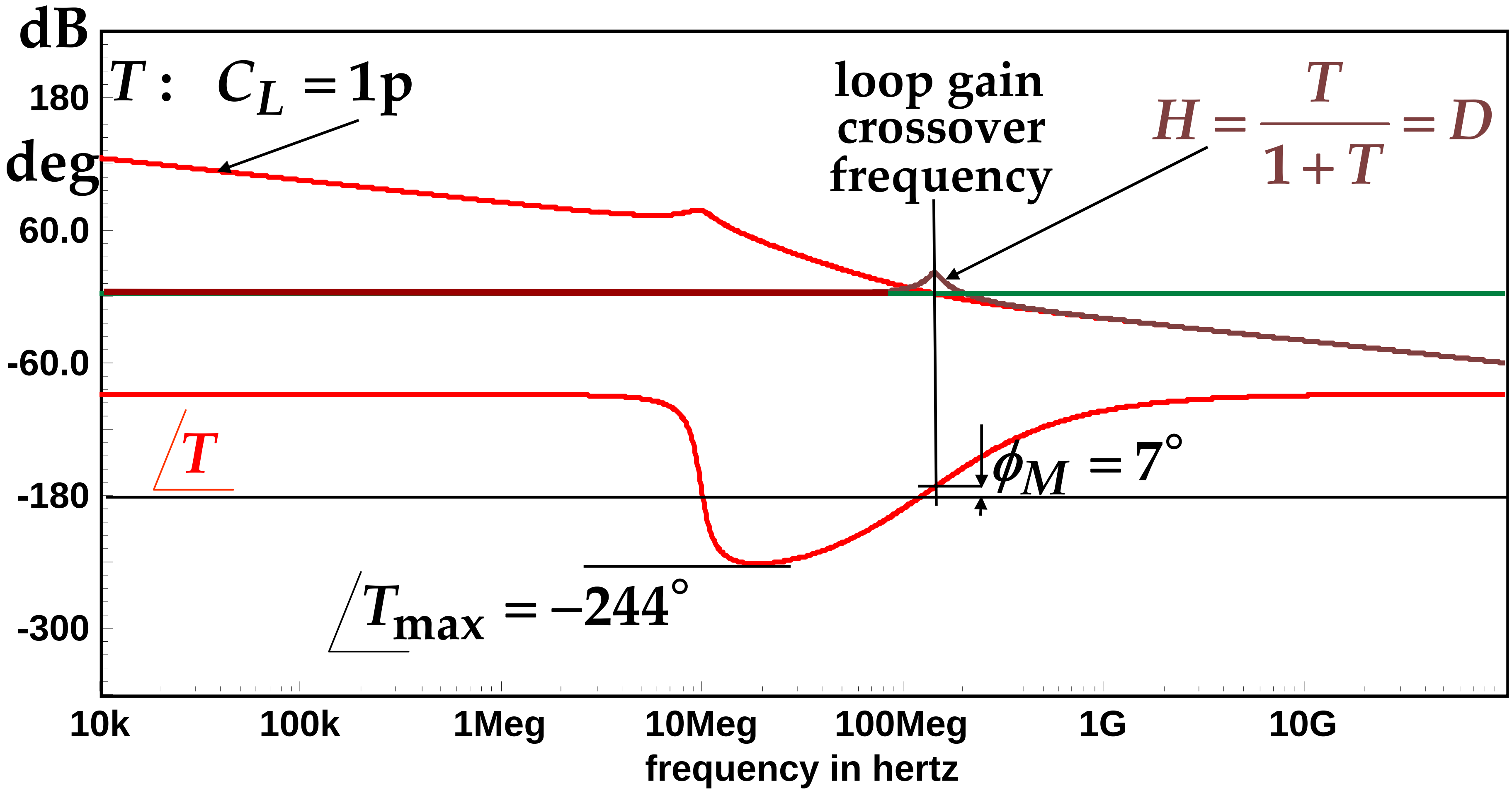


There is a range of C_L for which the phase margin ϕ_M is negative, and the Follower is unstable.

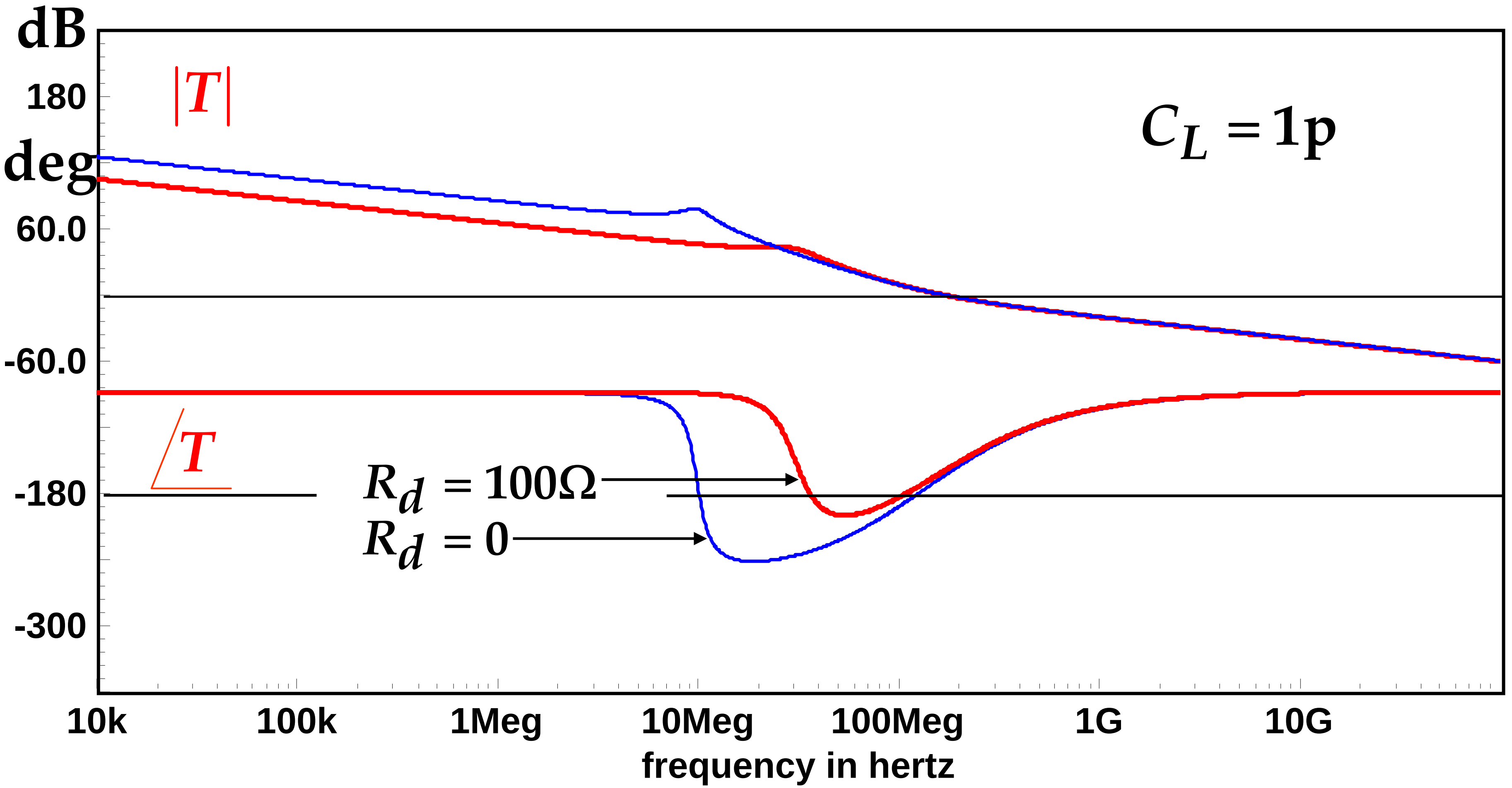
A design strategy is to add sufficient damping resistance R_d to Z_{o1} so that the maximum phase lag is less than 180° by some desired phase margin.



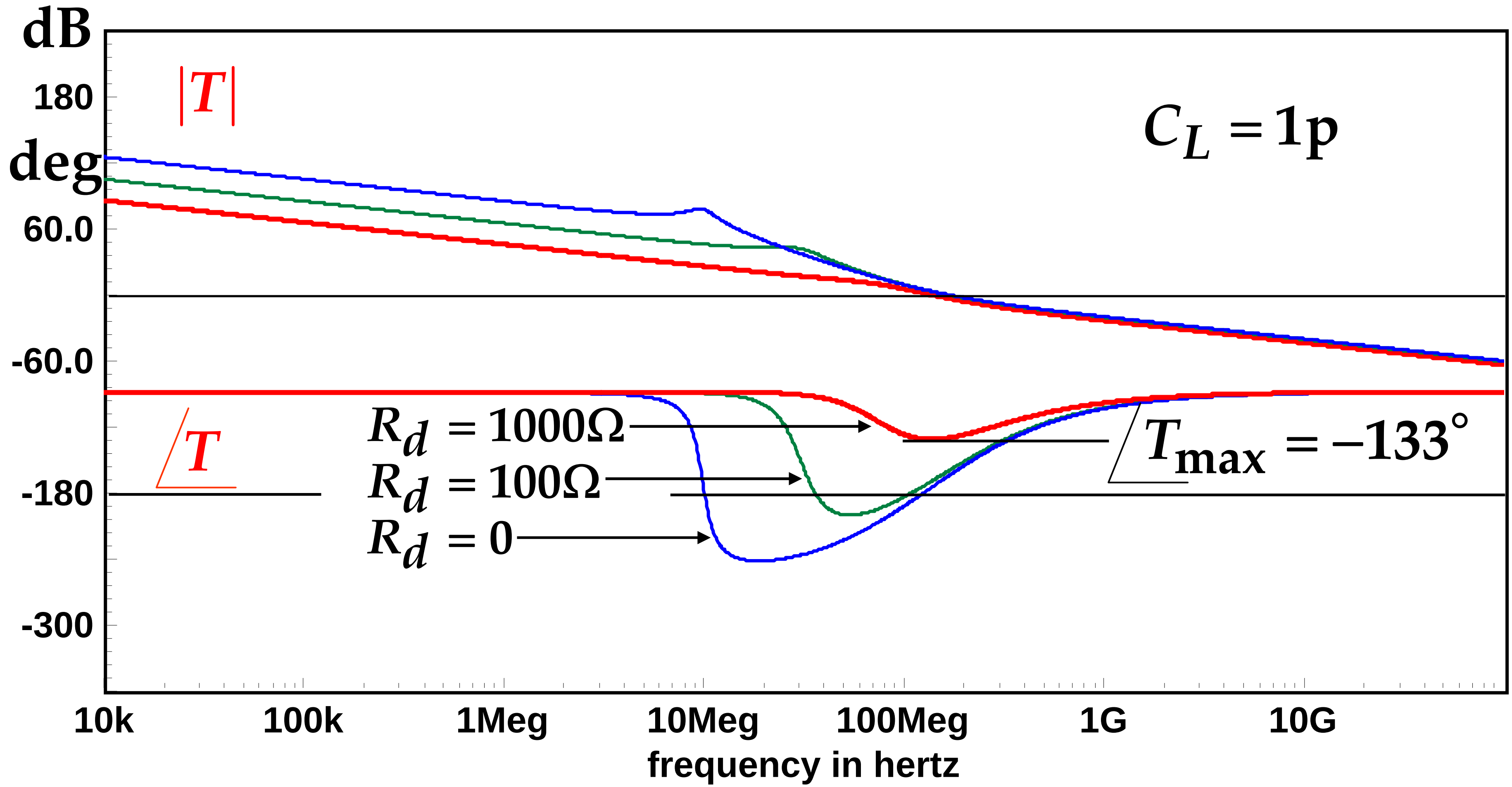
Damping resistance R_d added



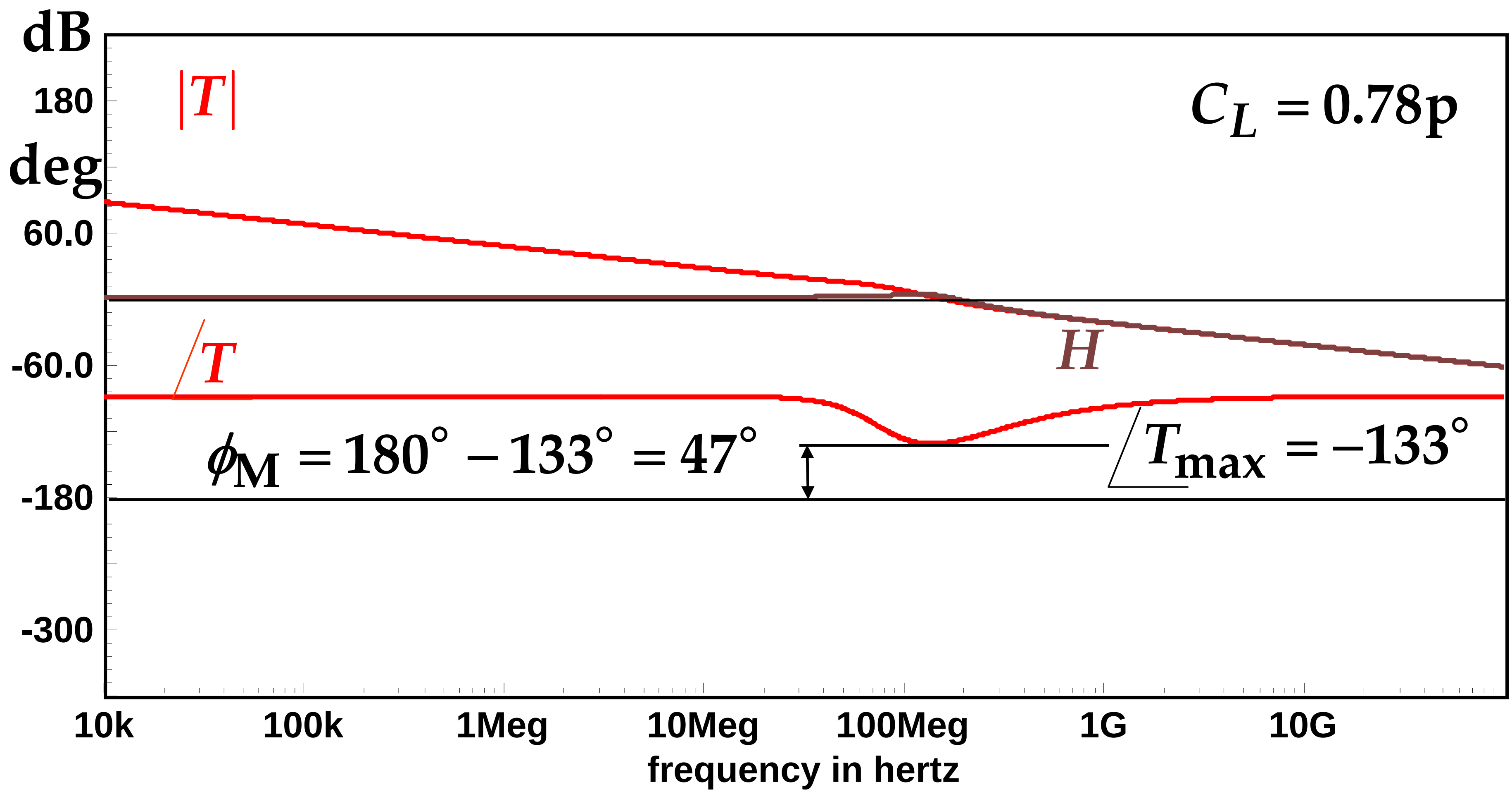
The phase margin ϕ_M is positive, so the Follower is stable. However, ϕ_M is small, so H has a large peak.



dB
180
deg
60.0
-60.0
-180
-300
|T|
$C_L = 1$p
T
$R_d = 100\Omega$
$R_d = 0$
10k
100k
1Meg
10Meg
100Meg
1G
10G
frequency in hertz

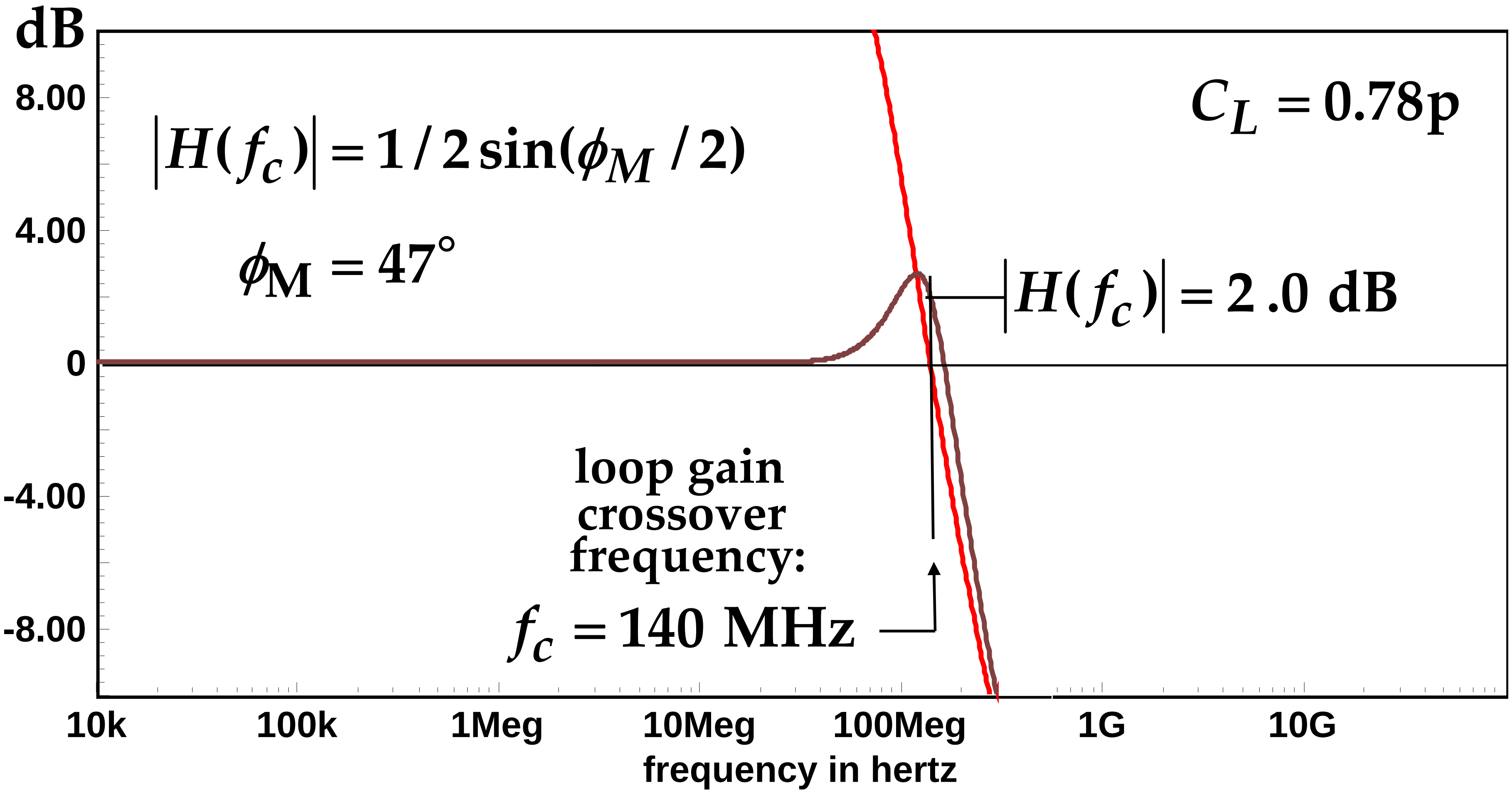


The worst-case phase margin thus occurs when C_L is such that the $|T|$ crossover frequency f_c is at the frequency where $\angle T$ is maximum:



Worst-case $\phi_M = 47^\circ$, and consequent greatest peaking in H, occurs for $C_L = 0.78\text{p}$.

