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PRACTICAL TECHNIQUES FOR ANALYZING, MEASURING, AND STABILIZING FEEDBACK CONTROL LOOPS IN SWITCHING REGULATORS AND CONVERTERS

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Abstract

Practical techniques are shown for analyzing the response of switching regulators and determining and realizing the desired feedback characteristics to achieve stable closed-loop operation. These techniques employ reactance paper to reduce the calculation time to minutes and increase understanding of circuit behavior. Measurement techniques are also shown which allow loop gain and phase margin to be determined in a few seconds while the circuit is operating.

1. INTRODUCTION

Many papers have been written in recent years concerning detailed mathematical analysis of the closed-loop frequency response of switching regulators. The value of these papers is without doubt, and a number of them are referenced herein. The purpose of this paper, however, is to take a step backward in complexity, and a step forward in understanding stability analysis.

2. THE BASICS

2.1 TRANSFER FUNCTIONS

The transfer function of a circuit is simply the output divided by the input. This function has a gain and a phase component and is usually a function of frequency. The most useful form of the transfer function is the Bode plot, named after H. W. Bode of Bell Telephone Laboratories. Typically there are two Bode plots required, gain and phase. The use of log scales greatly simplifies graphical analysis (see Figure 1).

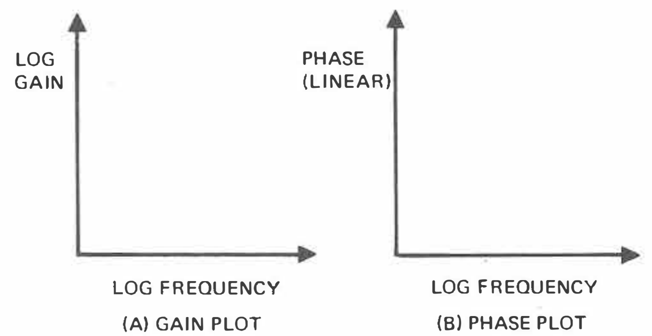


FIGURE 1 GAIN AND PHASE BODE PLOTS

The gain and phase shift around the loop of a closed-loop system is simply the product of the gains of all elements and the sum of the phase shifts of all elements. When the gains are plotted logarithmically, the product can be determined graphically just by summing the gain of each element. Typically in a switching regulator, there are only two elements to sum. One is the modulator, the device which takes a control voltage and causes the input power to be chopped and then smoothed to provide a d.c. output voltage, and the other is the feedback amplifier which senses the d.c. output voltage, compares it to a reference, and outputs a control voltage to the modulator.

2.2 POLES

Technically, poles are points on the s-plane where the denominator of the transfer function goes to zero. Realistically, they are points in frequency where the slope of the Bode gain plot breaks downward.

A very important point to know and remember is that in linear circuit analysis, the Bode plot can have only a limited number of discrete slopes and phase shifts except in a transition region near poles and zeros. (See Table 1) The "Phase Shift per Decade" column is the approximate slope of the phase curve near poles and zeros.

TABLE 1 SOME ALLOWABLE SLOPES AND PHASE SHIFTS ON A BODE PLOT

Slope	Gain Change per Octave	Gain Change per Decade	Associated Total Phase Shift	Phase Shift per Decade
+1	+6 dB	+20 dB	$+90^{\circ}$	$+45^{\circ}$
0	0 dB	0 dB	0°	0°
-1	-6 dB	-20 dB	-90°	-45°
-2	-12 dB	-40 dB	-180°	-90°
-3	-18 dB	-60 dB	-270°	-135°

A pole causes a transition from a +1 to a 0 slope, or 0 to -1, or -1 to -2, etc. This is accompanied by a 90° lag in phase shift. Some circuits which cause poles and their respective Bode plots are shown in Figure 2.

2.3 ZEROS

Zeros are points on the s-plane where the numerator of the transfer function goes to zero. Practically speaking, they are points in frequency where the slope of the Bode gain plot breaks upward. A zero causes a transition from a -1 to a 0 slope, or -2 to -1, or -3 to -2 etc. This is usually accompanied by a 90° lead in phase shift. Because of the nature of boost and buck-boost regulators, however, a right-half plane zero usually occurs in the transfer function of the modulator portion. A right-half plane zero causes the Bode gain plot to break upward the same as a left-half plane zero, but is accompanied by a 90° phase lag instead of lead. This phenomenon necessitates more care in the stabilization of boost and buck-boost regulators than in buck regulators, so that loop gain crossover occurs well below the frequency of the right-half plane zero. Circuits causing both poles and zeros are shown in Figure 3.

Figure 4 shows the difference between a left- and a right-half plane zero.

2.4 STABILITY CRITERIA

How do you make an oscillator? Simple! Just make a closed loop feedback control system that still has more than unity (0 dB) gain when the total phase shift around the loop is 360 degrees. Conversely, to make a system stable, make sure that the loop gain has fallen below unity by the time the total phase shift has reached 360°. The amount the gain is below unity when the total phase shift is 360° is called the gain margin. The closely related criterion, phase margin, is the difference between the actual phase shift and 360° when the loop gain is unity.

Most feedback control system texts talk about stability in terms of 180° of phase shift. This is because at d.c. the feedback is negative, that is, there is 180° of phase shift to start with. The stability criteria are then based on an additional 180° of phase shift.

Notice in Table 1 that in order for the gain to fall off with increasing frequency, a minimum of 90° of phase shift is required. This 90° is in addition to the 180° at d.c., so that a system with a single pole rolloff (-1 slope) at crossover will have 270° of total phase shift or a phase margin of 90°. Notice also that a system with a two pole rolloff (-2 slope) will have 360° of total phase shift and be on the verge of oscillation, if it does not in fact oscillate.

Notice especially in Table 1 that a system with a three pole rolloff (-3 slope) will have a whopping 450° of total phase shift and is sure to oscillate! The reason that this is important to note is that switching regulators inherently have a three pole rolloff in the closed loop feedback mode, 2 poles from the modulator and one additional pole from the feedback amplifier. A principal objective of this paper is to present techniques for achieving stability by reducing the closed loop gain rolloff rate to a -1 slope in the region of gain crossover (unity gain).

3. SWITCHING REGULATORS

All switching regulators operate on the principle of storing energy in an inductor on one portion of a cycle and then transferring the stored inductive energy to a capacitor in another portion of a cycle. The buck converter, shown in Figure 5, is the simplest example.