Expanded scale:



EXAMPLE

17. THE INPUT FILTER PROBLEM OF A SWITCHED-MODE REGULATOR

A different application of the EET is useful in design of the input filter for a switching regulator.

"The Input Filter Problem" was discussed in a 1976 paper:

R.D.Middlebrook, "Input Filter Considerations in Design and Application of Switching Regulators," IEEE Industry Applications Society Annual Meeting, 1976 Record, pp. 366 - 382.

Problem:

How to design an input filter for a switched-mode regulator without significantly disturbing its properties.

Conclusion:

The output impedance Z_s of the input filter should be much less than certain line input impedances of the regulator.

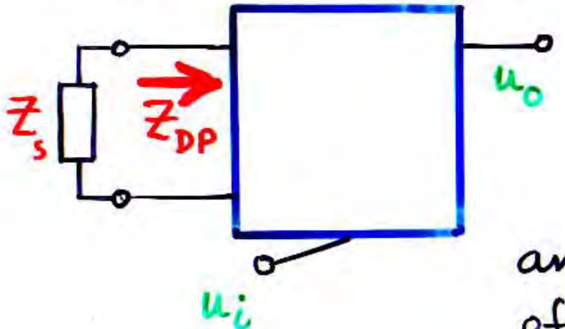
The design-oriented inequalities can easily be established by use of the Extra Element Theorem.

Objective

To find conditions on the input filter so that properties of the regulator are unaffected.

Approach

Use the Extra Element Theorem (EET)



$$\frac{u_o}{u_i} = \left.\frac{u_o}{u_i}\right|_{Z_s=0} \cdot \frac{1+\frac{Z_s}{Z_n}}{1+\frac{Z_s}{Z_d}}$$

any transfer function of a linear system, e.g. gain, loop gain, output impedance

same tranfer function with $Z_s = 0$.

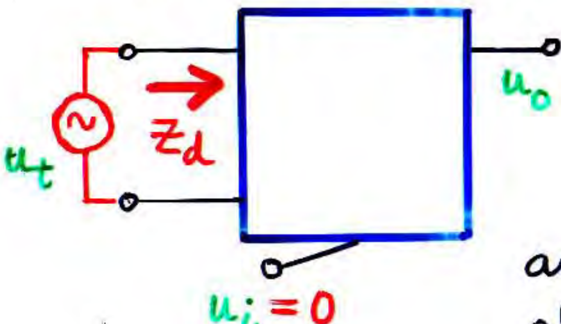
Z_n, Z_d are dpi's (driving point impedances) seen by a test signal u_t applied in place of Z_s:

Objective

To find conditions on the input filter so that properties of the regulator are unaffected.

Approach

Use the Extra Element Theorem (EET)



$$\frac{u_o}{u_i} = \left.\frac{u_o}{u_i}\right|_{Z_s=0} \cdot \frac{1 + \frac{Z_s}{Z_n}}{1 + \frac{Z_s}{Z_d}}$$

any transfer function of a linear system, e.g. gain, loop gain, output impedance

same tranfer function with $Z_s = 0$.

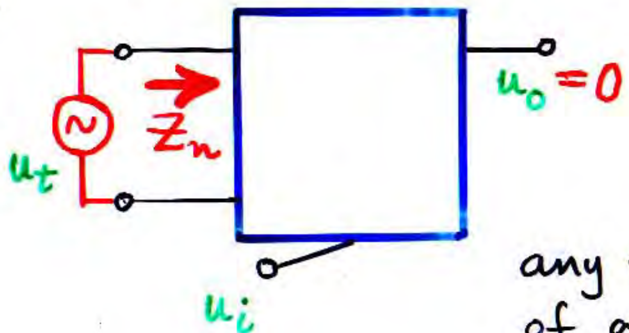
$Z_d = Z_{DP}$ seen by u_t with $u_i = 0$ (single injection)

Objective

To find conditions on the input filter so that properties of the regulator are unaffected.

Approach

Use the Extra Element Theorem (EET)



$$\frac{u_o}{u_i} = \left.\frac{u_o}{u_i}\right|_{Z_s=0} \cdot \frac{1 + \frac{Z_s}{Z_n}}{1 + \frac{Z_s}{Z_d}}$$

any transfer function of a linear system, e.g. gain, loop gain, output impedance

same tranfer function with $Z_s = 0$.

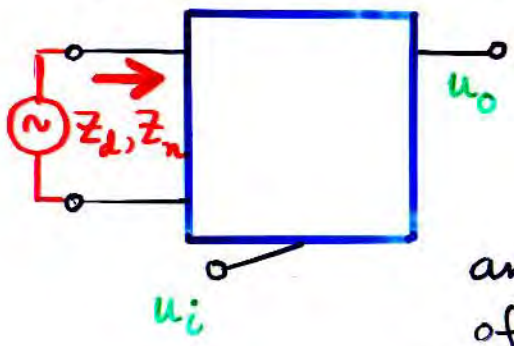
$Z_n = Z_{DP}$ seen by u_t in presence of u_i adjusted to make $u_o = 0$ (null double injection)

Objective

To find conditions on the input filter so that properties of the regulator are unaffected.

Approach

Use the Extra Element Theorem (EET)



$$\frac{u_o}{u_i} = \left.\frac{u_o}{u_i}\right|_{Z_s=0} \cdot \frac{1+\frac{Z_s}{Z_n}}{1+\frac{Z_s}{Z_d}}$$

any transfer function of a linear system, e.g. gain, loop gain, output impedance

same tranfer function with $Z_s = 0$.

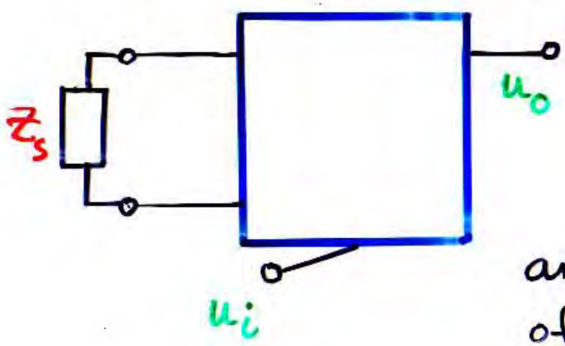
Note that, even for a given element Z_s, Z_n but not Z_d are different for different transfer functions of the same system (different u_i's and u_o's).

Objective

To find conditions on the input filter so that properties of the regulator are unaffected.

Approach

Use the Extra Element Theorem (EET)



$$\frac{u_o}{u_i} = \left.\frac{u_o}{u_i}\right|_{Z_s=0} \cdot \frac{1+\frac{Z_s}{Z_n}}{1+\frac{Z_s}{Z_d}}$$

any transfer function of a linear system, e.g. gain, loop gain, output impedance

same tranfer function with $Z_s = 0$.

Application of the EET to a regulator with input filter:

Identify Z_s as the output impedance of the input filter

Hence, the design criteria are:

$Z_s \ll Z_d, Z_n$ corresponding to each property (transfer function) of interest.

The EET for one extra element Z_s is

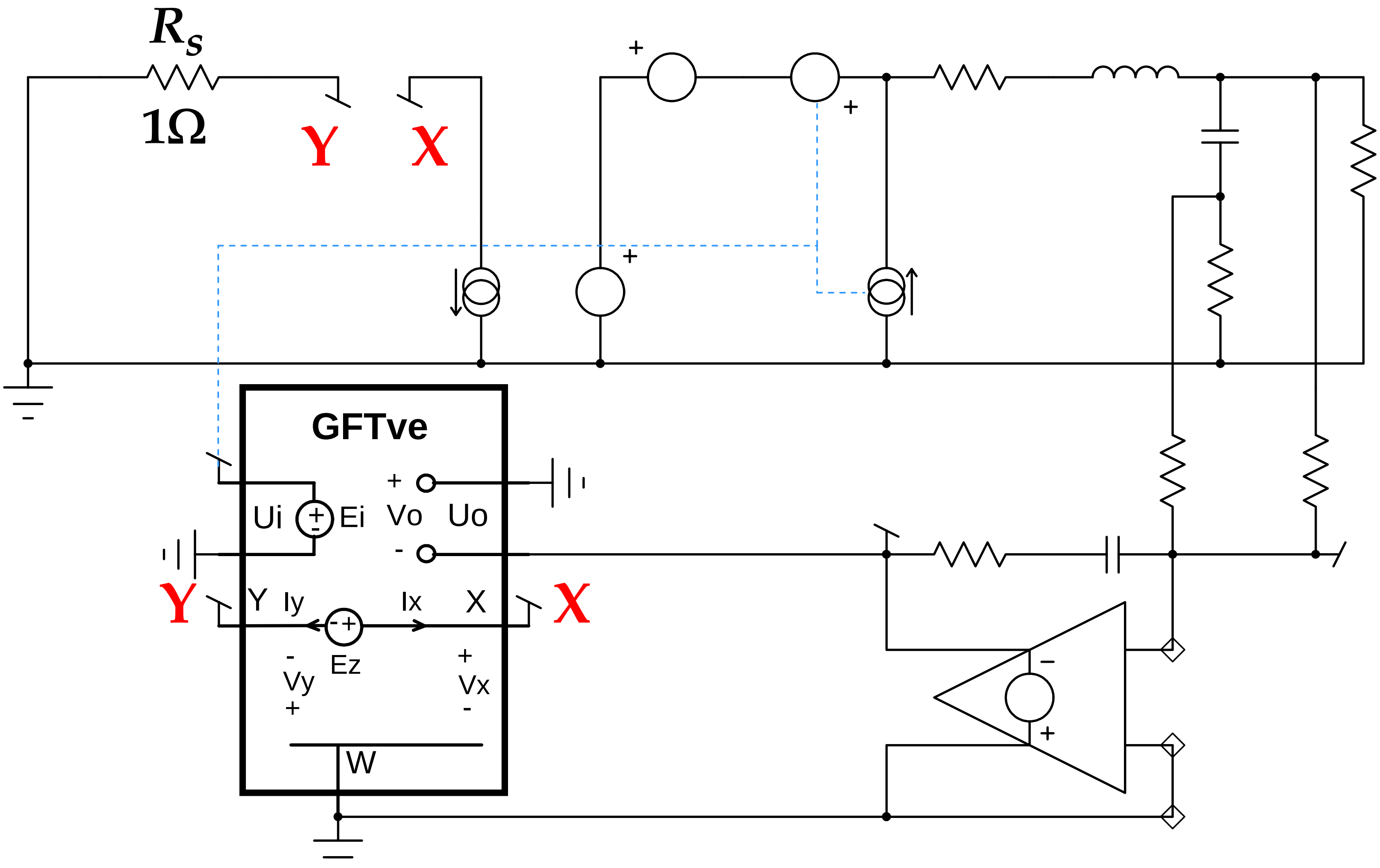
$$\frac{u_o}{u_i} = \left.\frac{u_o}{u_i}\right|_{Z_s=0} \frac{1+\frac{Z_s}{Z_n}}{1+\frac{Z_s}{Z_d}}$$

This is the same as the GFT for one injected test signal:

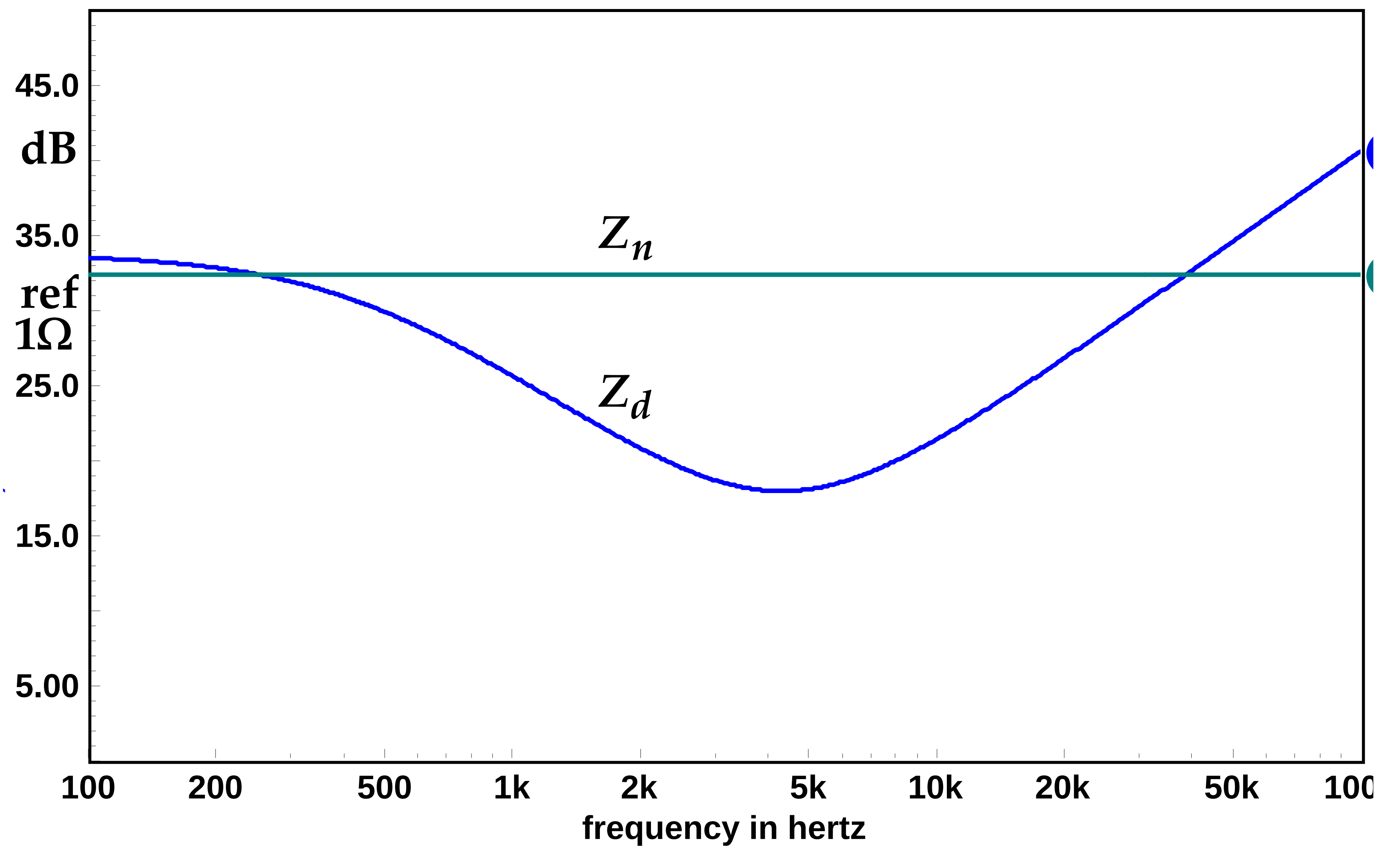
$$H = H_\infty \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}} = H_\infty D D_n$$

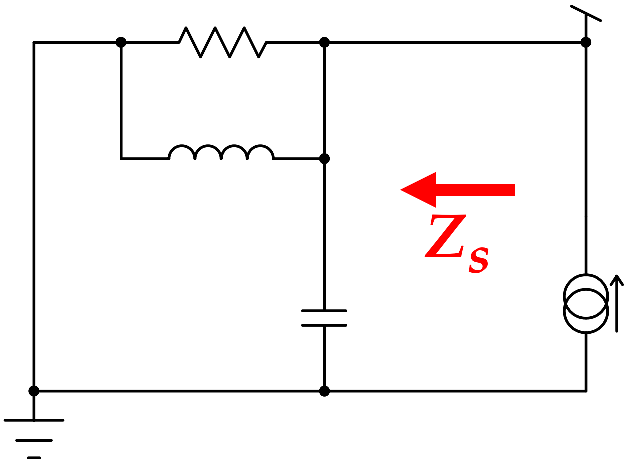
where $H = \text{loopgain[with } Z_s]$ and $H_\infty = \text{loopgain}[Z_s = 0]$.