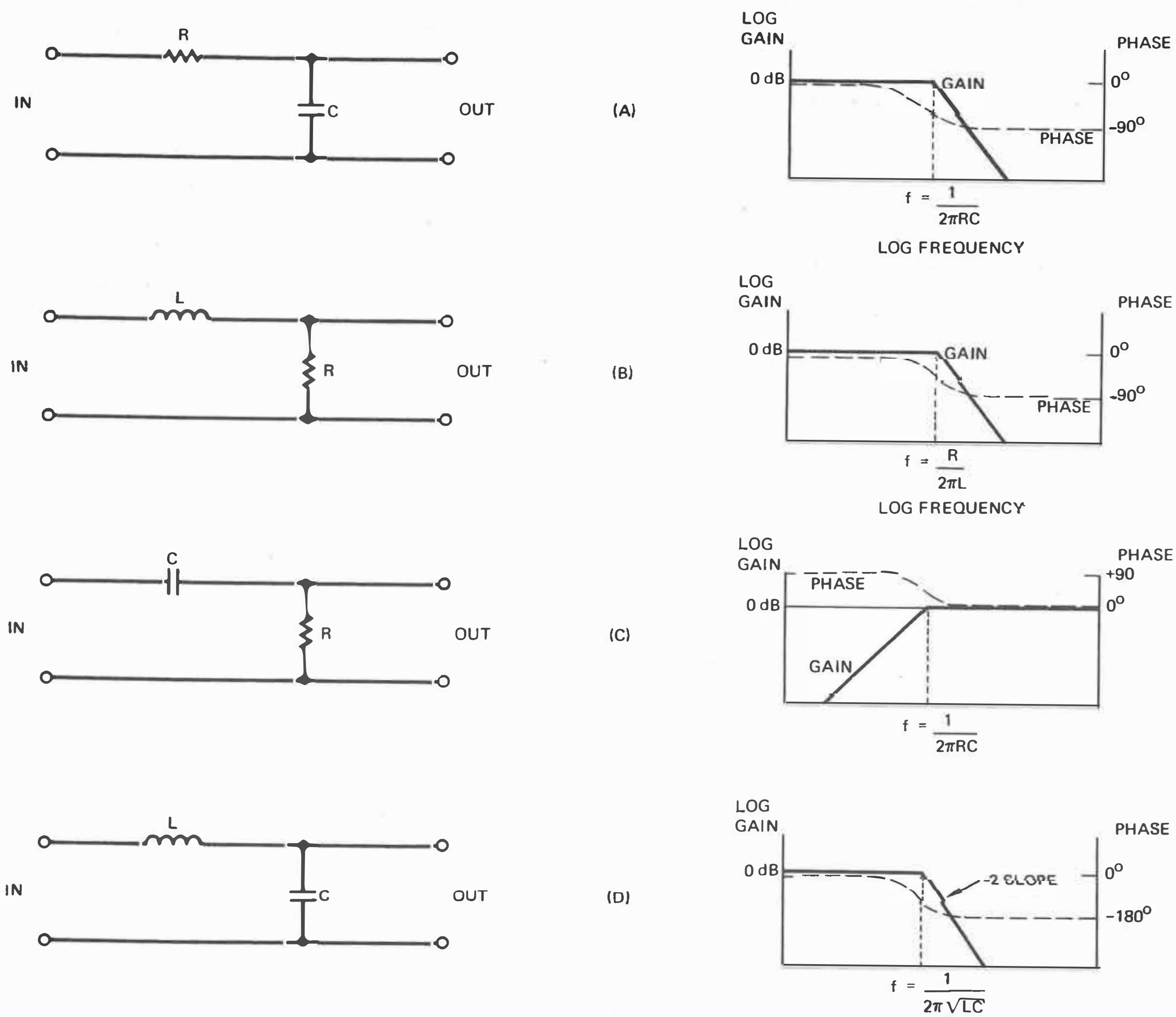


FIGURE 2 CIRCUITS WHICH CAUSE POLES

The input voltage is chopped by switch Q to produce a duty-cycle modulated a.c. waveform, the d.c. component of which is the only one passed by the low pass filter consisting of L and C.

The transfer function of the buck converter from control input to output typically consists of a d.c. gain which is flat out to the resonance of the L-C filter, and then falls at a -2 slope as shown in Figure 6.

The Bode plot of most switching regulators is similar. The objective is to determine the d.c. gain and the corner frequency, and the modifications caused by parasitic elements. Usually the gain and frequently the corner frequency are functions of the input and/or output voltage. Determination can be done by analysis or by measurement, preferably by both to get correlation and assurance that all significant parasitic elements have indeed been accounted for. Several excellent papers on

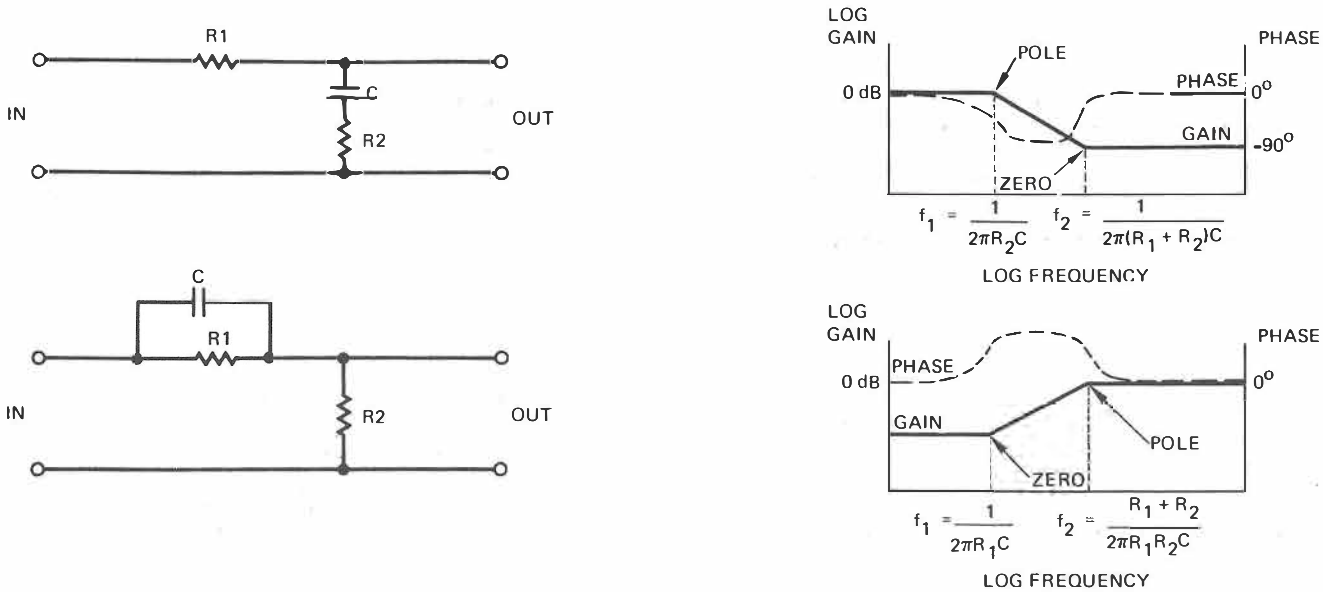


FIGURE 3 CIRCUITS WHICH CAUSE POLES AND ZEROS

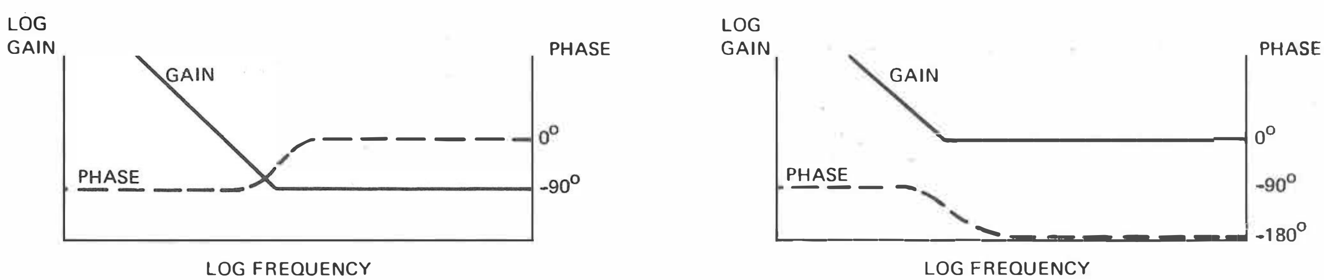


FIGURE 4 EFFECTS OF LEFT- AND RIGHT-HALF PLANE ZEROS

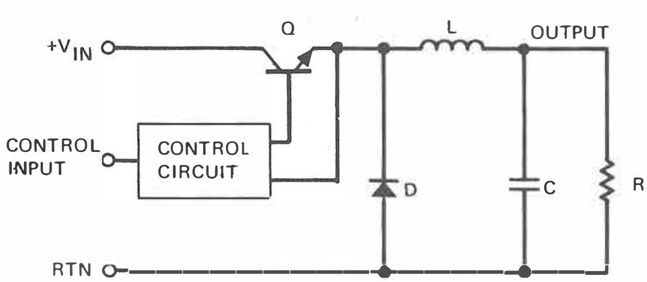


FIGURE 5 BUCK CONVERTER CIRCUIT

analysis techniques have been presented by Dr. R. D. Middlebrook et al. Measurement techniques are presented in Section 6 of this paper.

4. AMPLIFIERS

Amplifiers, usually in the form of integrated circuit operational amplifiers, are typically used to compare the output voltage of a regulator with some fixed reference, and to then generate an error signal to the control input of the modulator portion of a switching regulator. The gain, phase, and d.c. operating point can be selected by the use of external components. Although the number of amplifier configurations is virtually limitless, three are of special value as feedback amplifiers. The simplest form is shown in Figure 7.

A more complicated amplifier is shown in Figure 8. In this configuration, a zero-pole pair has been introduced to give a region of frequency where the gain is flat and no phase shift is introduced.

The algebra involved in calculating the transfer function is given in Appendix A. The results of the calculation are as follows:

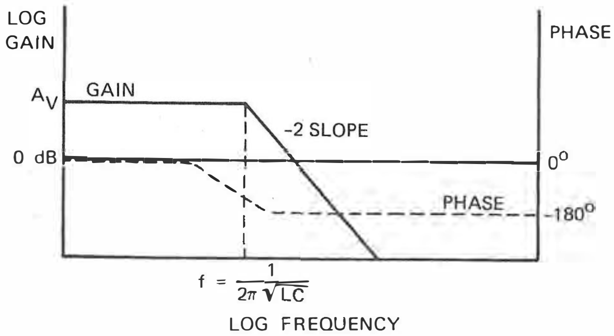


FIGURE 6 BUCK CONVERTER BODE PLOT

$$A_v = \frac{R2}{R1} \tag{1}$$

$$f_1 = \frac{1}{2\pi R2C1} \tag{2}$$

$$f_2 = \frac{C1 + C2}{2\pi R2C1C2} \approx \frac{1}{2\pi R2C2} \tag{3}$$

An even more complicated amplifier configuration is shown in Figure 9. In this configuration, two zero-pole pairs have been introduced to give a region of frequency where the gain is increasing at a +1 slope and 90^o of phase lead is introduced.

This configuration is also analyzed in Appendix A. The results are as follows:

$$A_{v1} = \frac{R2}{R1} \tag{4}$$

$$A_{v2} = \frac{R2(R1 + R3)}{R3R1} \approx \frac{R2}{R3} \tag{5}$$

$$f_1 = \frac{1}{2\pi R2C1} \tag{6}$$

$$f_2 = \frac{1}{2\pi(R1 + R3)\,C3} \approx \frac{1}{2\pi R1C3} \tag{7}$$

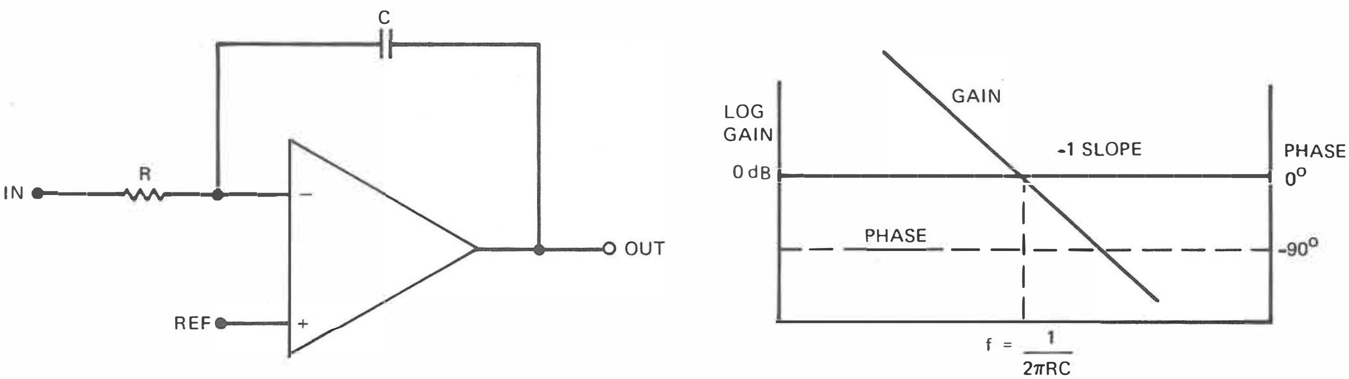


FIGURE 7 OP AMP WITH SINGLE POLE ROLLOFF

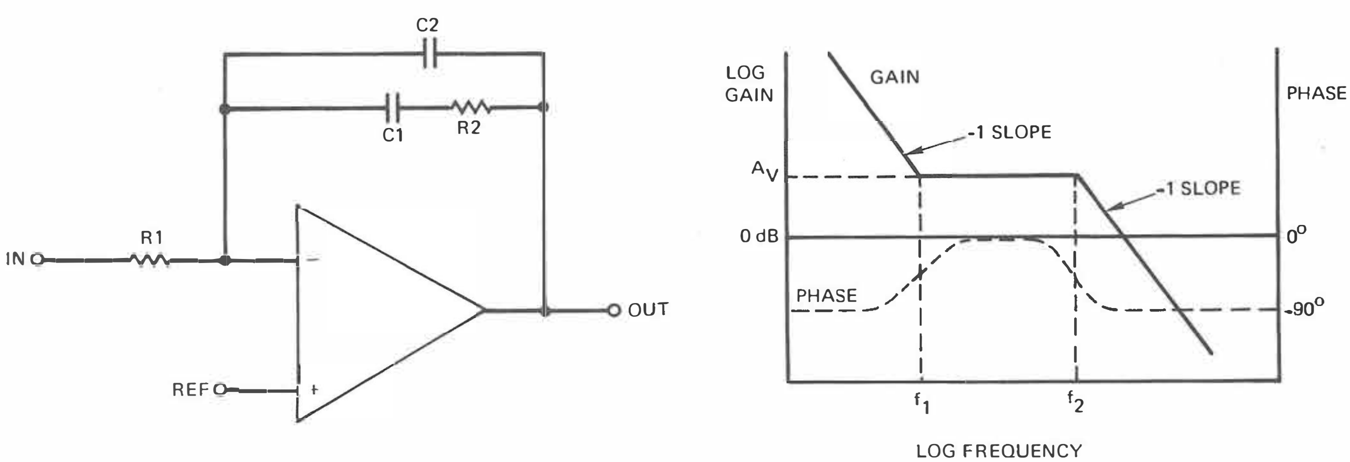
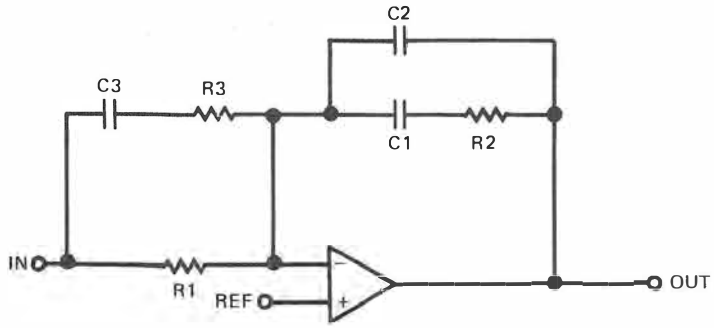


FIGURE 8 OP AMP WITH ZERO-POLE PAIR



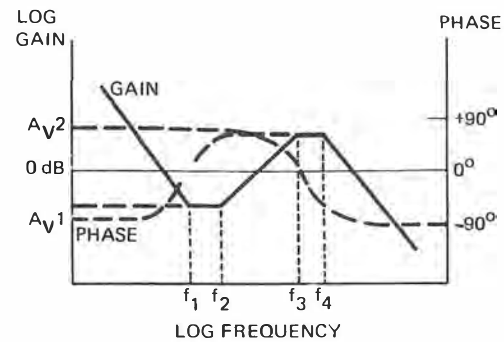


FIGURE 9 OP AMP WITH 2 ZERO-POLE PAIRS

$$f_3 = \frac{1}{2\pi R3C3} \quad (8)$$

$$f_4 = \frac{C1 + C2}{2\pi R2C1C2} \approx \frac{1}{2\pi R2C2} \quad (9)$$

One of the above three amplifier configurations should suffice for any switching regulator feedback scheme.

5. CLOSED-LOOP SYSTEMS

Remember from Section 2.1 that loop gain in a closed-loop system is simply the product of the gain of each of the elements around the loop. With logarithmic scales, this simply means adding the gains. Notice in Figure 6 that the gain falls at a -2 slope above the corner frequency of the L-C filter. Notice also that the gain of the basic amplifier shown in Figure 7 falls at a constant -1 slope. If these two elements are connected together to make a feedback system, the gain must be adjusted so that crossover (unity gain) occurs below the L-C corner frequency to prevent oscillation. This usually results in an unacceptably slow response time since the time the loop takes to respond to and correct for a disturbance is closely related to the frequency at which unity loop gain occurs. Section 7 will show a method to compensate the characteristic in Figure 6 using the amplifier shown in Figure 9 to achieve crossover beyond the L-C break frequency.

There are a number of ways to compensate a loop to achieve stability. Not only are there several amplifier configurations, but several modifications available for the basic modulator section. Figure 6 shows the Bode plot of the basic switching regulator. Figure 10 shows a modified circuit including some of the parasitics and a discrete modification for damping the L-C corner.

The Bode plot for the circuit of Figure 10 is shown in Figure 11.