Z_d and Z_n can be found from T and T_n by test voltage e_z injection at a source resistance $Z_s = R_s = 1\Omega$. Then, $Z_d = T$ and $Z_n = T_n$

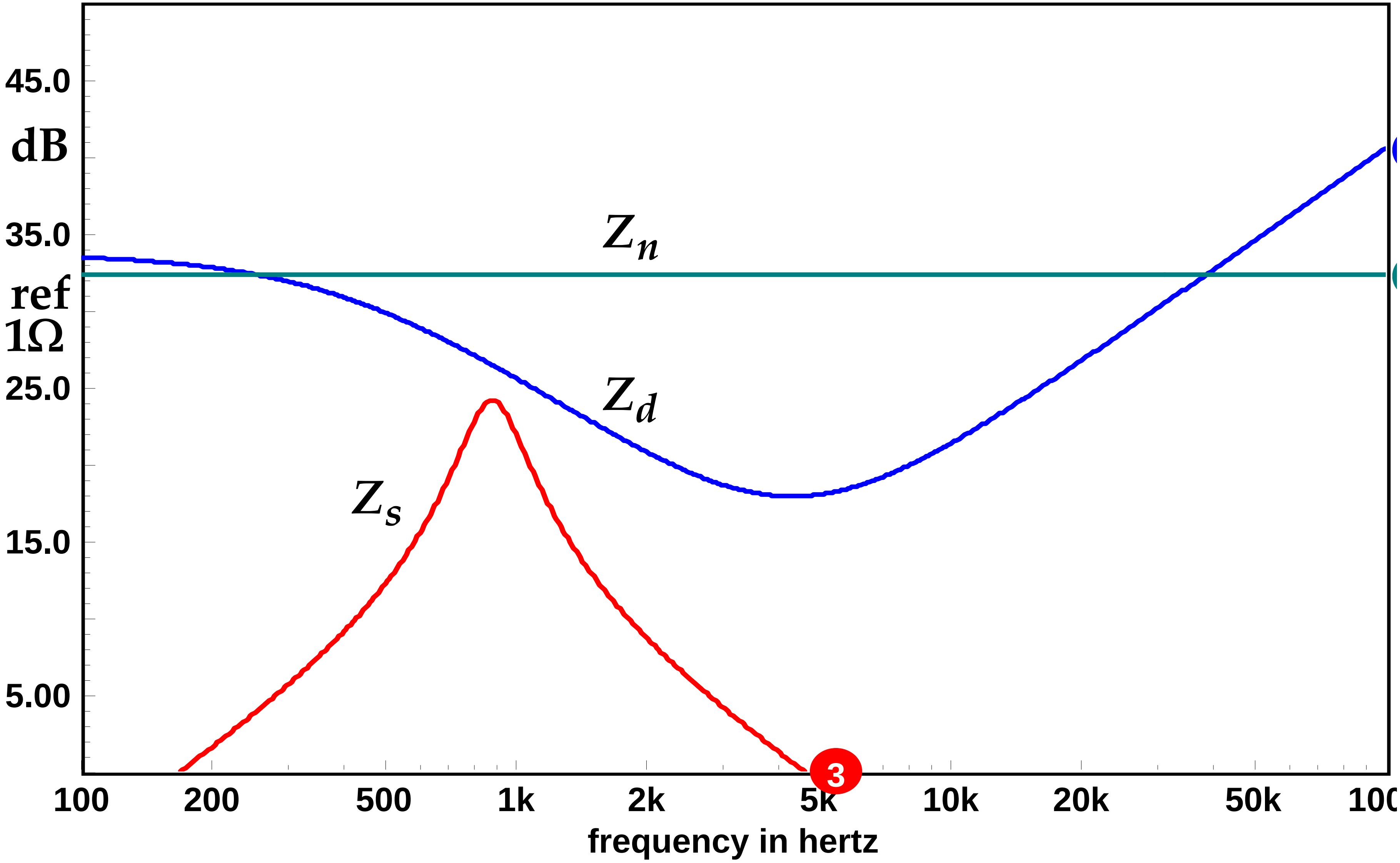


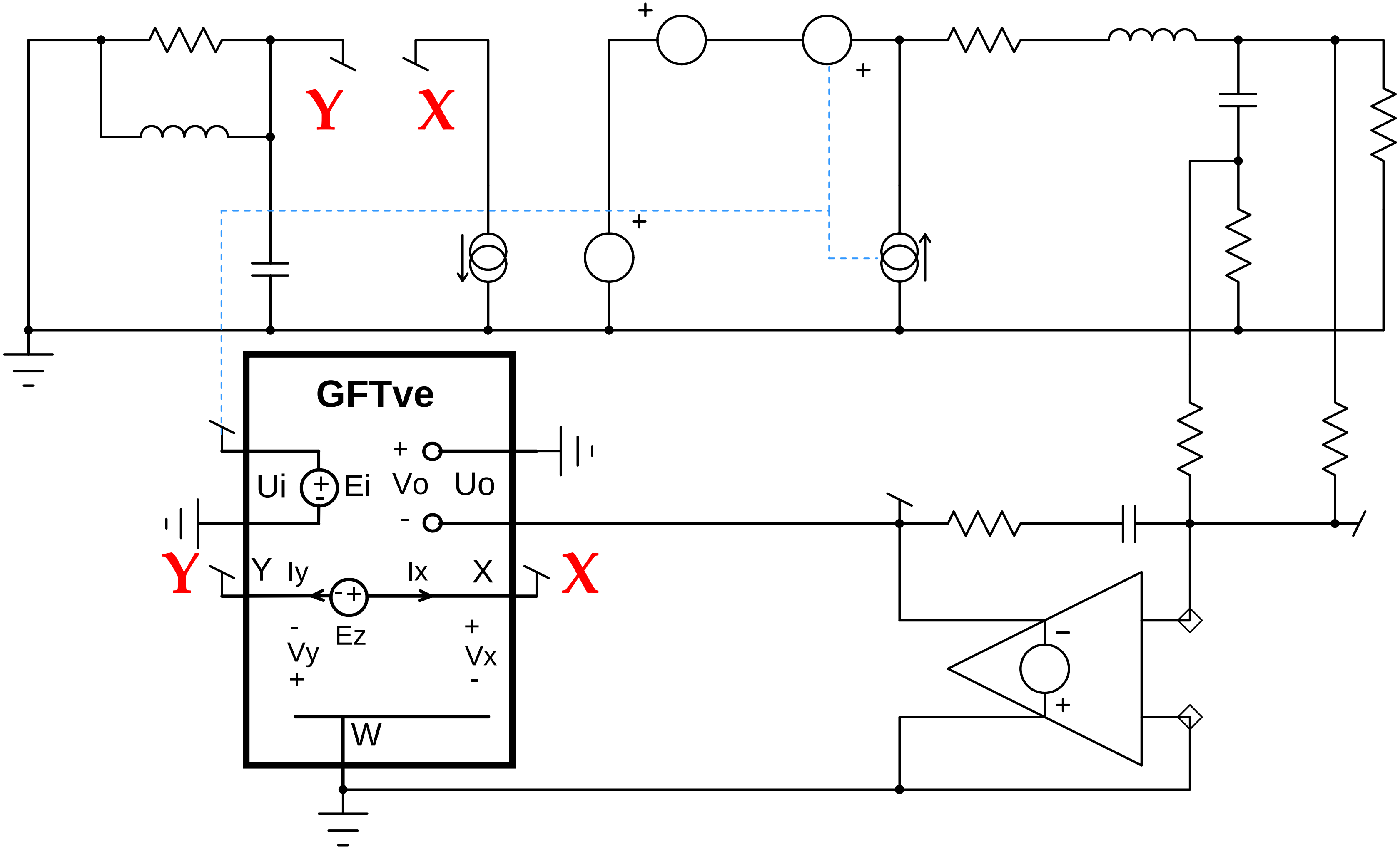
Values as in 1976 Paper Fig. 13

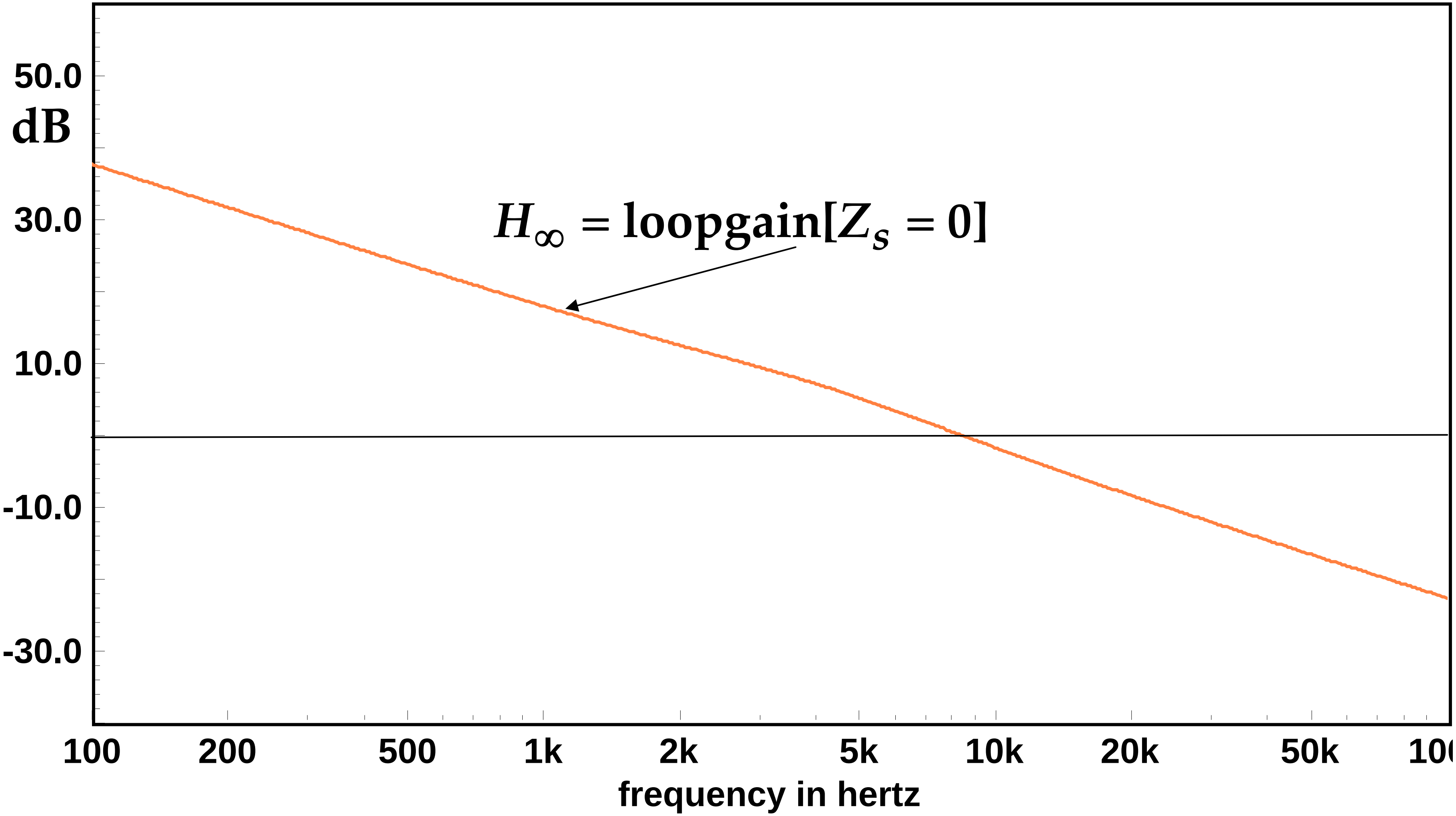

45.0
dB
35.0
ref
1Ω
25.0
15.0
5.00
Z_n
Z_d
100
200
500
1k
2k
5k
10k
20k
50k
100
frequency in hertz

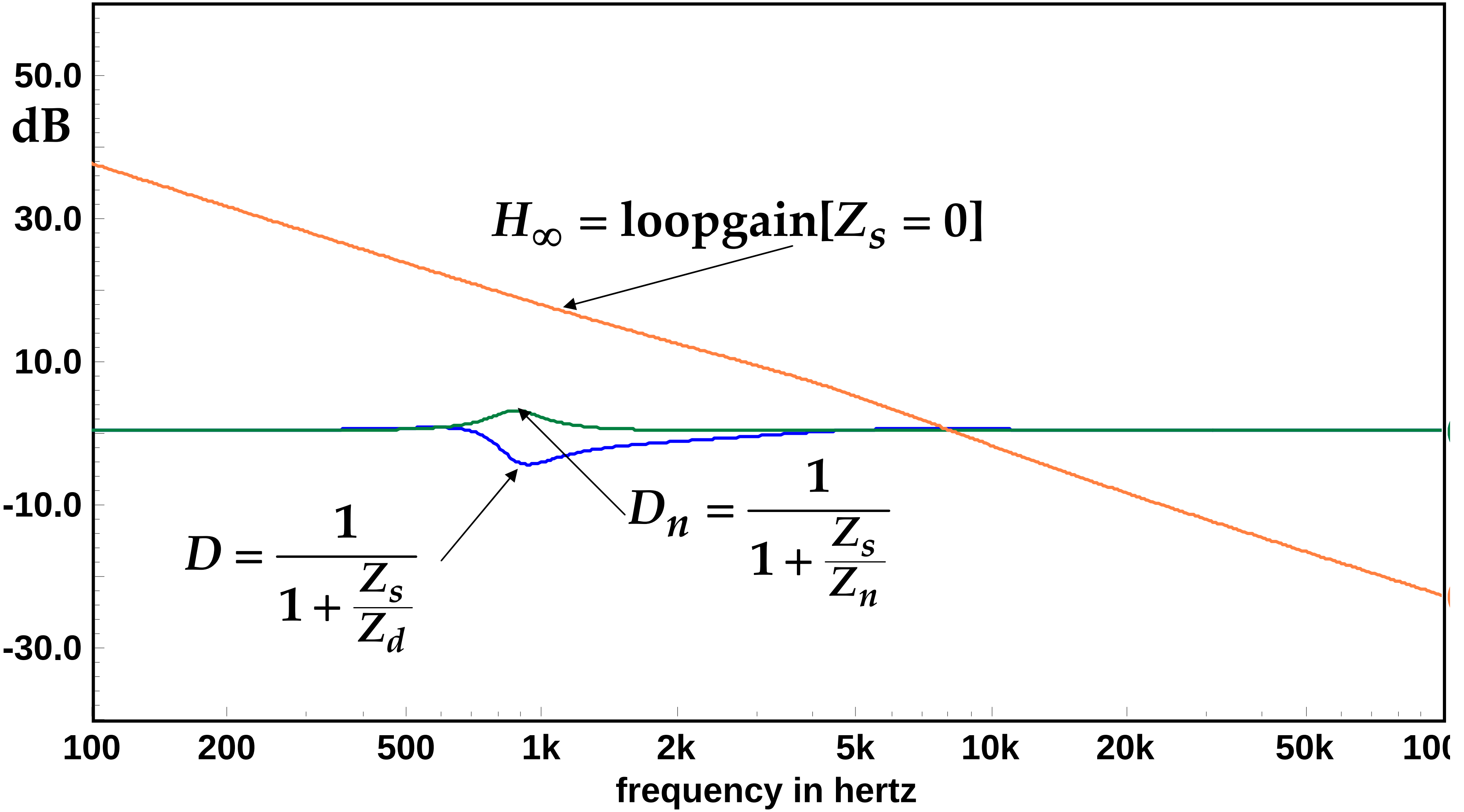


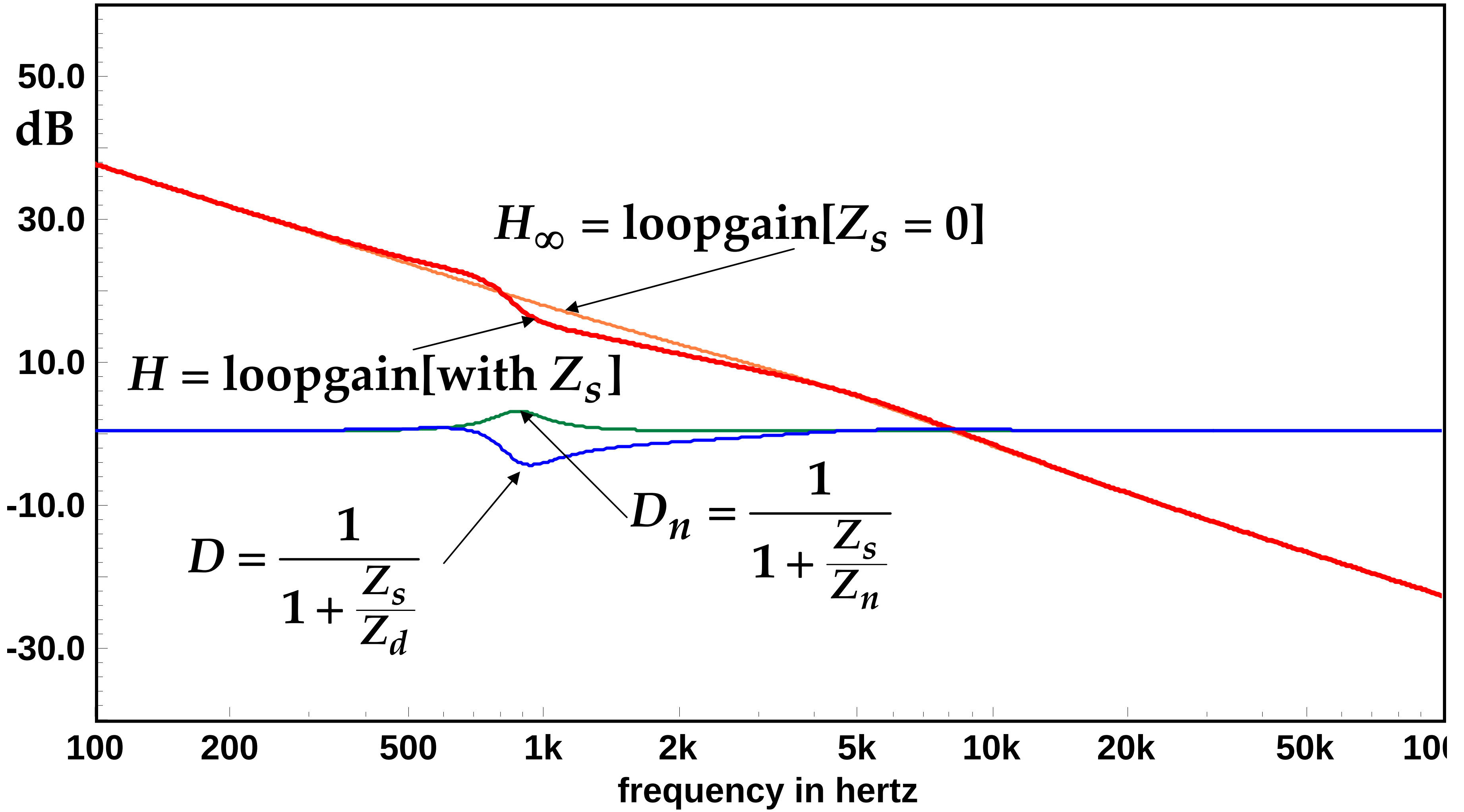
Values as in 1976 Paper Fig. 17, Filter B


45.0
dB
35.0
ref
1Ω
25.0
15.0
5.00
Z_n
Z_d
Z_s
3
100
200
500
1k
2k
5k
10k
20k
50k
100
frequency in hertz


Y
X
GFTve
Ui
Ei
Vo
Uo
Y
Iy
Ix
X
Ez
Vy
Vx
W


50.0
dB
30.0
10.0
-10.0
-30.0
$H_\infty = \text{loopgain}[Z_s = 0]$
100
200
500
1k
2k
5k
10k
20k
50k
frequency in hertz

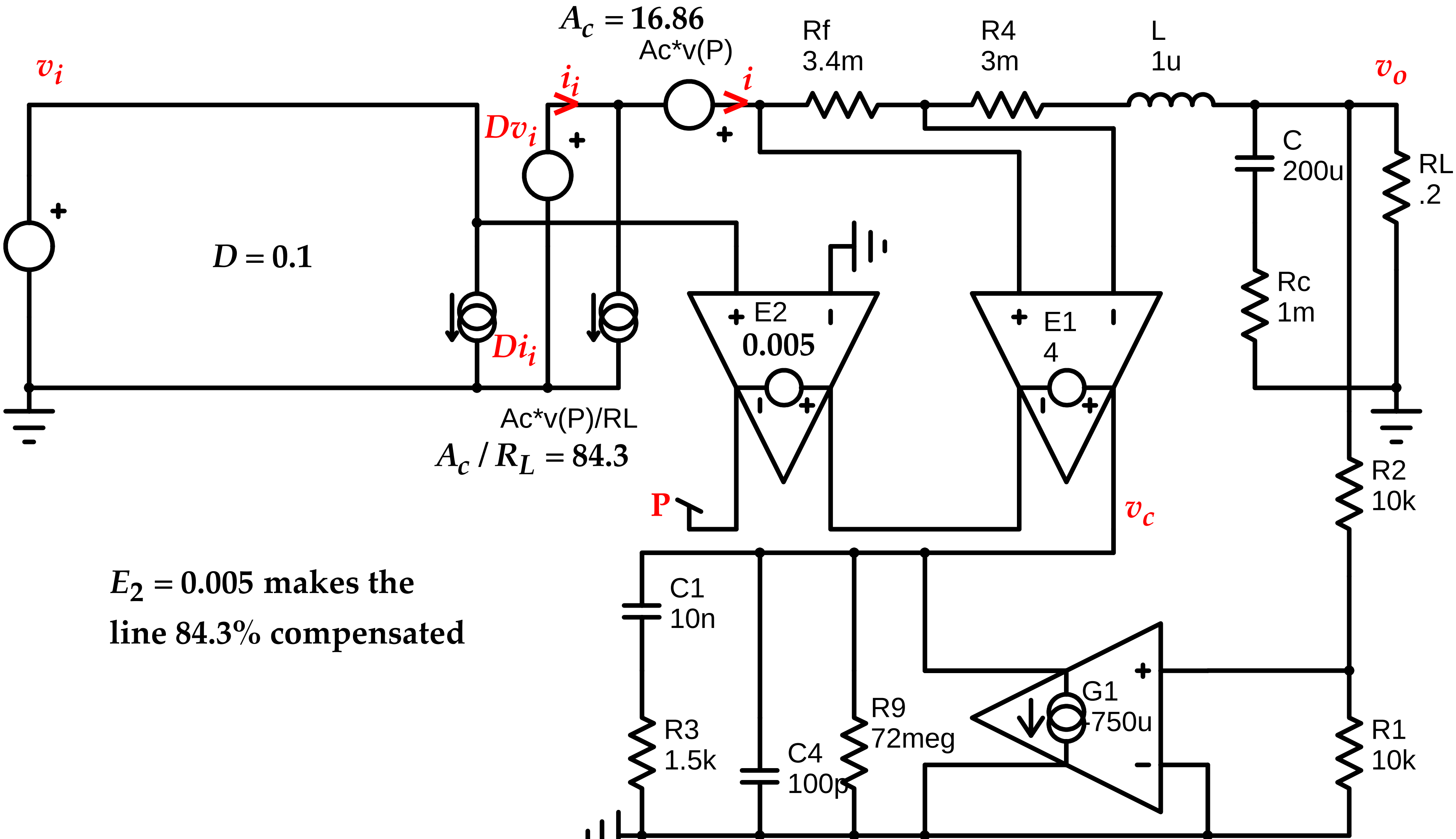

50.0
dB
30.0
10.0
-10.0
-30.0
$H_\infty = \text{loopgain}[Z_s = 0]$
$D_n = \dfrac{1}{1+\frac{Z_s}{Z_n}}$
$D = \dfrac{1}{1+\frac{Z_s}{Z_d}}$
100
200
500
1k
2k
5k
10k
20k
50k
100
frequency in hertz