If the transfer function of the entire modulator were shown, it would look just like Figure 11, except shifted up or down in gain. Notice that the slope changes several times, giving more options as to how to compensate the loop. These changes are due to the parasitic elements and to the damping resistor R_D. Capacitor C2 is larger than C1 and the break from 0 to -1 slope is due to the L/R_D pole. The second break from -1 to -2 slope is due to the C1 R_D pole and the zero which takes the slope back to -1 is due to R_{C1}, the equivalent series resistance (ESR) of capacitor C1.

There are now four methods of compensating the circuit of Figure 10. The first method is to use the amplifier of

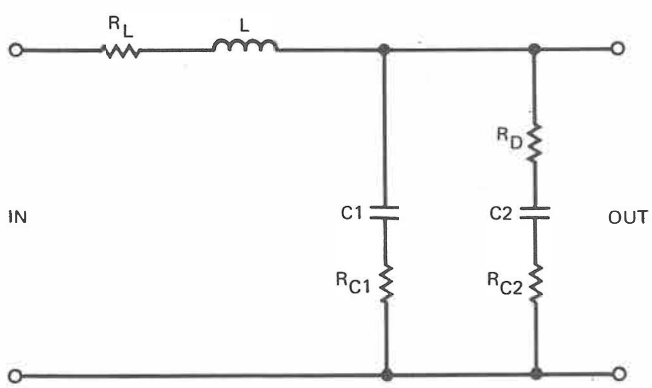


FIGURE 10 MODIFIED FILTER FOR SWITCHING REGULATOR SHOWING SOME PARASITICS

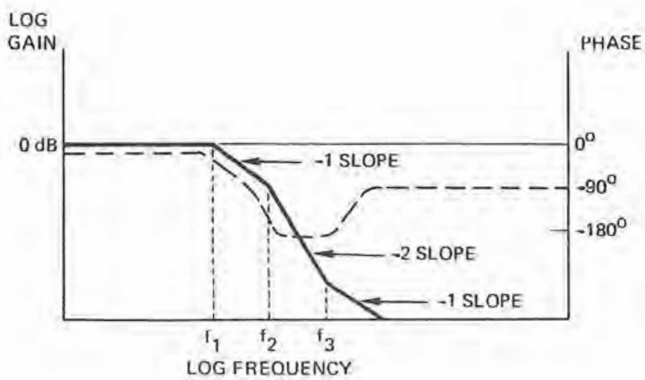


FIGURE 11 BODE PLOT OF THE CIRCUIT SHOWN IN FIGURE 10

Figure 7 and adjust the gain so that crossover is below f_1. This is a popular method but results in very slow transient response.

Better transient response can be obtained by using the circuit of Figure 8 and crossing over between f_1 and f_2. Notice that the object of stability analysis is to get the loop to cross over at a -1 slope.

This can be achieved at least three different ways as shown in Table 2.

TABLE 2 MODULATOR/AMPLIFIER COMBINATIONS TO ACHIEVE STABILITY

Modulator Gain Slope at Crossover	Amplifier Gain Slope at Crossover	Overall Loop Gain Slope at Crossover
0	-1	-1
-1	0	-1
-2	+1	-1

A third method of compensating the loop is to use the amplifier of Figure 9 and crossover between f_2 and f_3. This method gives the best transient response of the three approaches proposed so far.

What about pushing the frequency response out more and crossing over above f_3 using the amplifier of Figure 8? This is fine, and is probably the most widely used stabilization technique. The only problem is that f_3 is based on a parasitic element, the equivalent series resistance of C1, and can vary significantly with temperature, from lot to lot, and with aging. Also there is the danger of replacing C1 with a higher quality capacitor (lower ESR) at some point in a production run, and having the circuit oscillate because f_3 moved out beyond crossover. There are also inherent limitations on how high the loop crossover frequency can be relative to the switching frequency, so it may not be possible to cross over above f_3. The theoretical limit is half the switching frequency, but practical considerations limit most designs to less than 1/5 of the switching frequency.

6. MEASUREMENT TECHNIQUES

6.1 EQUIPMENT

The equipment needed to measure the frequency response of circuits consists mainly of a sweeping oscillator with leveled output and a narrow-band frequency selective voltmeter capable of locking on to the frequency of the oscillator. A number of companies manufacture this type of equipment, but to be really useful the equipment should have log voltage and log frequency outputs capable of driving an X-Y recorder. The selective voltmeter should be capable of reading low-level signals accurately in the presence of large amounts of out-of-band noise. Two instruments that have proven capable of doing the job are the Hewlett-Packard 3591A Selective Voltmeter with 3594A Sweeping Oscillator plug-in, and the Bafco Model 916H Frequency Response Analyzer.

6.2 HEWLET-PACKARD 3591A/3594A

The Hewlett-Packard 3591A/3594A is a single channel selective voltmeter with remarkable selectivity. It is capable of reading microvolts of signal in the presence of volts of noise, and has a dynamic range of about 70 dB. It is autoranging and has a leveled beat frequency oscillator (BFO) output for injecting a signal into a loop, and logarithmic recorder outputs for both amplitude and frequency. Frequency range is 20 Hz to 620 kHz. Unfortunately, this remarkable instrument has been dropped from the 1980 Hewlett-Packard catalog and may not be available except on the used-equipment market. Three new H-P instruments are required to perform the functions of the 3591A/3594A.

6.3 BAFCO MODEL 916H

The Bafco Model 916H is a two channel instrument which can simultaneously measure gain and phase. It uses all digital techniques and is exceptionally stable and repeatable. Recorder outputs are provided for an X-Y-Y recorder to simultaneously plot log gain and linear phase versus log frequency. Its principal limitation is an upper frequency capability of 50 kHz, although very few switching regulators have crossovers in excess of this frequency. It is also autoranging and has a leveled output with selectable sine, square, or triangular wave output.

6.4 MAKING BODE PLOTS

Hewlett-Packard published an application note about making closed-loop Bode plots. The technique described here is much quicker, but the mathematics of the technique are identical. The application note (No. 59) is considered obsolete and is out of print, but is included for reference as Appendix B of this paper. The test set-up used at Hughes Aircraft is shown in Figure 12.

The oscillator is the BFO oscillator in the selective voltmeter and the voltages V1, V2, and V3 are read with the selective voltmeter. Transformer T1 is a 600 ohm to 10 ohm matching transformer designed to have a very flat frequency response over the frequency range of interest. By using a leveled BFO and a transformer (T1) which has a flat frequency response, the injected voltage V3 can be made constant with frequency.

The point in the circuit where the signal is injected is critical to the success of this technique. Accuracy is based on the assumption that the input impedance is very much larger than the output impedance at the point of injection. Two points that usually meet these criteria are: a) immediately following the error amplifier, in series with the control voltage to the modulator, and b) immediately following the output filter capacitor, in series with the input to the error amplifier. In Figure 12, the input impedance is the impedance looking into the switching regulator and the output impedance is the output impedance of the error amplifier. The injection of signal V3 causes an error signal V1 with respect to ground. This signal is then amplified through the switching regulator and error amplifier and comes back as V2. The loop gain is then V2/V1, provided the input and output impedance criteria are met.

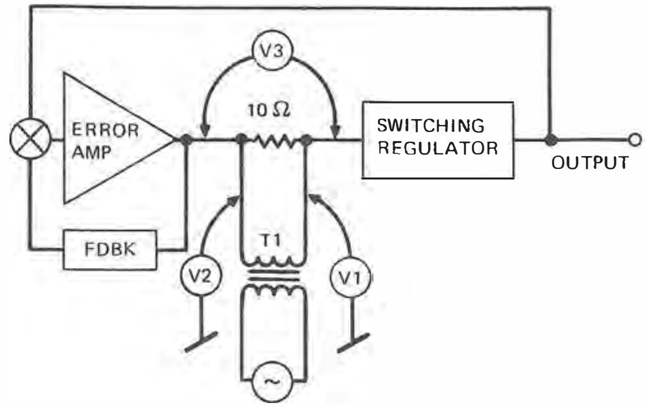


FIGURE 12 BODE PLOT TEST SET-UP

At low frequency, loop gain is typically high and V2 is much larger than V1. Since V3 is constant amplitude, and the three voltages V1, V2, and V3 make a vector triangle, at low frequency V2 ≈ V3. Figure 13 shows the relationship at low frequency.

At high frequency, loop gain is typically low and V2 is much smaller than V1. Again, since V3 is constant amplitude, at high frequency V1 ≈ V3. Figure 14 shows the relationship at high frequency.

At crossover, by definition, V1 = V2 the phase margin is the angle between them and this can be determined by solving the isosceles vector triangle shown in Figure 15.

If a single channel voltmeter such as the H-P 3591A is used, two plots of response versus frequency are made.

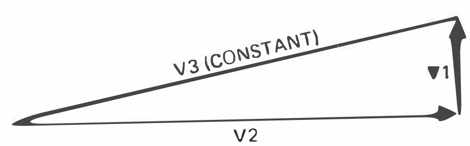


FIGURE 13 PHASE AND AMPLITUDE RELATIONSHIP OF V1, V2, V3 AT LOW FREQUENCY

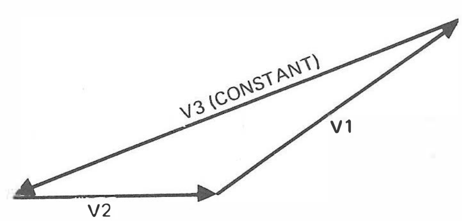


FIGURE 14 PHASE AND AMPLITUDE RELATIONSHIP OF V1, V2, AND V3 AT HIGH FREQUENCY

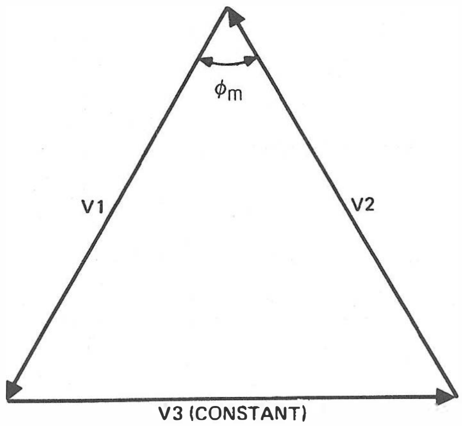


FIGURE 15 ISOSCELES TRIANGLE AT CROSSOVER

One is a plot of V1 and the other of V2. V3 is assumed constant and equal to V2 at low frequency and V1 at high frequency. Figure 16 shows a typical plot.

The point at which the two curves cross is the unity gain frequency and the phase margin is related to the difference in amplitude between the crossover point and the level of V3. If the crossover amplitude is equal to V3, the phase margin is 60 degrees. If the crossover is above (larger than) V3, the phase margin is less than 60 degrees and if the crossover amplitude is below (less than) V3, the phase margin is greater than 60 degrees. Figure 17 shows the crossover area in more detail, Table 3 lists some discrete points of amplitude difference versus phase margin, and Figure 18 is a plot of amplitude difference versus phase margin.

A conventional Bode plot can be obtained from the two curves of Figure 16 by measuring the vertical spread

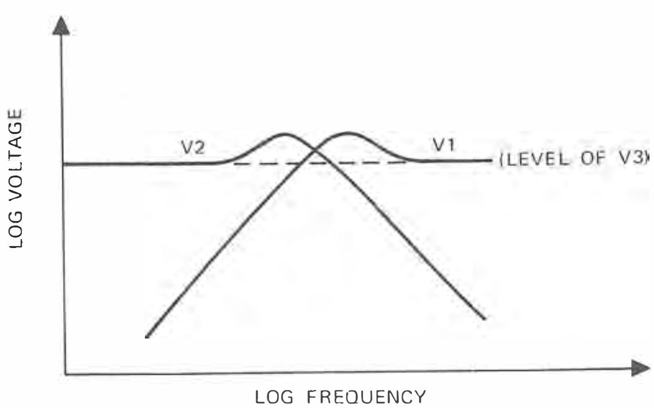


FIGURE 16 BODE PLOT USING SINGLE CHANNEL VOLTMETER

TABLE 3
AMPLITUDE DIFFERENCE VERSUS PHASE MARGIN

V1 (or V2) - V3 (dB)	Phase Margin (degrees)
+6	29.0
+5	32.7
+4	36.8
+3	41.5
+2	46.8
+1	52.9
0	60.0
-1	68.3
-2	78.0
-3	90.0

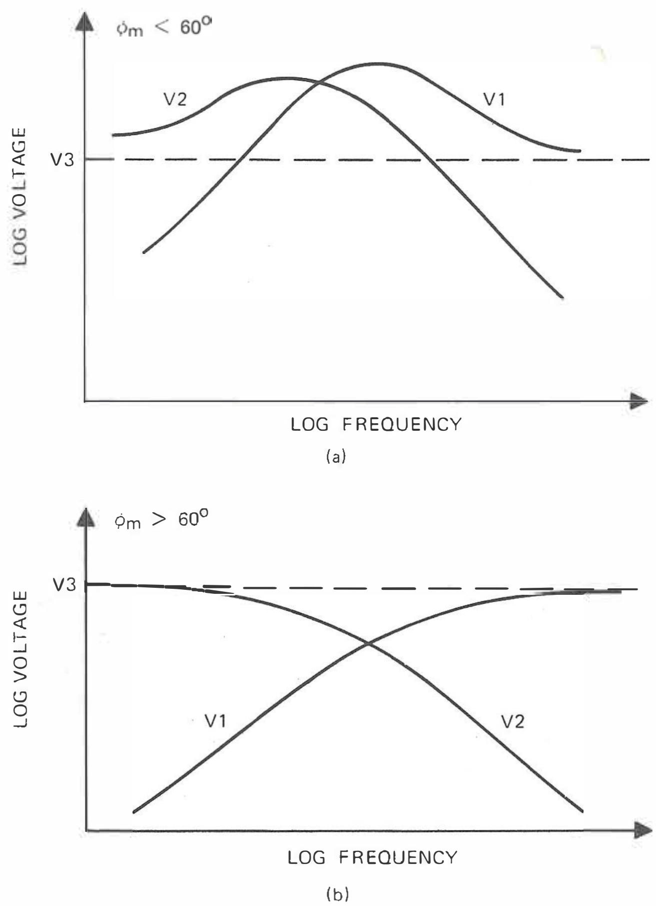


FIGURE 17 DETAIL OF CROSSOVER AREA

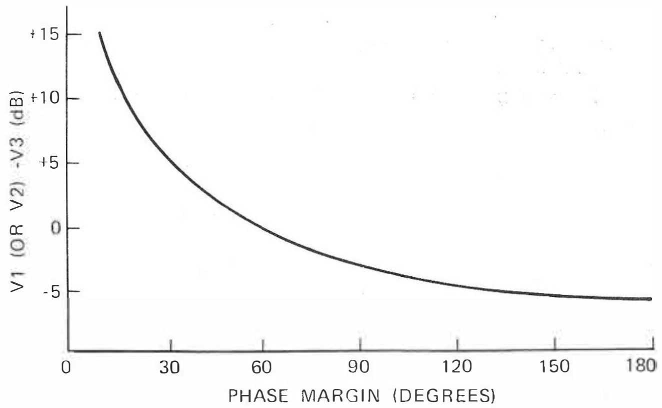


FIGURE 18 PLOT OF AMPLITUDE DIFFERENCE VERSUS PHASE MARGIN

between the V2 and V1 curves and re-plotting this difference versus frequency, but this extra step is not necessary.

A two-channel gain/phase meter such as the Bafco Model 916H will plot gain and phase directly in conventional form. Phase information at points other than crossover can be very useful and probably justifies the extra cost of the two-channel unit. In addition, the requirement of a constant injected signal level is no longer valid and the signal may be injected with a current probe as described in Appendix B. A Tektronix P6021 probe works well for this purpose.

7. GRAPHICAL ANALYSIS

The use of reactance paper such as K+E 46 7963 greatly simplifies stability analysis. The reactance of inductors and capacitors at any given frequency can be taken right from the graph paper. This makes the paper very useful for synthesizing amplifier characteristics and choosing values to achieve the desired characteristics.