50.0
dB
30.0
10.0
-10.0
-30.0
$H_\infty = \text{loopgain}[Z_s = 0]$
$H = \text{loopgain}[\text{with } Z_s]$
$D_n = \dfrac{1}{1+\dfrac{Z_s}{Z_n}}$
$D = \dfrac{1}{1+\dfrac{Z_s}{Z_d}}$
100
200
500
1k
2k
5k
10k
20k
50k
frequency in hertz

EXAMPLE

18. CURRENT-PROGRAMMED SWITCHED-MODE REGULATOR

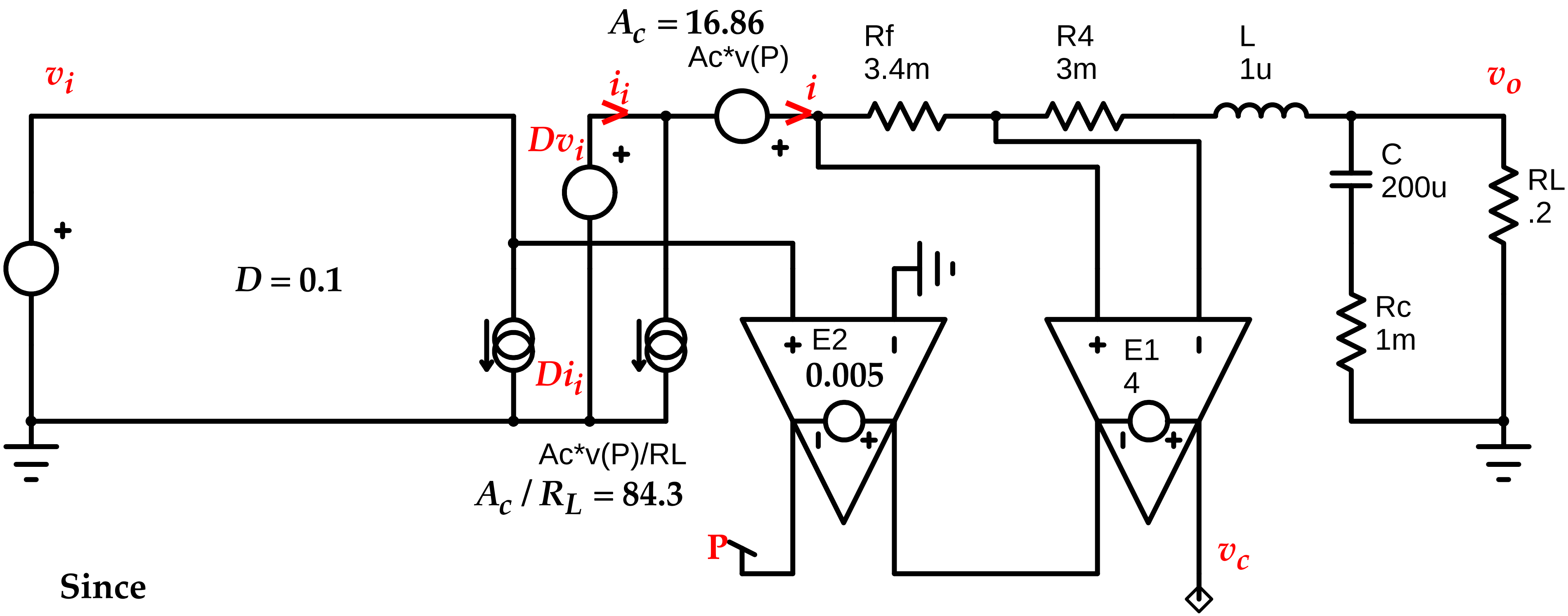


$E_2 = 0.005$ makes the line 84.3% compensated

The power stage has an internal current loop and a voltage feedforward loop.

STEP 1

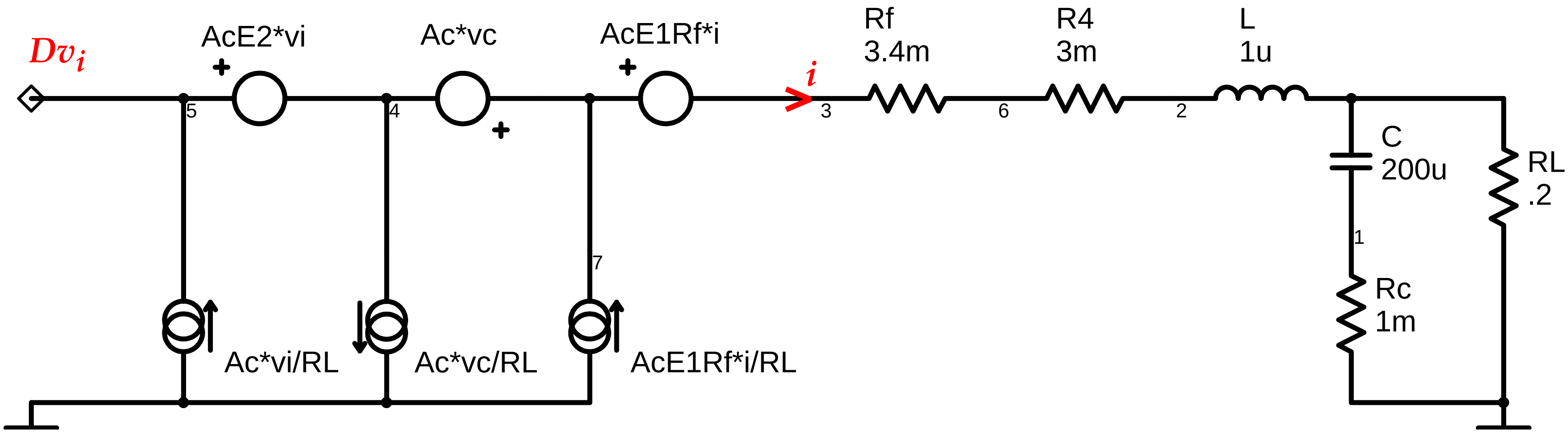
Strategy: absorb both loops into an equivalent power stage model, by Doing Some Algebra on the Circuit Diagram (Ch.2)



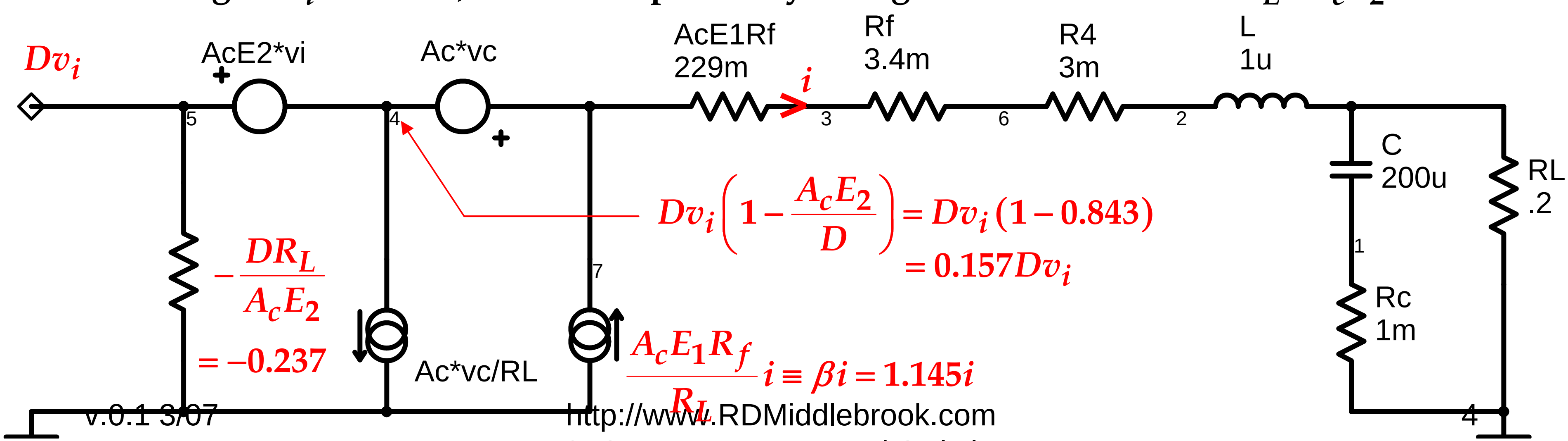
Since

$$v(P) = -E_2 v_i + v_c - E_1 R_f i$$

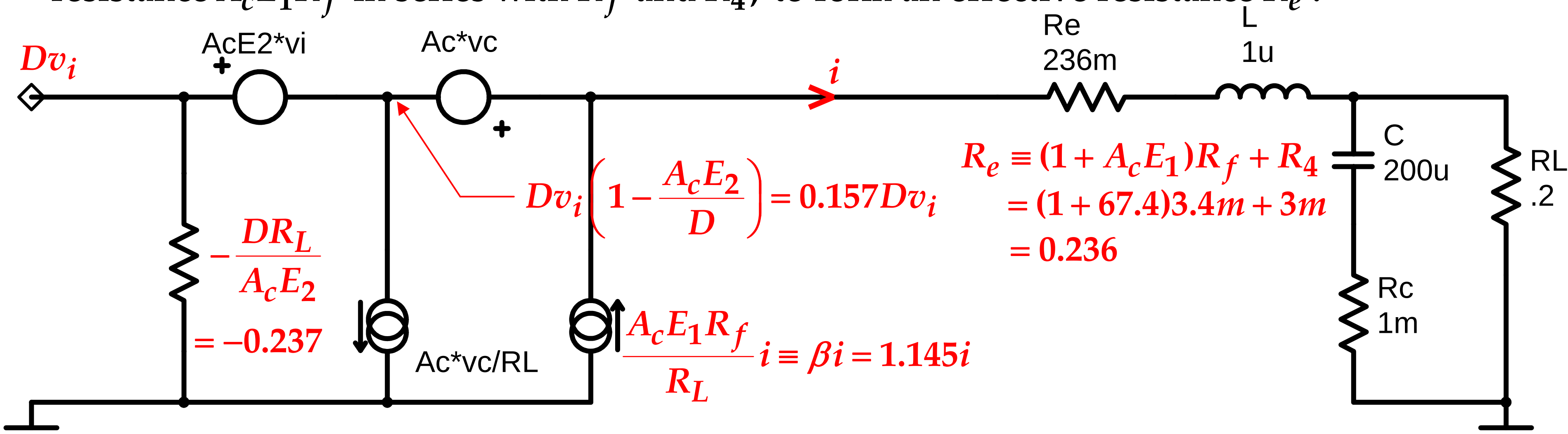
split the voltage and current modulation generators into three:



Since the voltage generator $A_c E_1 R_f i$ is proportional to the current through it, it can be replaced by a resistance $A_c E_1 R_f$. Also, since the current generator $A_c E_2 v_i / R_L$ is proportional to the voltage Dv_i across it, it can be replaced by a (negative) resistance $-DR_L / A_c E_2$.



The consequence of the current loop is that it introduces a (lossless) damping resistance $A_c E_1 R_f$ in series with R_f and R_4, to form an effective resistance R_e :

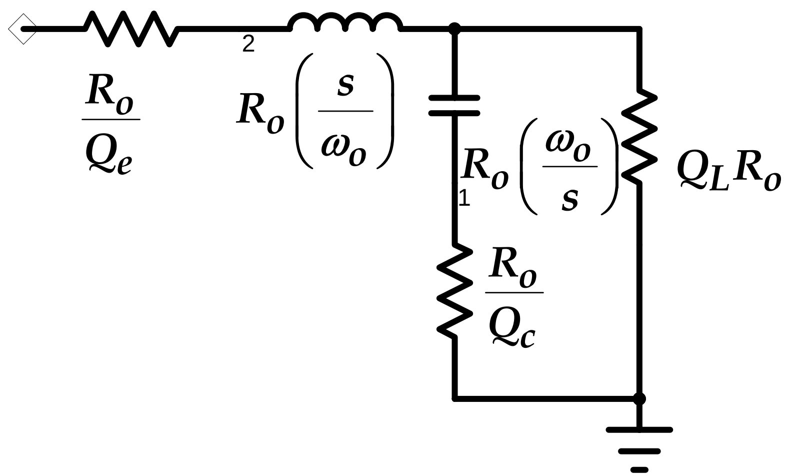


STEP 2

Normalize the element values in the filter (Ch. 5):

$$\omega_o \equiv \frac{1}{\sqrt{LC}} \qquad f_o = 11.2 kHz \qquad R_o \equiv \sqrt{\frac{L}{C}} = 70.7m$$

$$Q_e \equiv \frac{R_o}{R_e} = 0.300 \qquad Q_c \equiv \frac{R_o}{R_c} = 70.7 \qquad Q_L \equiv \frac{R_L}{R_o} = 2.83$$



The voltage transfer function *H* for this triple-damped RLC filter was obtained in Ch. 12:

$$\frac{1}{Q_cQ_L}$$

$$H = \frac{1}{1+\frac{1}{Q_eQ_L}} \; \frac{1+\frac{1}{Q_c}\left(\frac{s}{\omega_0}\right)}{1+\left(\frac{1}{Q_L\left(1+\frac{1}{Q_eQ_L}\right)}+\frac{1}{Q_c}+\frac{1}{Q_e}\,\frac{1}{1+\frac{1}{Q_eQ_L}}\right)\left(\frac{s}{\omega_0}\right)+\frac{1+\frac{1}{Q_eQ_c}}{1+\frac{1}{Q_eQ_L}}\left(\frac{s}{\omega_0}\right)^2}$$

$$\frac{1}{Q_cQ_L}$$

$$= \frac{1}{1+\frac{1}{Q_eQ_L}} \; \frac{1+\frac{1}{Q_c}\left(\frac{s}{\omega_0}\right)}{1+\frac{\frac{1}{Q_e}+\frac{1}{Q_L}+\frac{1+1/Q_eQ_L}{Q_c}}{1+\frac{1}{Q_eQ_L}}\left(\frac{s}{\omega_0}\right)+\frac{1+\frac{1}{Q_eQ_c}}{1+\frac{1}{Q_eQ_L}}\left(\frac{s}{\omega_0}\right)^2}$$

Q_cQ_L

For $Q_eQ_c \gg 1$, $Q_eQ_L \gg 1$, the result reduces to that previously obtained by extrapolation of the result for $Q_c = \infty$:

$$H \approx \frac{1+\frac{1}{Q_c}\left(\frac{s}{\omega_0}\right)}{1+\left(\frac{1}{Q_e}+\frac{1}{Q_c}+\frac{1}{Q_L}\right)\left(\frac{s}{\omega_0}\right)+\left(\frac{s}{\omega_0}\right)^2}$$

For the LM3495, however, the inequality $Q_cQ_L = 70.7 * 2.83 = 200 >> 1$ still holds, but the inequality $Q_eQ_L = 0.300 * 2.83 = 0.848 >> 1$ does not.