For example, take a buck converter in the continuous conduction mode with an input of +15 volts, output of +5 volts, and L-C corner frequency of 1 KHz. Assume that an SG1524 is used as a control element. The control voltage swings 2.5 volts to change the duty cycle from 0 to 1, which results in an output voltage change of 15 volts. The d.c. gain is therefore 15/2.5 = 6. Assume also that the L-C output filter resonance is at 1 KHz. The transfer function of the modulator is then as shown in Figure 19. Suppose the desired crossover frequency is 3 KHz. The modulator gain at this point is 0.7, so the

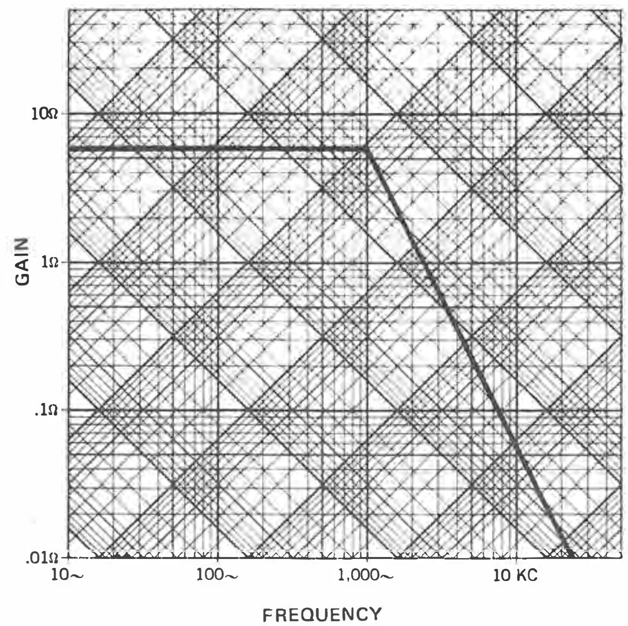


FIGURE 19 MODULATOR TRANSFER FUNCTION

feedback amplifier must have a gain of 1/0.7 = 1.43 at 3 KHz. In order to compensate the 2 pole rolloff, an amplifier of the type shown in Figure 9 can be used. Caution! The output impedance of the error amplifier internal to the SG1524 has too high an output impedance to work successfully in the circuit of Figure 9. A conventional op amp such as an LM108 should be used as the error amplifier, with the SG1524 internal amplifier wired as a non-inverting unity gain buffer. Assume the following characteristics are desired as shown in Figure 20:

Gain = 1.43 @ 3 KHz
Double zero at 1 KHz
Pole at 10 KHz
Pole at 30 KHz

The value of R1 is arbitrary, related to the desired sense current and the input current of the op amp. Assume a value of 10K for R1. The remaining values may be calculated as follows:

From Figure 20,

$$A_{V1} = 0.5$$

$$A_{V2} = 4.5$$

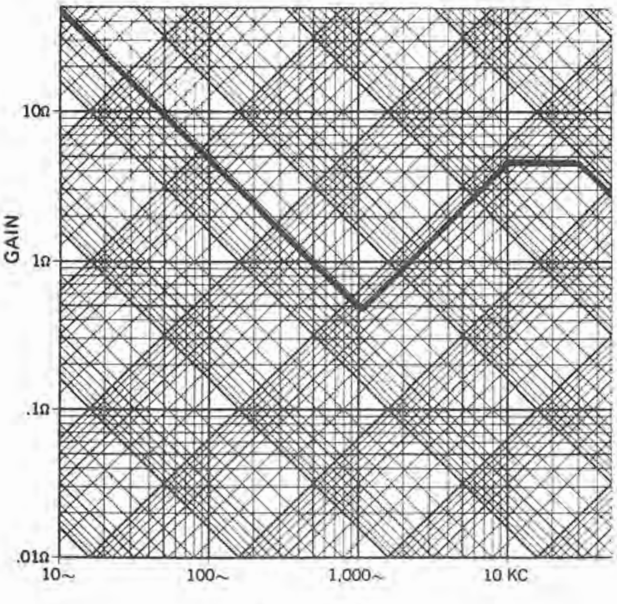


FIGURE 20 DESIRED AMPLIFIER CHARACTERISTIC

f_1 = 1 KHz

f_2 = 1 KHz

f_3 = 10 KHz

f_4 = 30 KHz

From Equation (4), assuming R1 = 10K

R2 = 5K

From Equation (5),

R3 = 1.25K

From Equation (6) or the reactance paper,

C1 = 0.032 μf

From Equations (7) or (8) or the reactance paper,

C3 = 0.014 μf

From Equation (9) or the reactance paper,

C2 = 0.0011 μf

If more accuracy is required, practical values can be selected for the components (i.e., C1 = 0.033 etc.) and Equations (4-9) can be re-evaluated to "fine-tune" the graph although this step is usually not necessary. The resultant overall loop gain plot can be obtained by adding the curves of Figures 19 and 20, or by starting with the now known crossover frequency of 3 KHz and plotting by slopes as shown in Figure 21.

A "mirror image" technique can also be used which may be helpful. This involves drawing a mirror image (around the unity gain line) of the desired amplifier characteristic. The vertical difference between the modulator curve and the mirrored amplifier curve is then a direct Bode plot of the resulting overall loop as shown in Figure 22.

8. CONCLUSION

Stability analysis of switching regulators has been a subject akin to the mountains of Tibet -- shrouded in smoke and clouds. This paper has been an attempt to clear away some of the smoke and clouds and show why circuits oscillate or are stable, and how to design them so

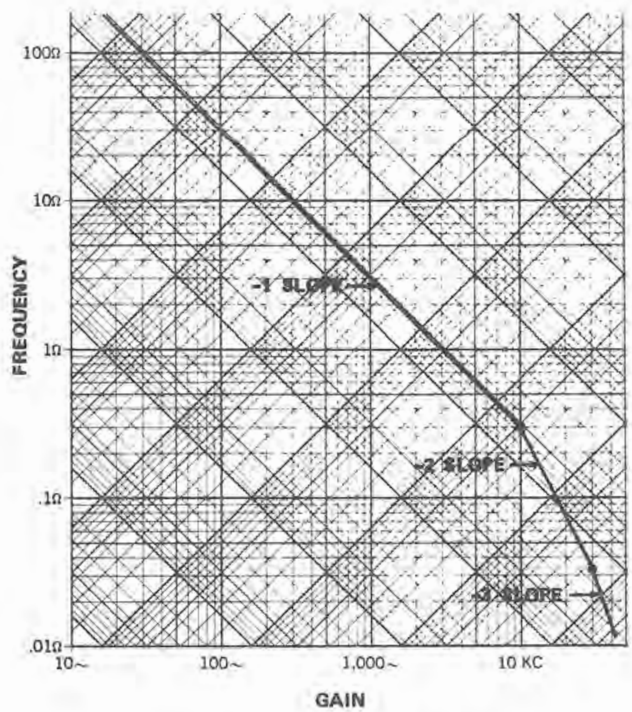


FIGURE 21 OVERALL LOOP GAIN

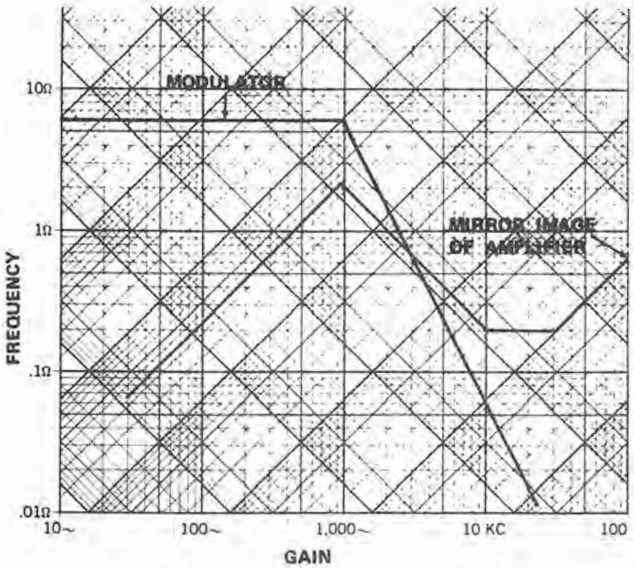


FIGURE 22 MIRROR IMAGE TECHNIQUE

that they behave in the desired fashion. Specific feedback amplifiers were presented, one of which may be used to compensate the loop gain characteristic of almost any switching regulator or converter. Measurement techniques were presented for quickly and easily determining the loop gain of a circuit or any portion thereof. Graphical analysis techniques were given for compensating the overall loop transfer function, given the transfer function of the modulator. These techniques enable the switching regulator designer to be in control of his design, to understand what it does and why, and to make the circuit work when the loop is closed, with a minimum of trial-and-error.

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3) R.D. Middlebrook and Slobodon Ćuk, "A General Unified Approach to Modeling Switching-Converter Power Stages," IEEE Power Electronics Specialists Conference, 1976 Record, pp. 18-34, (IEEE Publication 76 CH1084-3 AES); also International Journal of Electronics, Vol. 42, No. 6, pp. 521-550, June 1977.

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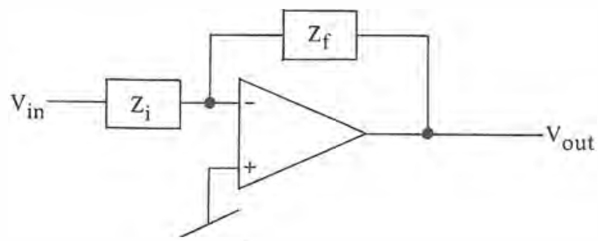
APPENDIX A
DERIVATION OF TRANSFER FUNCTIONS

Writing the transfer functions of the circuits of Figures 8 and 9. Any of the amplifiers of Figures 7, 8, and 9 may be simplified to the following form:



whose transfer function can be shown to be:

$$\frac{V_{out}}{V_{in}} = \frac{Z_f}{Z_i} = \frac{\text{feedback impedence}}{\text{input impedence}}$$

A desirable format to write (Z_f/Z_i) is going to be

$$\frac{(\tau_1 s + 1)}{(\tau_2 s)(\tau_3 s + 1)} \qquad \text{(A-1)}$$

where $(\tau_1 s + 1)$ represents a "zero" because it is in the numerator, $(\tau_2 s)$ represents a "pole-at-the-origin" because it is in the denominator and it lacks a "+1" term, and $(\tau_3 s + 1)$ represents a "pole" because it is in the denominator, and τ is a time constant, RC.

Refer to Figure 8 in the main text. We write by inspection:

$$\frac{V_{out}}{V_{in}} = \frac{[Z_f]}{[Z_i]} = \frac{\left[\dfrac{\left(\frac{1}{C_2 s}\right)\left(R_2 + \frac{1}{C_1 s}\right)}{\left(R_2 + \frac{1}{C_1 s} + \frac{1}{C_2 s}\right)}\right]}{R_1} \qquad \text{(A-2)}$$

where s represents the complex operator jw and (1/Cs) represents the impedance of a capacitor as a function of frequency. Now we want to re-write Equation (A-2) in the form of Equation (A-1).

$$\frac{V_{out}}{V_{in}} = \frac{\left(R_2 + \frac{1}{C_1 s}\right)}{(R_1)\left(R_2 C_2 s + \frac{C_2}{C_1} + 1\right)}$$

$$= \frac{(R_2 C_1 s + 1)}{(R_1 C_1 s)\left(\frac{R_2 C_1 C_2 s + (C_1 + C_2)}{C_1}\right)}$$

$$= \frac{(R_2 C_1 s + 1)}{(R_1 (C_1 + C_2) s)\left(R_2 \frac{C_1 C_2}{C_1 + C_2} s + 1\right)} \qquad \text{(A-3)}$$

which is in the form of Equation (A-1).

$$\tau_1 = R_2 C_1$$

$$\tau_2 = R_1 (C_1 + C_2)$$

$$\tau_3 = R_2 \frac{C_1 C_2}{C_1 + C_2}$$

The circuit shown in Figure 9 of the main text may be treated in similar fashion:

$$\frac{V_{out}}{V_{in}} = \frac{\left[\dfrac{\left(\frac{1}{C_2 s}\left(R_2 + \frac{1}{C_1 s}\right)\right)}{\left(R_2 + \frac{1}{C_1 s} + \frac{1}{C_2 s}\right)}\right]}{\left[\dfrac{\left(R_1\left(R_3 + \frac{1}{C_3 s}\right)\right)}{\left(R_1 + R_3 + \frac{1}{C_3 s}\right)}\right]}$$

$$= \frac{\left[\dfrac{\left(R_2 + \frac{1}{C_1 s}\right)}{R_2 C_2 s + \frac{C_2}{C_1} + 1}\right]}{\dfrac{R_1 (R_3 C_3 s + 1)}{(R_1 + R_3) C_3 s + 1}}$$

$$= \left[\frac{(R_2 C_1 s + 1)}{(R_2 C_1 C_2 s^2 + C_2 s + C_1 s)}\right]\left[\frac{(R_1 + R_3) C_3 s + 1}{R_1 (R_3 C_3 s + 1)}\right]$$

$$= \frac{(R_2 C_1 s + 1)((R_1 + R_3) C_3 s + 1)}{(R_1 (C_1 + C_2) s)\left(R_2 \frac{C_1 C_2}{C_1 + C_2} s + 1\right)(R_3 C_3 s + 1)} \qquad \text{(A-4)}$$

which is the desired form.

APPENDIX B

HEWLETT-PACKARD APPLICATION NOTE 59

LOOP GAIN MEASUREMENTS WITH H-P WAVE ANALYZERS

Until recently, measurements of loop gain in negative feedback circuits required that the feedback loop be opened at a particular point and then a load impedance be inserted to duplicate the closed loop impedance at that point. The loop gain could then be determined by the conventional technique of measuring the ratio of the output to the input voltage of the open loop.

These two steps are both inconvenient and time consuming. A new method, utilizing a Hewlett-Packard Model 302 or 310 Wave Analyzer and a 1110A Clip-On Current Probe, allows loop gain to be measured without breaking the loop. Consequently, the measurement can be made in a very small fraction of the time previously needed.

The loop gain measurement is made in four simple steps:

1. The probe, which is attached to the BFO output of the Wave Analyzer, is clipped around a lead in the feedback circuit where the signal is confined to a single path.
2. The narrow-band, tuned voltmeter of the Wave Analyzer is connected to one side of the clip-on probe.
3. The BFO Output Amplitude control is adjusted to set a convenient reference level, such as 0 dB, on the Wave Analyzer Voltmeter.
4. A voltage reading is taken on the other side of the probe.

This reading will be the loop gain of the feedback circuit. expressed directly in dB, at the particular BFO frequency.

Accurate measurements are made with this method, especially when the loop gain is small, because the Wave Analyzer's narrow-band, selectively-tuned voltmeter avoids the spurious measurement of hum and noise that are usually present at low signal levels. Also, the BFO output is automatically adjusted to coincide with the frequency to which the input is sensitive. Since signal levels are necessarily small to avoid saturating the circuit, the narrow bandwidth and high sensitivity of the Wave Analyzer are essential requirements for measurements of this type.

MEASUREMENT OF FEEDBACK

In a typical feedback amplifier, such as shown in Figure B-1, the gain around the feedback loop, $A\beta$, can be measured in the conventional way. To do this, however, means breaking the feedback circuit at some point, such as at "X", applying a signal to amplifier A_1, and measuring the output of Amplifier A_o. But for this measurement to be valid, A_o must be terminated in the load that A_o would normally see when the loop is not broken. Besides the inconvenience of breaking the circuit, it may be difficult to design a load that will equal the input impedance of A_1 over the frequency range for which $A\beta$ is to be measured. With the new technique the loop need not be broken and, consequently, the difficulty of designing the proper termination is completely avoided.

This new method for measuring loop gain utilizes the magnetic coupling capability of the current probe to insert a signal into the feedback loop -- without breaking it -- by simply clipping it around a circuit lead. The current probe does not alter the feedback loop since it has very low output impedance (0.01 ohm). The Models 302 or 310 Wave Analyzers can be very conveniently used for both the signal source and voltage measurement device when used in the BFO mode. The advantage of the Wave Analyzer is that both the source and the measurement

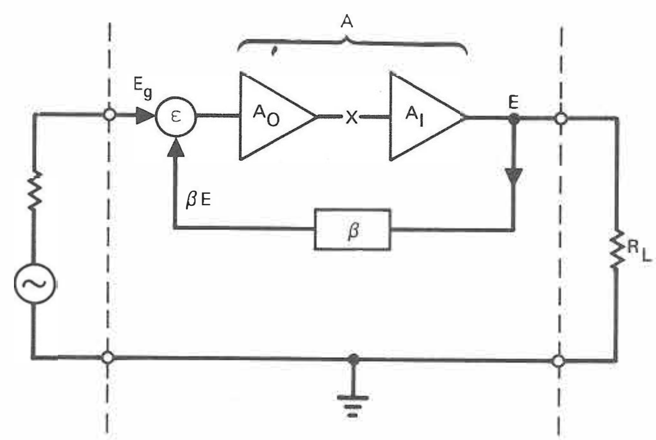


FIGURE B-1

circuits are tuned simultaneously so that only the gain at the frequency of interest is being measured. The narrow bandwidths of the Wave Analyzers also ensure a high degree of noise and spurious signal rejection.