The result therefore becomes

$$\frac{1}{Q_cQ_L}$$

$$H = \frac{1}{1+\frac{1}{Q_eQ_L}} \frac{1+\frac{1}{Q_c}\left(\frac{s}{\omega_o}\right)}{1+\frac{\frac{1}{Q_e}+\frac{1}{Q_L}+\cancel{\frac{1+1/Q_eQ_L}{Q_c}}}{1+\frac{1}{Q_eQ_L}}\left(\frac{s}{\omega_o}\right)+\frac{1+\cancel{\frac{1}{Q_eQ_c}}}{1+\frac{1}{Q_eQ_L}}\left(\frac{s}{\omega_o}\right)^2}$$

$$H = \frac{1}{1+\frac{1}{Q_eQ_L}} \frac{1+\frac{1}{Q_c}\left(\frac{s}{\omega_o}\right)}{1+\frac{1}{1+\frac{1}{Q_eQ_L}}\frac{1}{Q_e\|Q_L}\left(\frac{s}{\omega_o}\right)+\frac{1}{1+\frac{1}{Q_eQ_L}}\left(\frac{s}{\omega_o}\right)^2}$$

where $Q_e\|Q_L = 0.300\|2.83 = 0.271$ and $\dfrac{1}{1+\frac{1}{Q_eQ_L}} = \dfrac{1}{2.18} = 0.459$

The Q_t of the denominator quadratic is (Ch. 4)

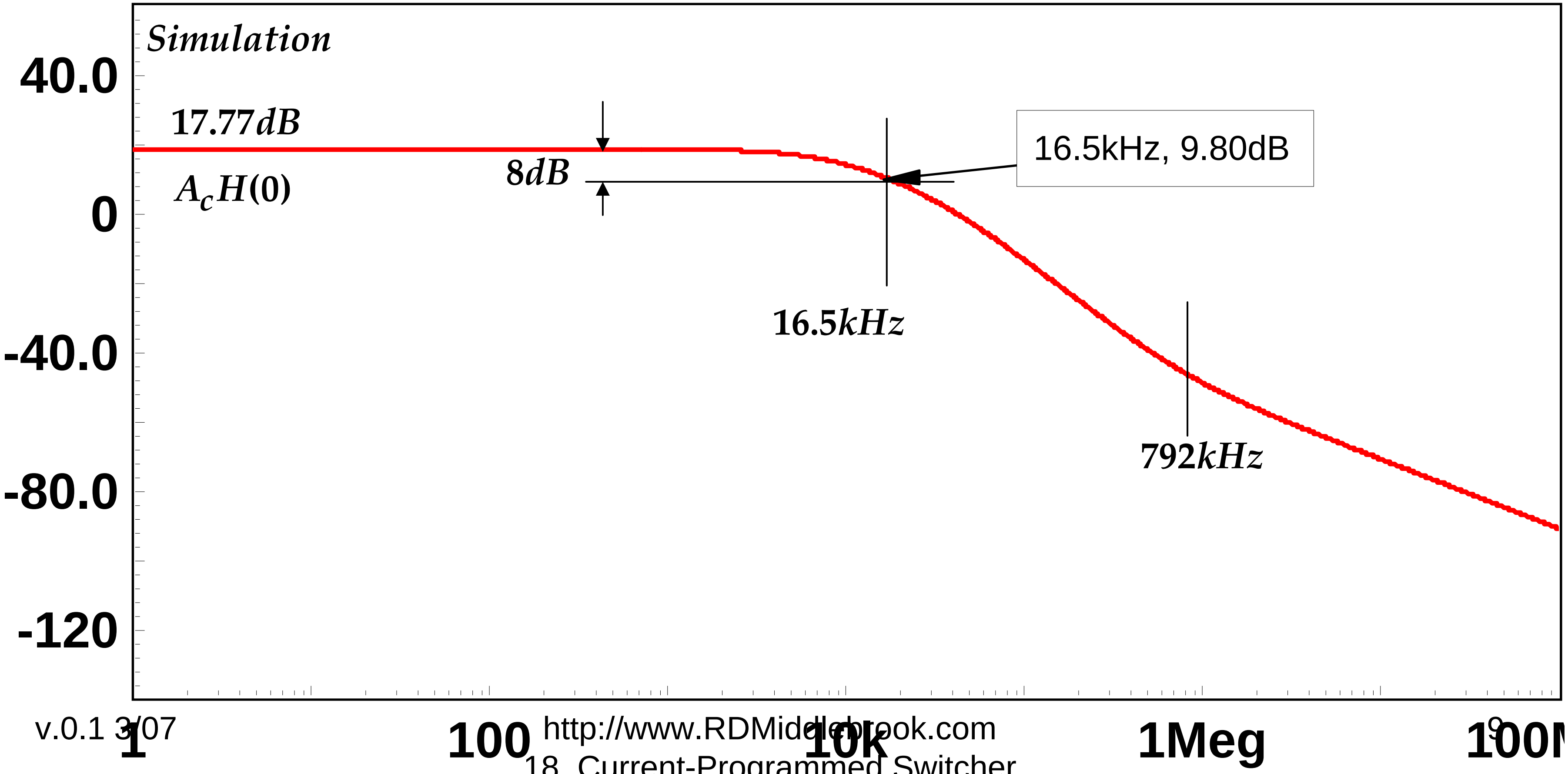
$$Q_t \equiv \frac{\sqrt{ac}}{b} = \sqrt{\left(1+\frac{1}{Q_eQ_L}\right)}\left(Q_e\|Q_L\right) = 0.400,\ Q_t = -7.96dB$$

$$\omega_o' \equiv \sqrt{\left(1+\frac{1}{Q_eQ_L}\right)}\omega_o, \qquad f_o' = \sqrt{2.18} * 11.2kHz = 16.5kHz$$

The power stage control-to-output voltage gain is $v_o / v_c \big|_{v_i=0} = A_c H$ where $A_c = 16.86$.

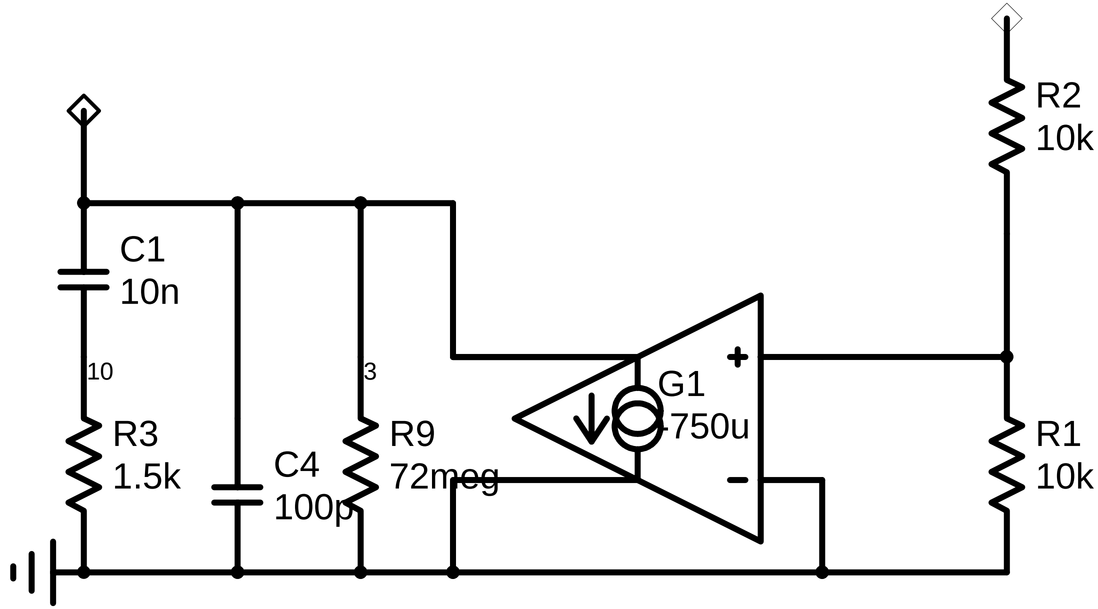
Insertion of numbers gives

$$A_c H = 17.77dB \frac{1 + \frac{s/2\pi}{792kHz}}{1 + \frac{1}{0.400}\left(\frac{s/2\pi}{16.5kHz}\right) + \left(\frac{s/2\pi}{16.5kHz}\right)^2}$$



STEP 3

Error Amplifier



The flat gain that sets the loop gain crossover occurs when C_1 is short, and C_4 open, so take this gain as reference (Ch. 3). The two poles are obviously well separated, so the gain can be written by inspection as

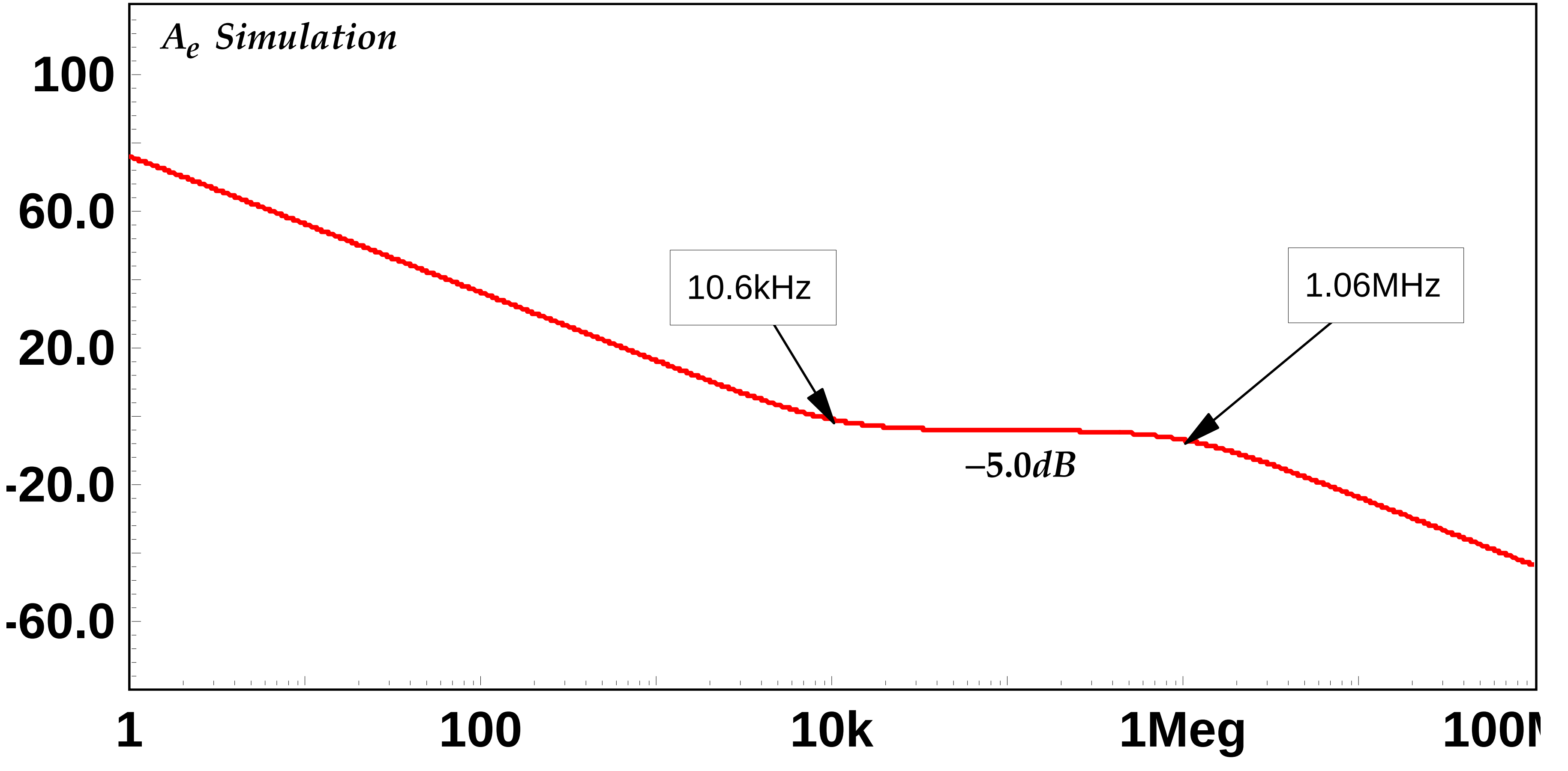
$$A = A_{em} \frac{1}{\left(\frac{1}{sC_1R_3}+1\right)\left(1+sC_4R_3\right)}$$

where

$$A_{em} = \frac{R_1}{R_1+R_2} G1R_3 = 0.563 \Rightarrow -5.0dB$$

With substitution of numbers,

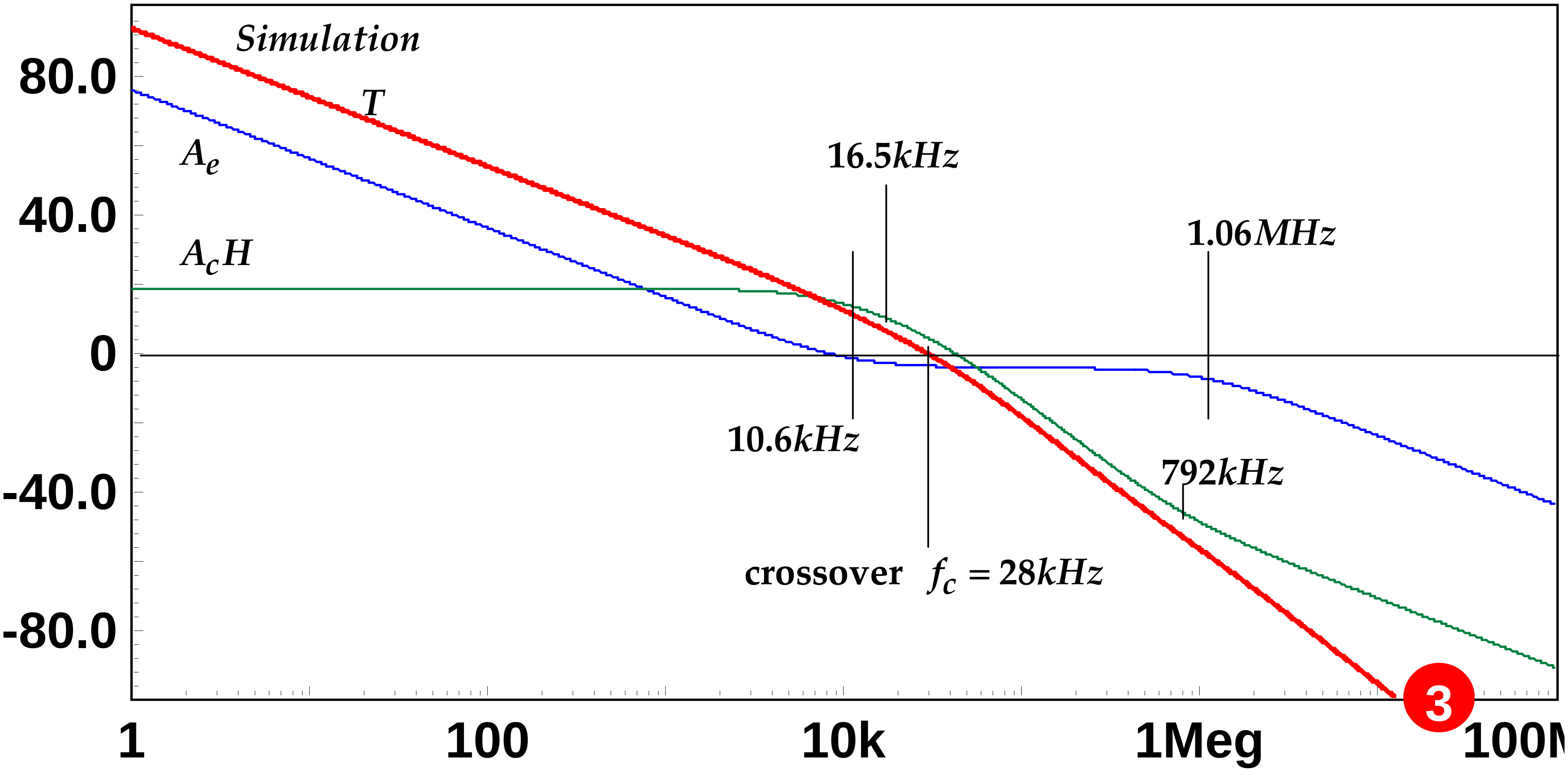
$$A_e = -5dB \frac{1}{\left(\frac{10.6kHz}{s/2\pi}+1\right)\left(1+\frac{s/2\pi}{1.061MHz}\right)}$$


A_e Simulation
100
60.0
20.0
-20.0
-60.0
10.6kHz
1.06MHz
$-5.0dB$
1
100
10k
1Meg

STEP 4

The loop gain T is:

$$T = A_e A_c H = 12.77dB \frac{1+\frac{s/2\pi}{792kHz}}{\left(\frac{10.6kHz}{s/2\pi}+1\right)\left[1+\frac{1}{0.400}\left(\frac{s/2\pi}{16.5kHz}\right)+\left(\frac{s/2\pi}{16.5kHz}\right)^2\right]\left(1+\frac{s/2\pi}{1.06MHz}\right)}$$



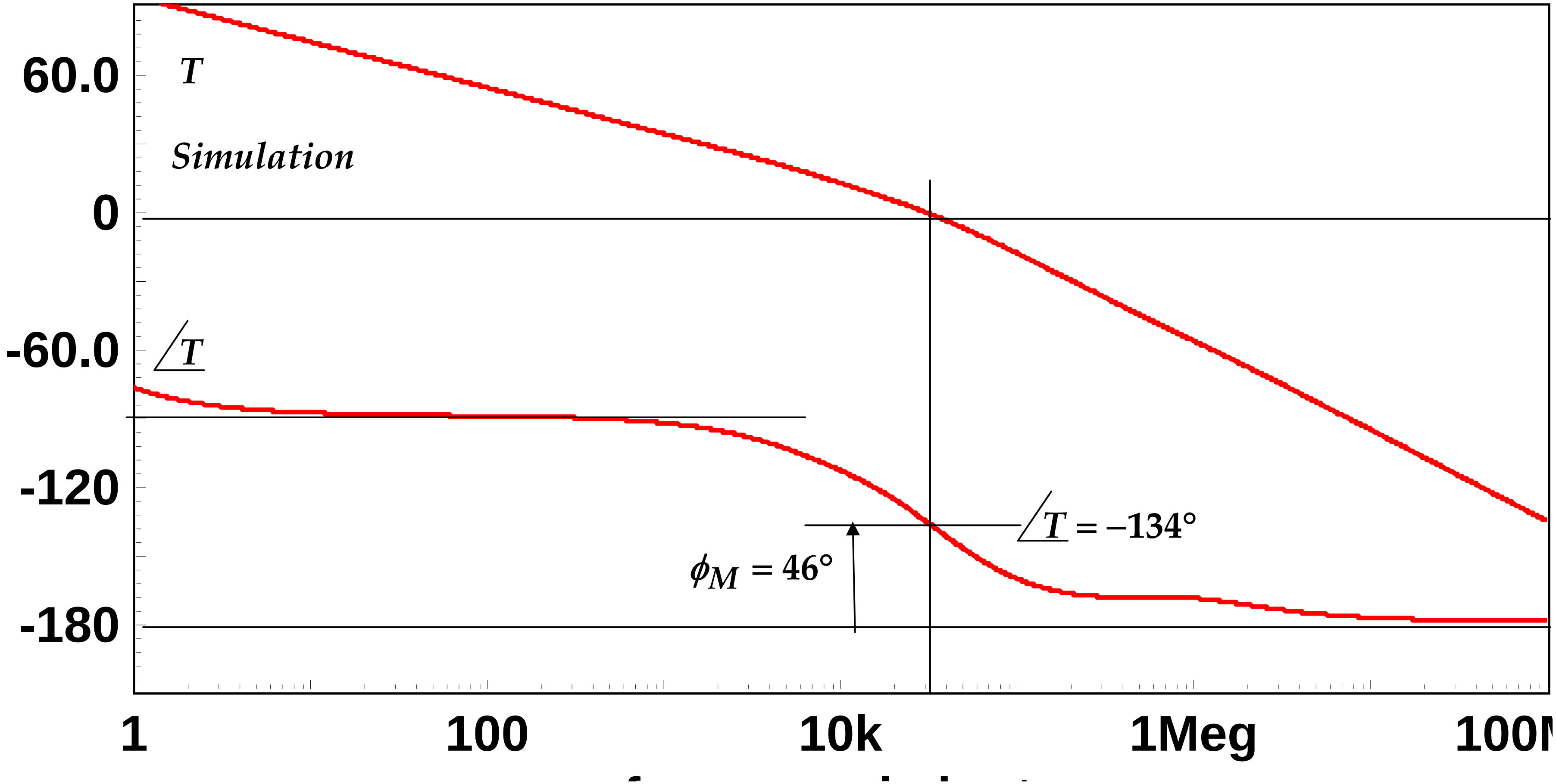
Since three poles close together determine the crossover frequency f_c, there is no point in trying to predict its value. Instead, the simulation shows it to be $f_c = 28kHz$.
From the predicted corner frequencies of T, $\angle T$ can then be calculated as:

$$\angle T = -90° + \tan^{-1}\frac{28}{10.6} - \left(180° + \tan^{-1}\frac{\frac{1}{0.4}\frac{28}{16.5}}{1-\left(\frac{28}{16.5}\right)^2}\right) + \tan^{-1}\frac{28}{792} - \tan^{-1}\frac{28}{1060}$$

$$= -90° + 70° - (180° - 66°) + 2° - 2°$$

$$= -134°$$

This agrees with the simulation:



STEP 5: Closed-loop gain G

The closed-loop gain $G = G_\infty D$ where $G_\infty = \dfrac{R_1 + R_2}{R_1} = 2 = 6.02dB$ is the ideal closed-loop gain, and $D = \dfrac{T}{1+T}$ is the discrepancy factor (Ch. 10).

Since D is a unique function of T, D can be evaluated at the loop gain crossover frequency in terms of the phase margin φ_M.

In polar form,

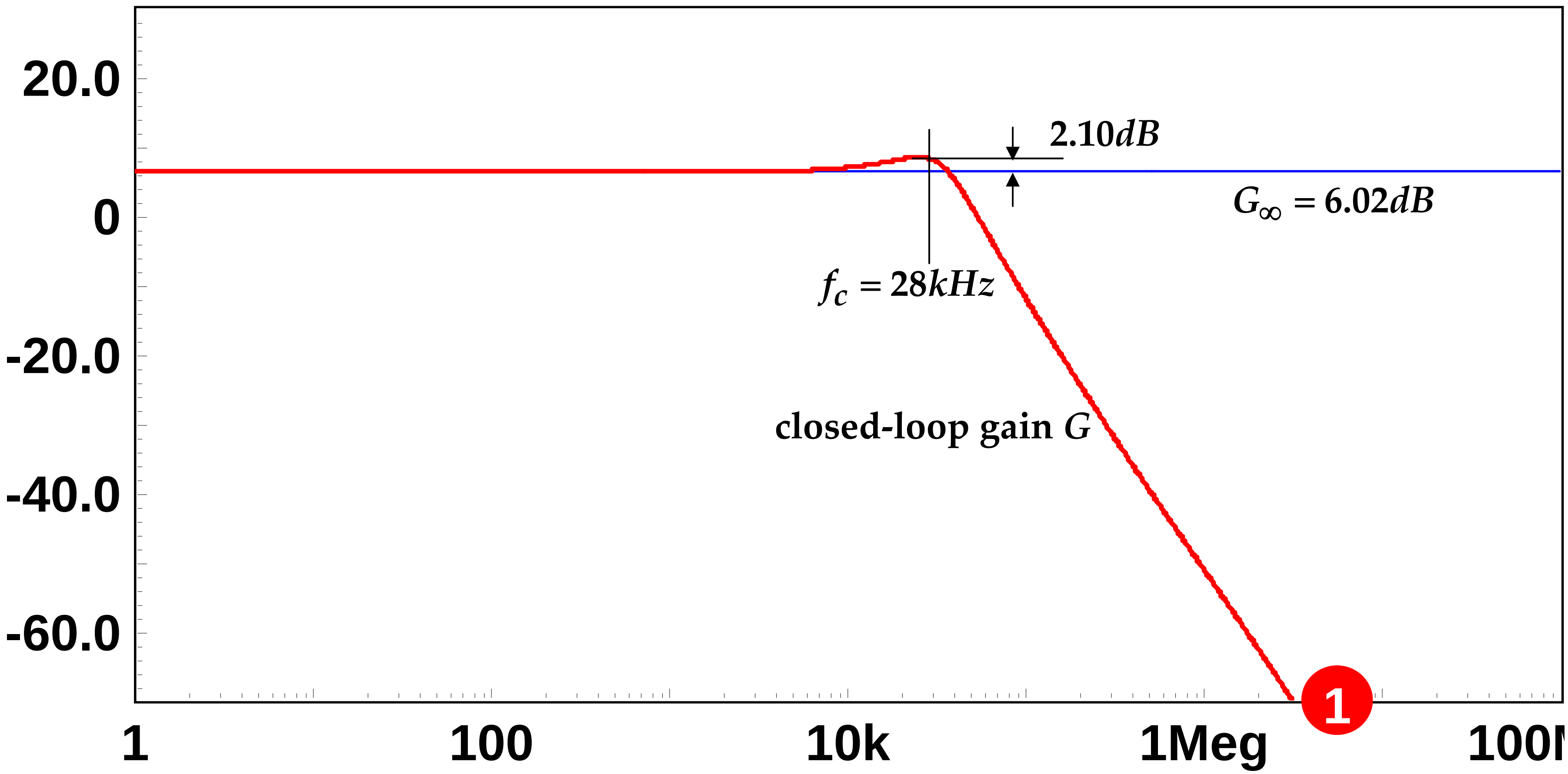
$$T = |T| e^{j\angle T}, \quad |D| = \left| \frac{T}{1+T} \right| = \left| \frac{1}{1+\frac{1}{T}} \right| = \left| \frac{1}{1+\frac{1}{|T|} e^{-j\angle T}} \right|$$

If the phase margin is φ_M, then $-\angle T = (\pi - \varphi_M)$ at the crossover frequency where $|T| = 1$. Substitute in $|D|$:

$$|D|_{f_c} = \left| \frac{1}{1+e^{j(\pi-\varphi_M)}} \right| = \left| \frac{1}{1+e^{-j\varphi_M}} \right| = \left| \frac{1}{1-(\cos\varphi_M - j\sin\varphi_M)} \right| = \frac{1}{\sqrt{(1-\cos\varphi_M)^2 + \sin^2\varphi_M}}$$

$$= \frac{1}{\sqrt{2(1-\cos\varphi_M)}} = \frac{1}{2\sin\frac{\varphi_M}{2}}$$

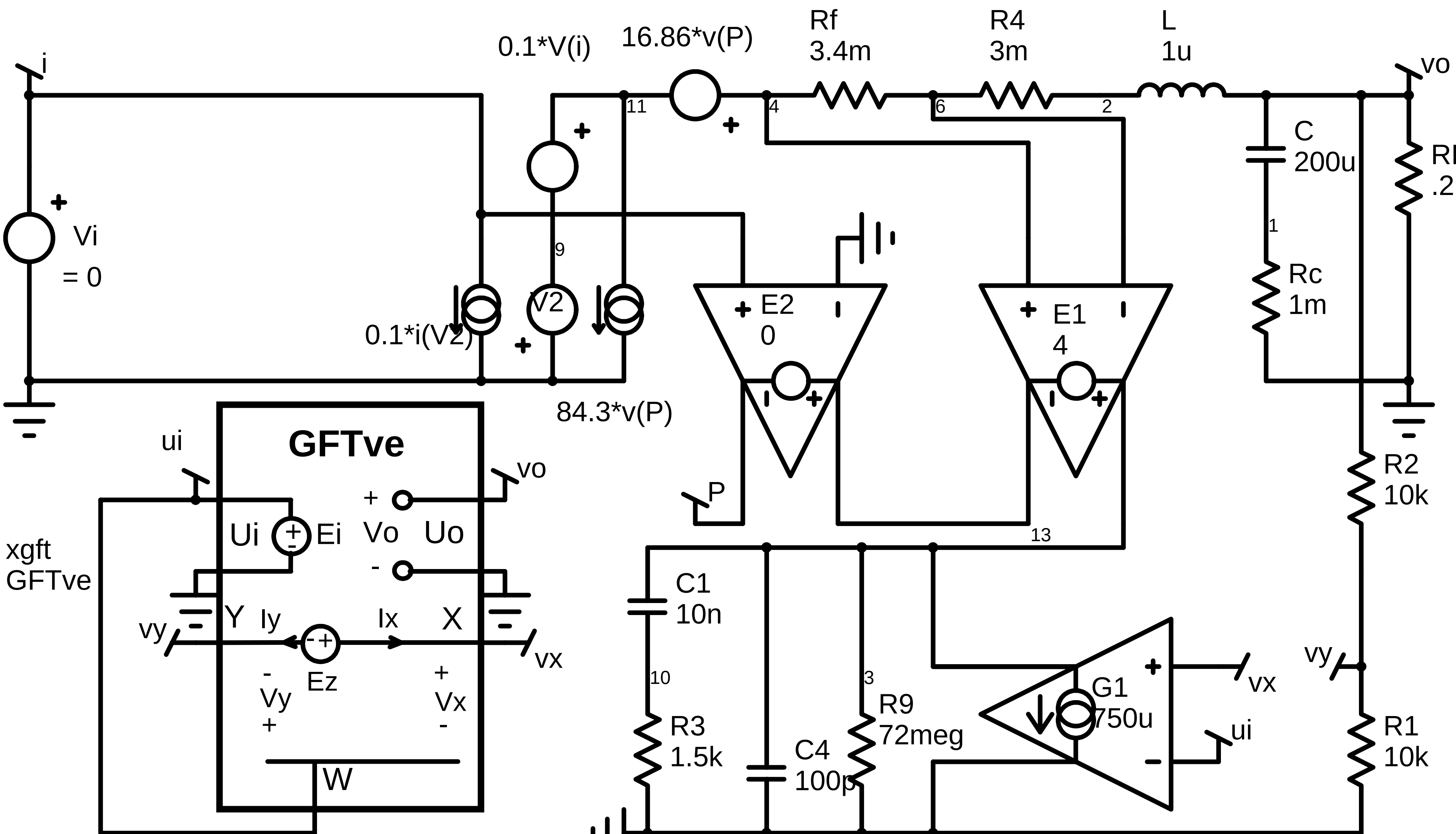
In the LM3495, $G_\infty = 2 = 6.02dB$ and $\varphi_M = 46°$ at the crossover frequency $f_c = 28kHz$. Hence, $|D|_{28kHz} = 1.28 \Rightarrow 2.10dB$. These results are in agreement with the simulation using the Intusoft ICAP/4 GFT Template



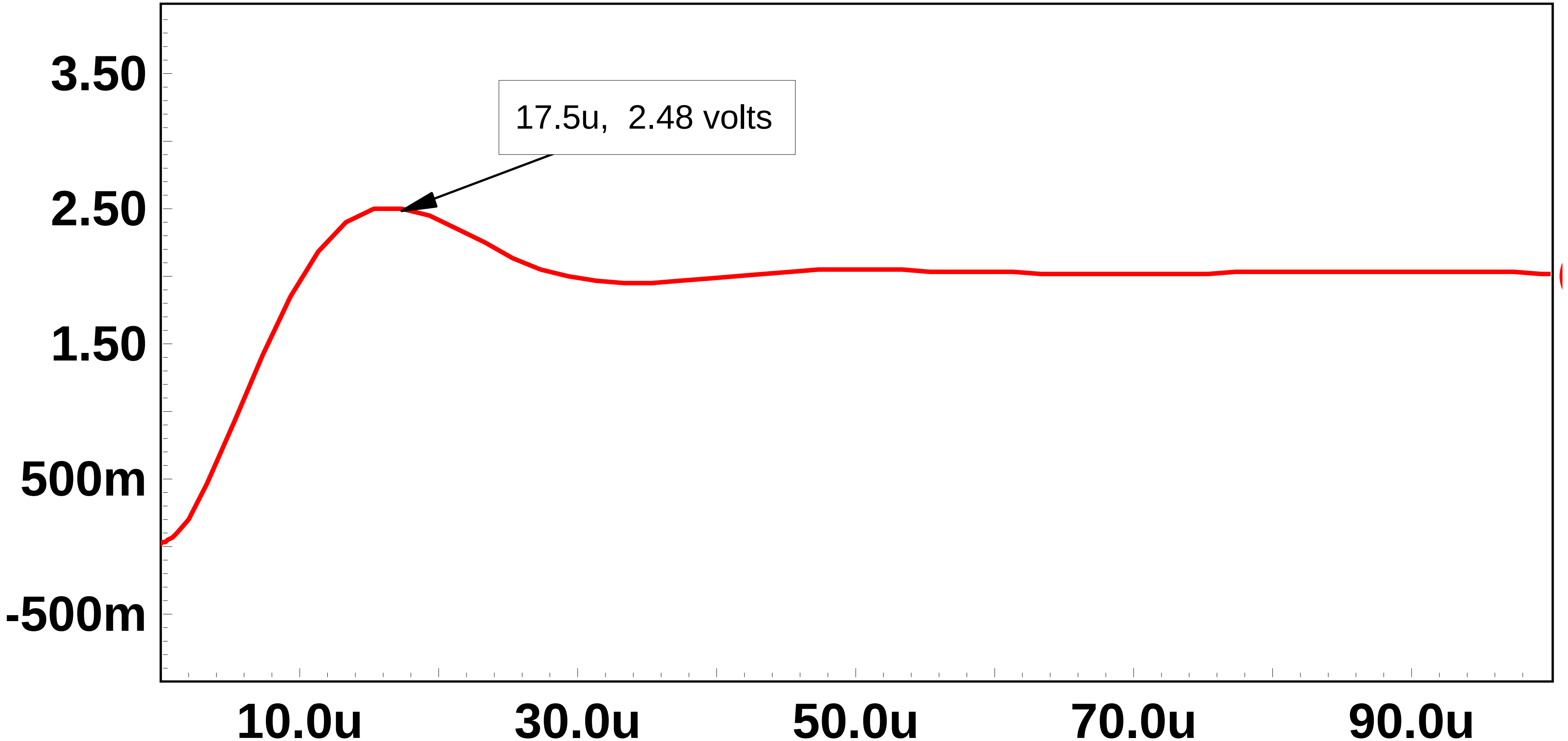
20.0
0
-20.0
-40.0
-60.0
1
100
10k
1Meg
100M
2.10dB
$G_{\infty} = 6.02dB$
$f_c = 28kHz$
closed-loop gain G
1

Model for simulation using the Intusoft ICAP/4 GFT Template.

(No input filter)

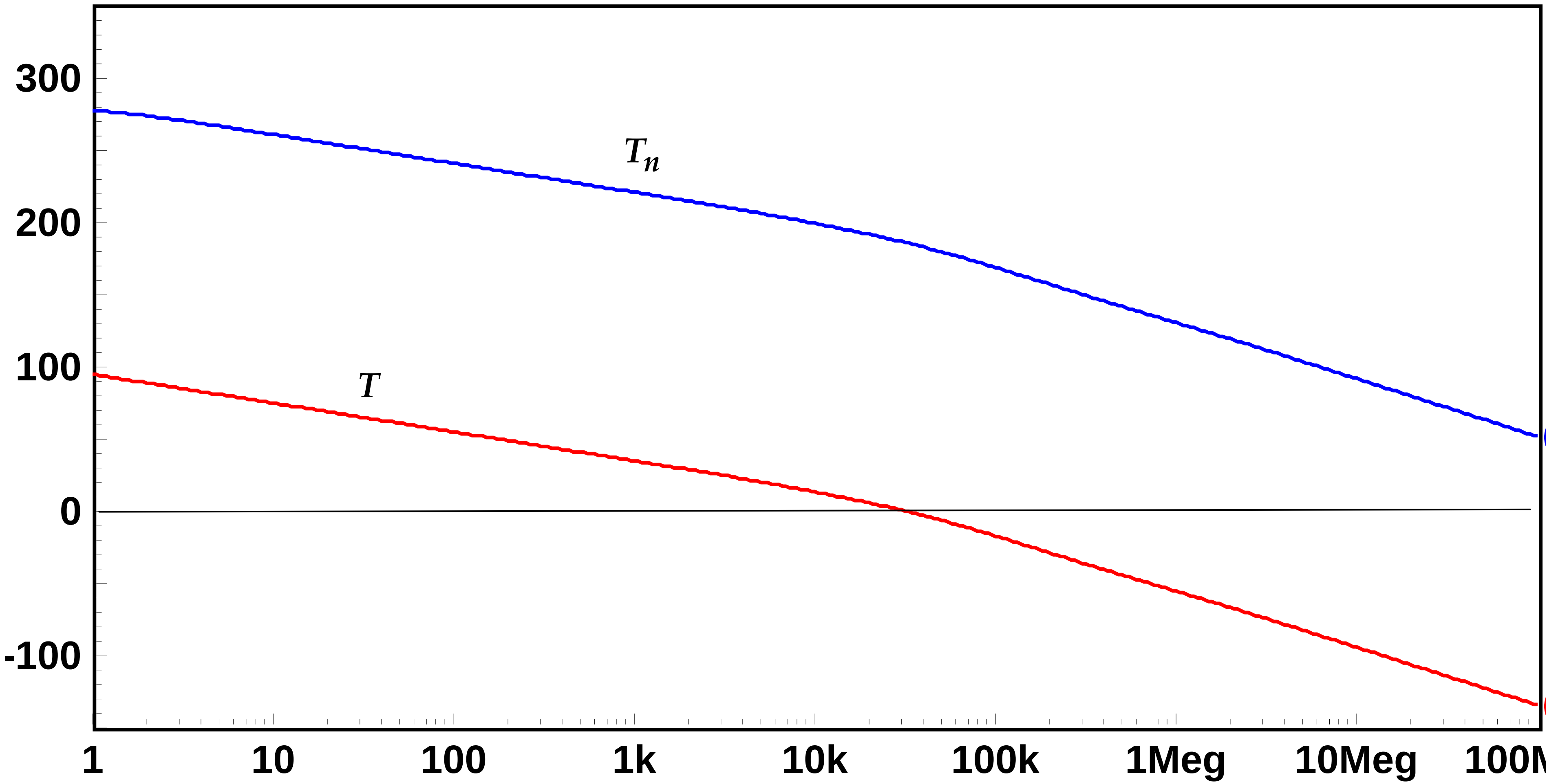


Output voltage response to a 1v step in reference voltage



STEP 6: Check for nonidealities

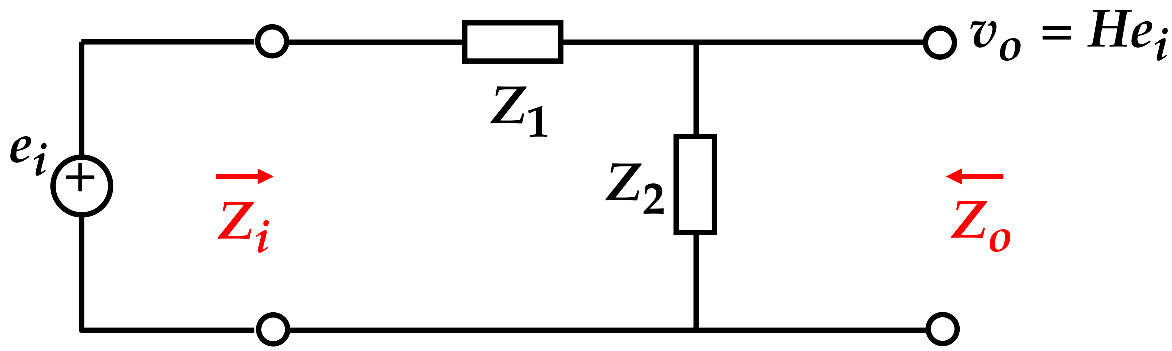
The ICAP/4 simulation also delivers the null loop gain T_n :



Since $T_n >> 1$ at all frequencies of interest, the nonidealities are negligible.

STEP 7: Output Impedance

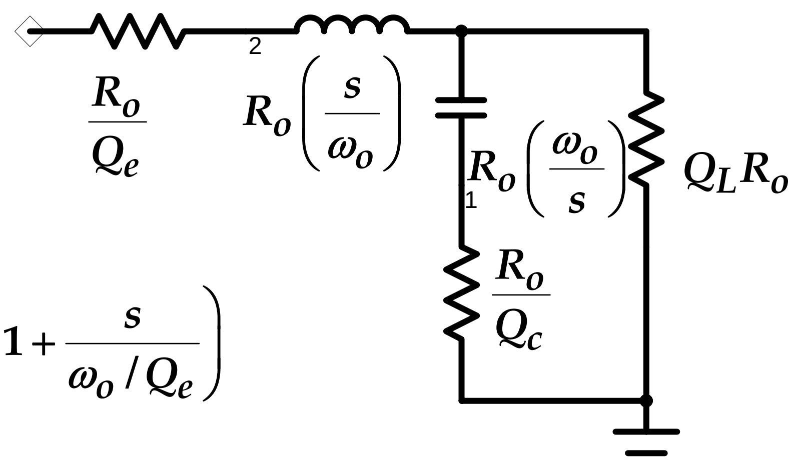
As seen in Ch. 7, for a ladder network such as



the output impedance is

$$Z_o = Z_1 H$$

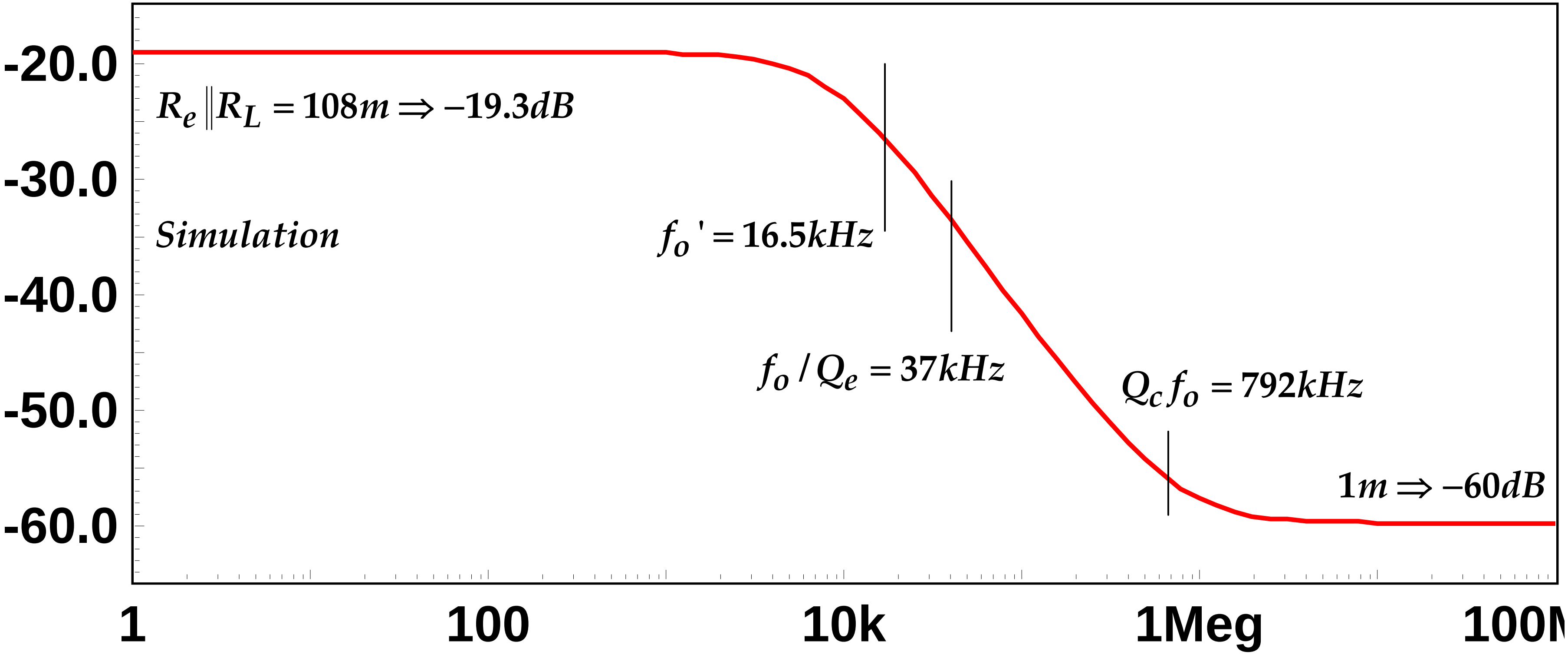
Here,



$$Z_1 = R_o \frac{1}{Q_e}\left(1 + \frac{s}{\omega_o / Q_e}\right)$$

and H is already known, so

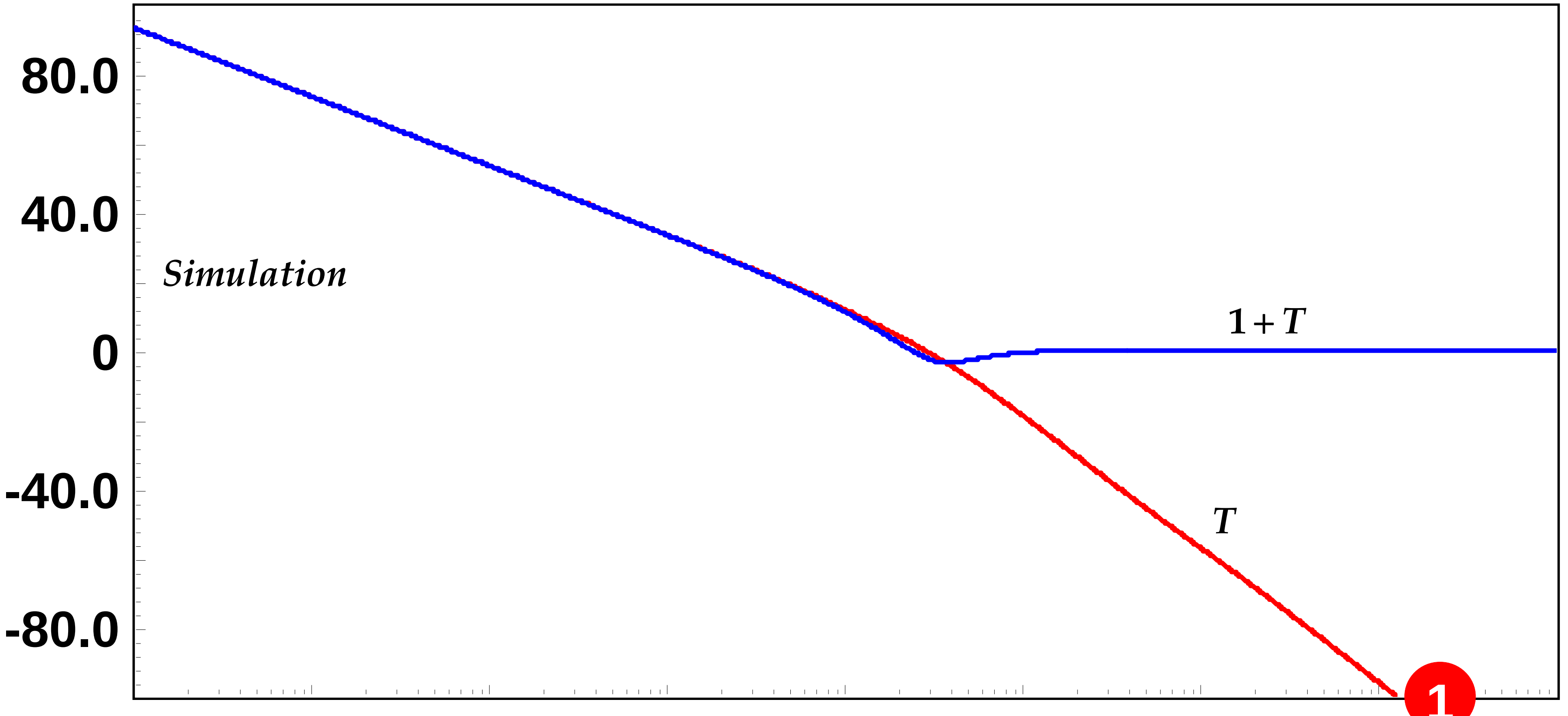
$$Z_o = \left(R_e \| R_L\right) \frac{\left(1+\frac{s}{\omega_o/Q_e}\right)\left(1+\frac{s}{Q_c\omega_o}\right)}{1+\frac{1}{1+\frac{1}{Q_eQ_L}}\frac{1}{Q_e\|Q_L}\left(\frac{s}{\omega_o}\right)+\frac{1}{1+\frac{1}{Q_eQ_L}}\left(\frac{s}{\omega_o}\right)^2}$$

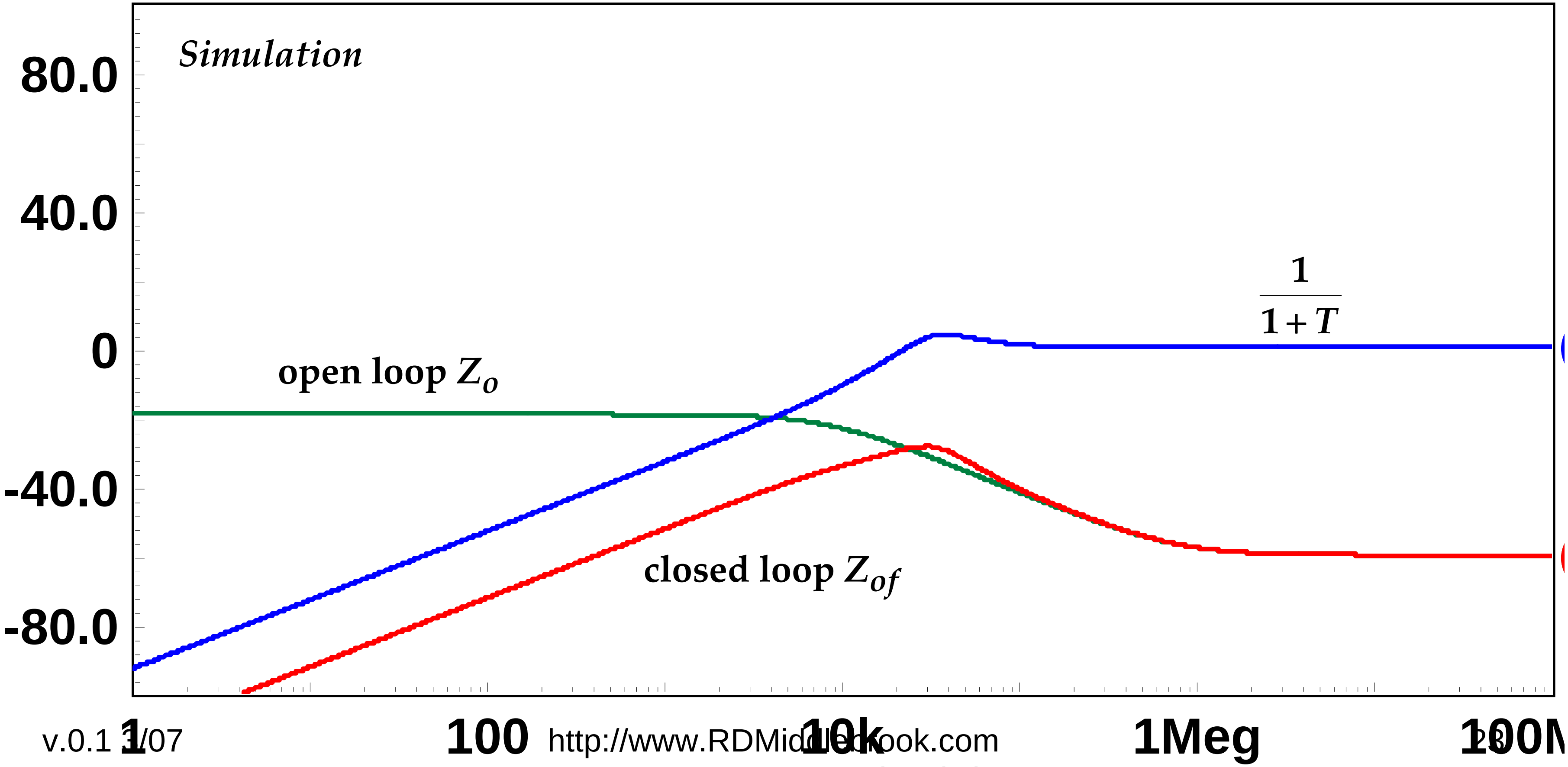


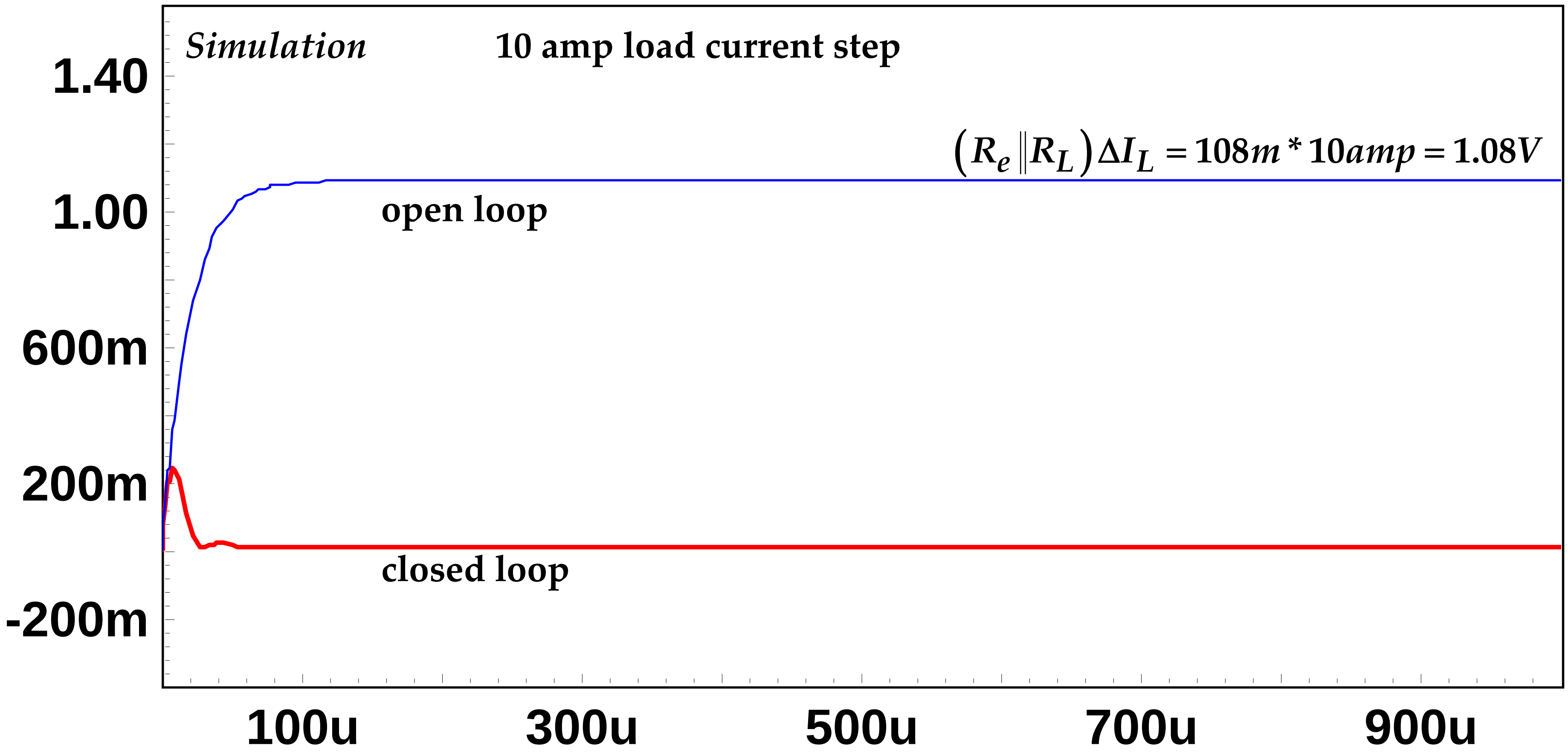
Closed loop output impedance Z_{of}

$$Z_{of} = \frac{Z_o}{1+T}$$

Since T is already known, $1+T$ and $\dfrac{1}{1+T}$ can be found by the methods of Ch. 6.



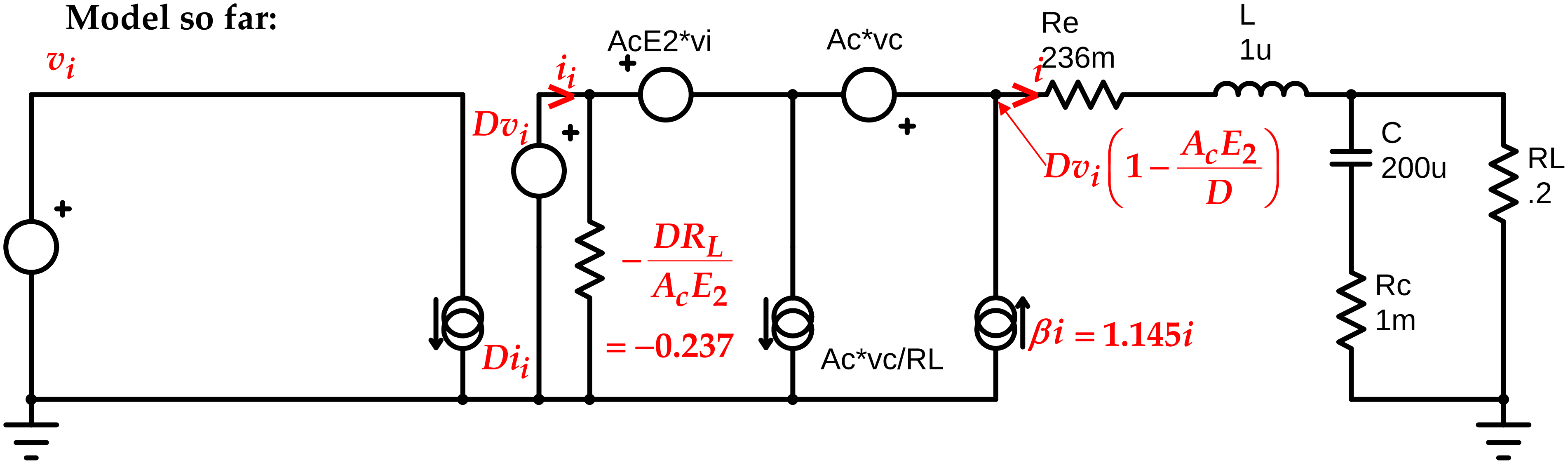

Simulation
80.0
40.0
0
-40.0
-80.0
open loop Z_o
closed loop Z_{of}
$\frac{1}{1+T}$
1
100
10k
1Meg
100M



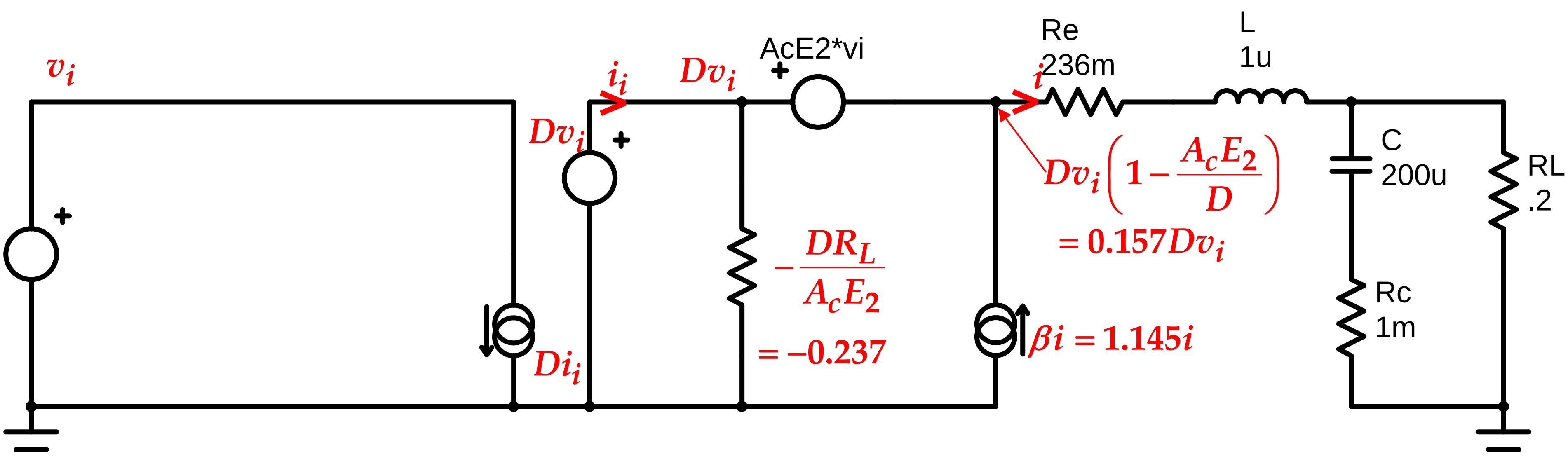
Simulation
10 amp load current step
$(R_e \| R_L)\Delta I_L = 108m * 10amp = 1.08V$
open loop
closed loop
1.40
1.00
600m
200m
-200m
100u
300u
500u
700u
900u

STEP 8: Line-to output TF

Model so far:



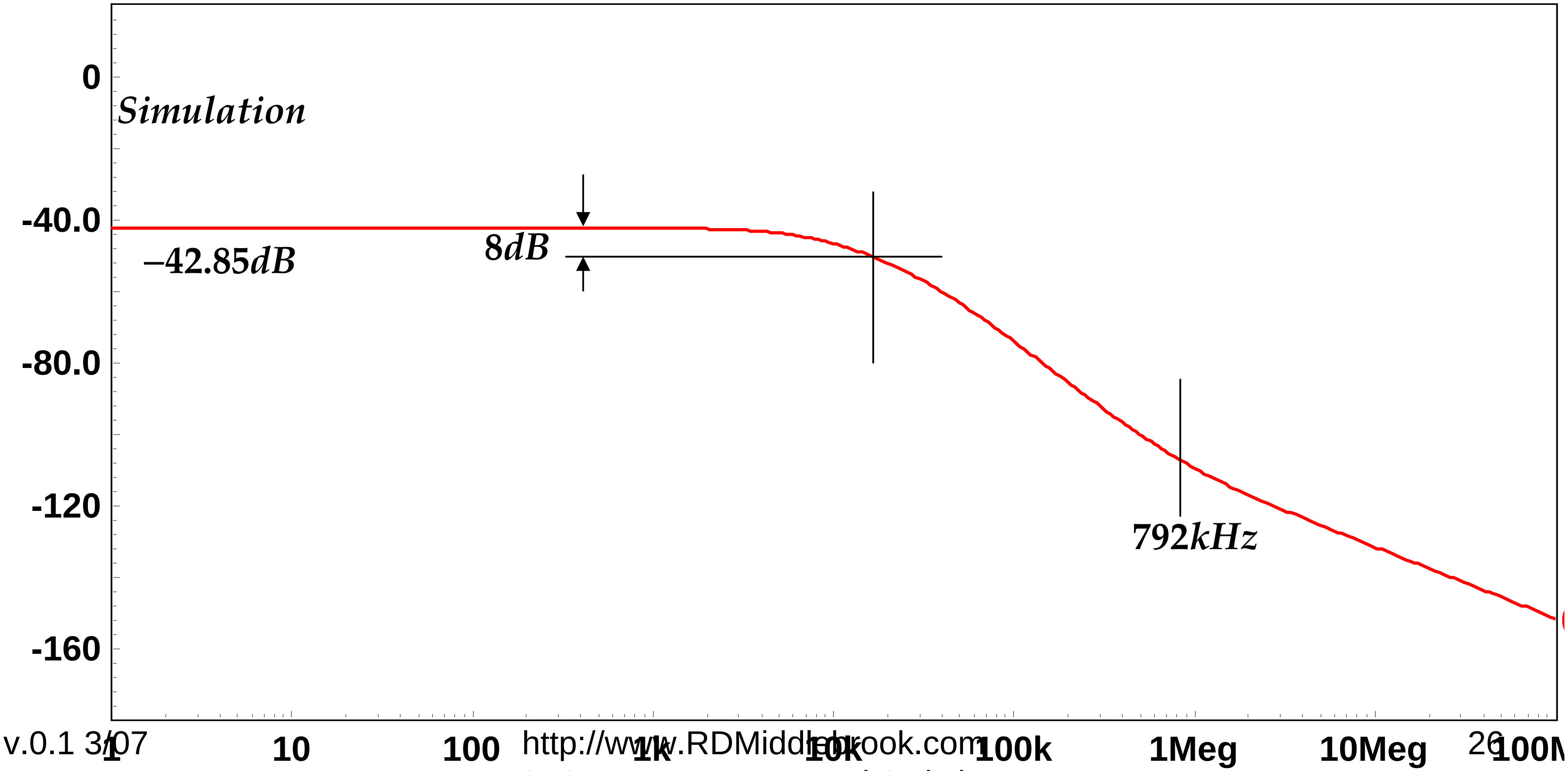
Since $v_c = 0$, the voltage and current modulation generators are zero:



The power stage line-to-output voltage gain is $v_o / v_i \Big|_{v_c=0} = Dv_i \left(1 - \frac{A_c E_2}{D}\right) H = 0.157 DHv_i$

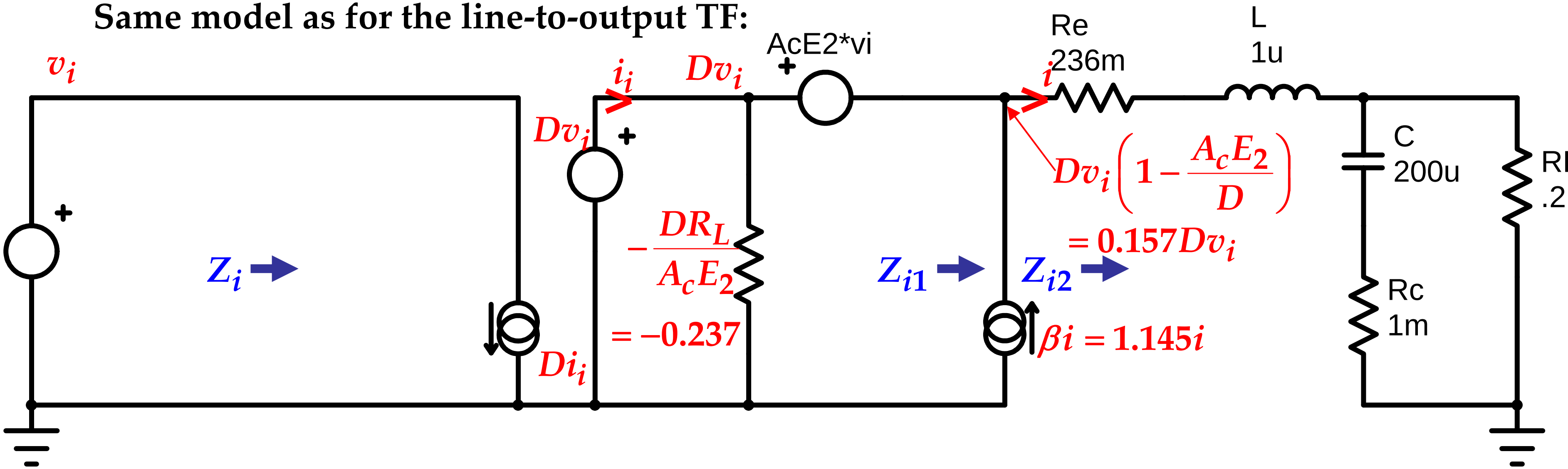
Insertion of numbers gives

$$v_o / v_i \Big|_{v_c=0} = -42.85dB \frac{1 + \frac{s/2\pi}{792kHz}}{1 + \frac{1}{0.400}\left(\frac{s/2\pi}{16.5kHz}\right) + \left(\frac{s/2\pi}{16.5kHz}\right)^2}$$



STEP 9: Line input impedance TF Z_i

Same model as for the line-to-output TF:



$$Z_i = \frac{1}{D^2}\left[-\frac{DR_L}{A_cE_2} \Bigg\| \frac{Z_{i1}}{1-\frac{A_cE_2}{D}}\right]$$ **where** $$Z_{i1} = \frac{1}{1-\beta} Z_{i2}$$ **in which**

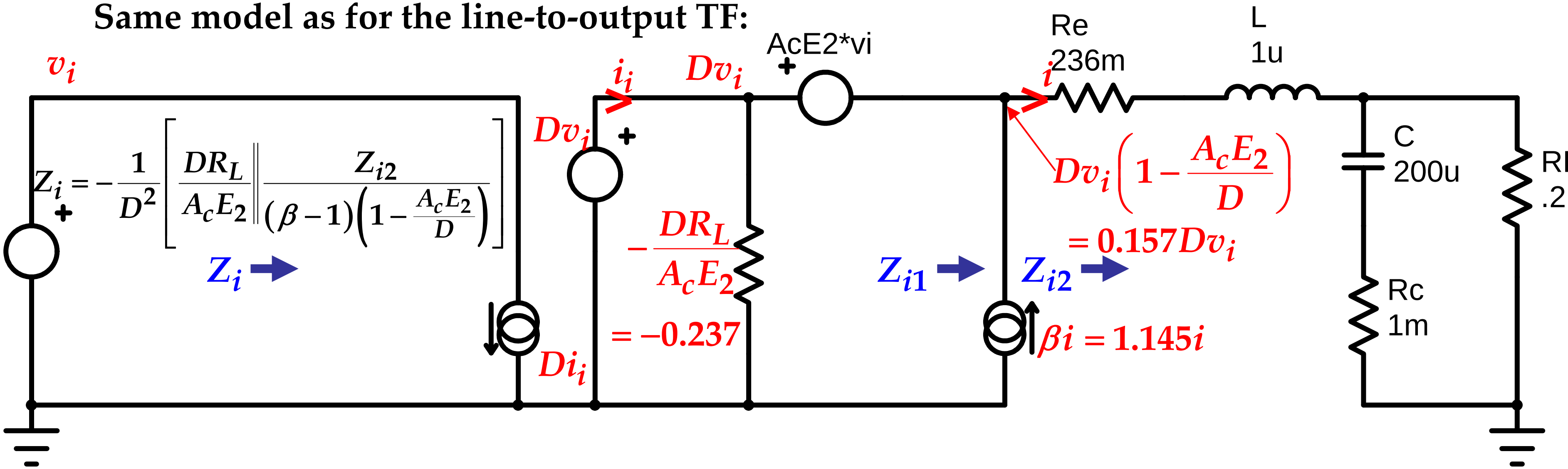
Z_{i2} = input impedance of the filter, which is already known.

Since β>1, the result can be written

$$Z_i = -\frac{1}{D^2}\left[\frac{DR_L}{A_cE_2} \Bigg\| \frac{Z_{i2}}{(\beta-1)\left(1-\frac{A_cE_2}{D}\right)}\right]$$

STEP 9: Line input impedance TF Z_i

Same model as for the line-to-output TF:



Since $Z_{i2}(0) = R_e + R_L = 0.236 + 0.2 = 0.436$,

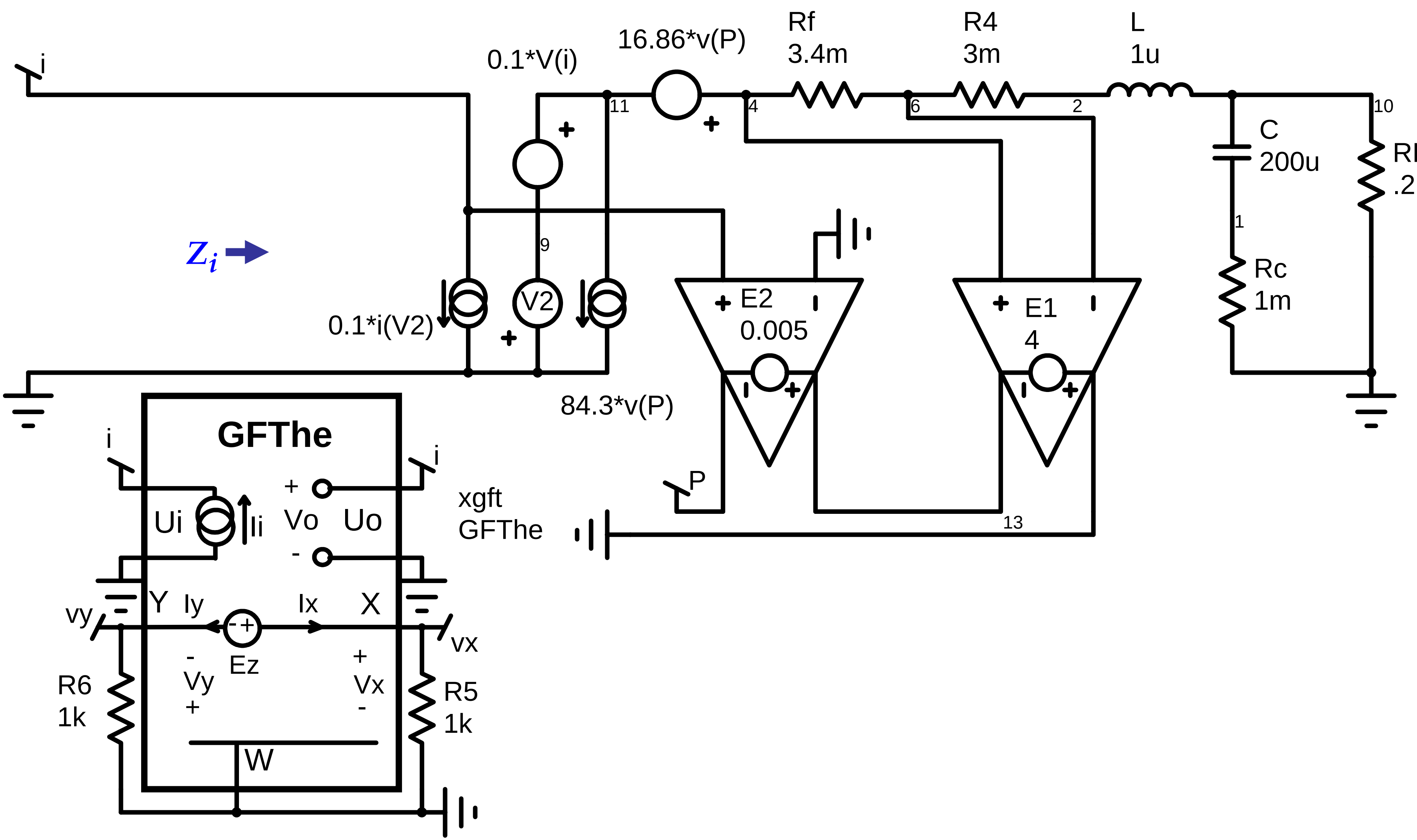
$$Z_i(0) = -\frac{1}{0.1^2}\left[0.237 \Big\| \frac{0.436}{(1.146-1)0.157}\right] = -100[0.237 \| 19] = -100[0.234] = -23.4 \Rightarrow 27.38dB$$

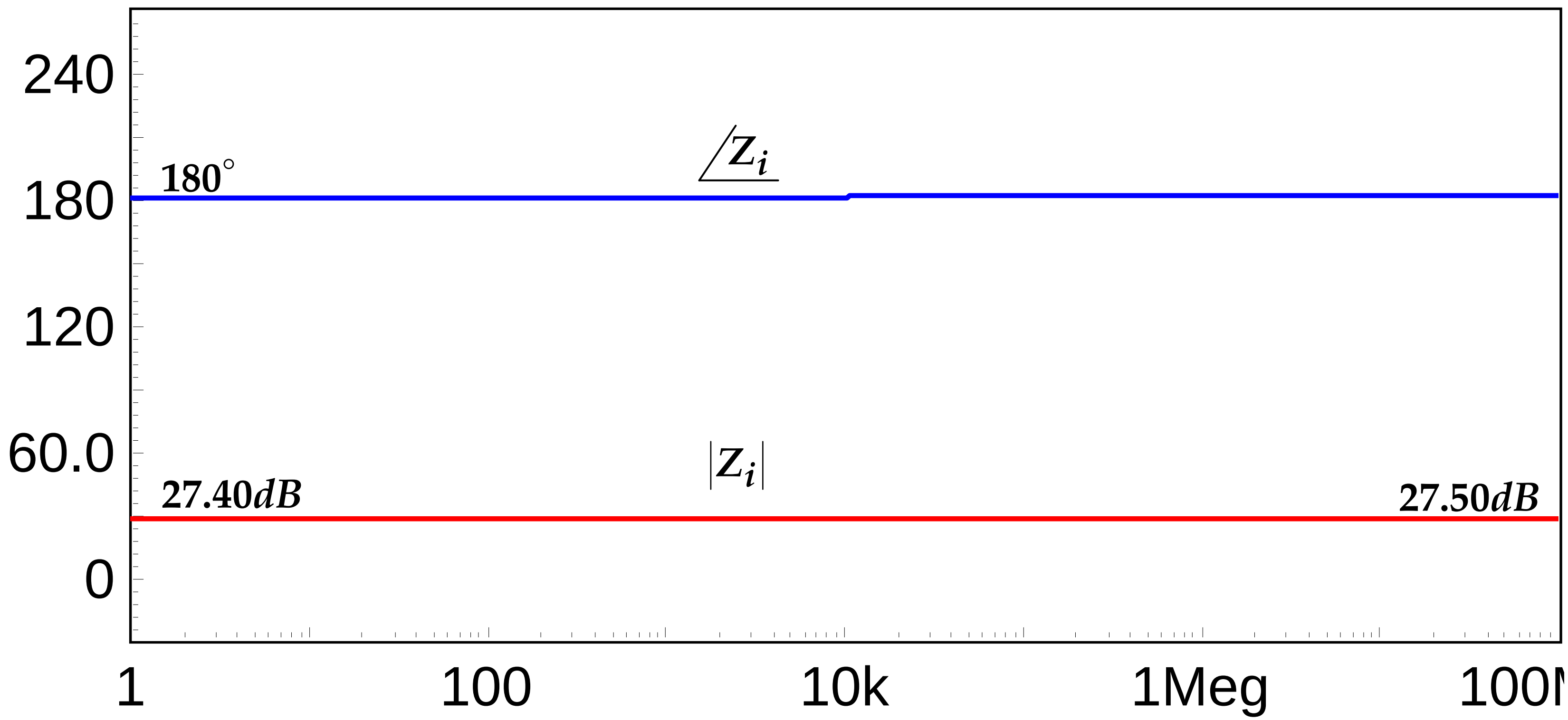
Since $Z_{i2}(\infty) = \infty$,

$$Z_i(\infty) = -\frac{1}{0.1^2}[0.237 \| \infty] = -100[0.237] = -23.7 \Rightarrow 27.49dB$$

Conclusion: the current-programming loop makes the filter input impedance look high, and the (negative) line input impedance is dominated by the term due to the line feedforward.

Input impedance Z_i by the ICAP/4 GFT Template:





The result is in agreement with that predicted by the modified model.