The signal from the BFO output of the Wave Analyzer, which is coupled into the feedback loop by the current probe, establishes voltages in the loop which allow Aβ to be measured directly. To understand how Aβ can be measured in this matter, consider first the feedback loop of Figure B-1 as an open loop amplifier with a gain of Aβ. The amplifier is opened at "X", terminated with the input impedance of A_1 (A_{in}), and a signal E_{in} is applied to A_1.

From Figure B-2(b)

$$A\beta = \frac{E_{out}}{E_{in}} \quad \text{definition of loop gain} \qquad \text{(B-1)}$$

$$E_{out} = IZ_{zin} \qquad \text{(B-2)}$$

$$I = \frac{E}{Z_{out} + Z_{in}} \qquad \text{(B-3)}$$

Substituting in Equation (B-2)

$$E_{out} = E - \frac{EZ_{out}}{Z_{out} + Z_{in}} \qquad \text{(B-4)}$$

$$E_{out} = \frac{EZ_{in}}{Z_{in} + Z_{out}}$$

Substituting in Equation (B-1)

$$A\beta = \frac{E}{E_{in}}\left(\frac{Z_{in}}{Z_{in} + Z_{out}}\right) = \frac{E/E_{in}}{1 + \frac{Z_{out}}{Z_{in}}} \qquad \text{(B-5)}$$

Now consider the situation in Figure B-3 where the feedback loop is closed and a voltage source is connected in series with it by the probe. The clip-on probe can be inserted at any point in the feedback loop where the signal is confined to a single path.

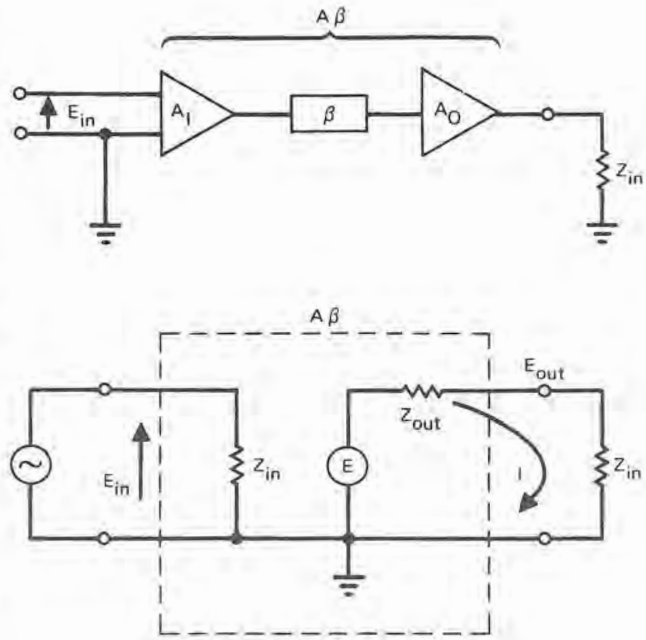


FIGURE B-2

From Figure B-3

$$E_{out} = E - IZ_{out} \qquad \text{(B-6)}$$

$$\text{But, } I = \frac{E_{in}}{Z_{in}} \qquad \text{(B-7)}$$

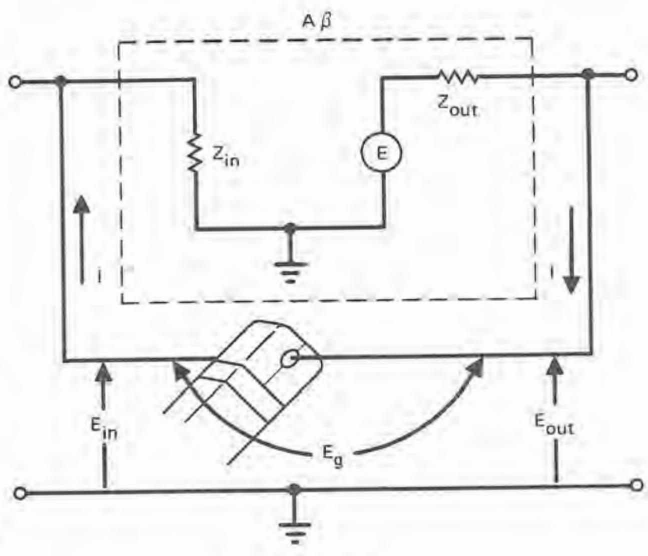


FIGURE B-3

Substituting in Equation (B-6)

$$E_{out} = E - E_{in}\frac{Z_{out}}{Z_{in}} \qquad \text{(B-8)}$$

$$E = E_{out} + E_{in}\frac{Z_{out}}{Z_{in}}$$

Substituting in Equation (B-5)

$$A\beta = \frac{\frac{E_{out}}{E_{in}} + \frac{Z_{out}}{Z_{in}}}{1 + \frac{Z_{out}}{Z_{in}}} \cong \frac{E_{out}}{E_{in}} \text{ for } Z_{out} << Z_{in} \qquad \text{(B-9)}$$

As can be seen from Equation (B-9), if a signal is coupled into the loop at a point where the output impedance is much less than the input impedance, the the loop gain reduces to the simple ratio of E_{out}/E_{in}.

To read the ratio of E_{out} to E_{in} directly in dB, set E_{out} to the 0 dB level on the Wave Analyzer meter by adjusting the level of the signal to the feedback loop with the BFO Amplitude control. E_{in} will now be read directly in dB units and will represent the loop gain directly in dB. If the loop gain is less than one, E_{in} is set to the 0 dB reference level and then the dB reading of E_{out} represents Aβ.

A dual of the above method exists and may be used when it is not possible to find a point where Z_{out} is much less than Z_{in}. If Z_{out} is much greater than Z_{in}, then Aβ can be determined from the ratio of currents flowing into the feedback loop when the source of excitation is a current source. This is shown in Figure B-4. Aβ equals I_2/I_1 for Z_{out} much greater than Z_{in}. The current source can be simulated by connecting a resistor (much larger than Z_{in}) in series with the Wave Analyzer's BFO output.

DETERMINING POINT OF SIGNAL INSERTION

To determine whether the output impedance is much less than the input impedance, on would normally have to make an approximate calculation based on the particular feedback circuit. However, either of the above

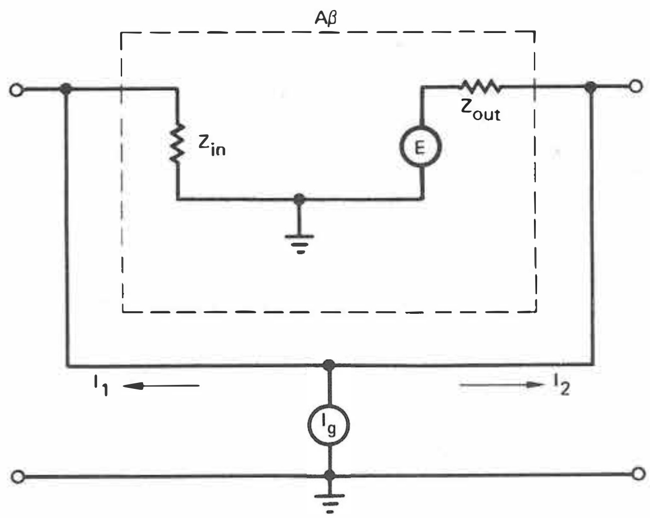


FIGURE B-4

techniques lends itself very easily to measuring the ratio of Z_{out} to Z_{in}. These impedances are the output and input impedances without feedback so that it is necessary to connect a large capacitor from a convenient point in the feedback loop to ground to short out the feedback path. Then with a signal applied with the clip-on probe as in Figure B-3, the ratio of E_{out} to E_{in} is the ratio of Z_{out} to Z_{in} since the same current flows through both impedances. Alternatively, a current could be applied and the ratio of I_{out} to I_{in} would be the ratio of Z_{in} to Z_{out}.

MEASUREMENT OF PHASE ANGLE OF Aβ

It is also important to know the phase angle of Aβ, or E_{out}/E_{in}, since when the voltages are in phase oscillations can result. From Figure B-3 it is clear that $E_{out} = E_g + E_{in}$. If the amplitudes of these three quantities are known then, by trigonometry, the phase angles between each quantity can be determined. This is easily done by constructing a vector diagram of the amplitudes of each voltage as shown in Figure B-5. From the above relationship between these voltages, the graphical construction must form a closed triangle. The phase angle between E_{out} and E_{in} can then be measured directly from the diagram.

Determining E_g, the voltage induced in the feedback loop by the clip-on probe, is easily done with the Wave Analyzer. Without changing the BFO Amplitude Output control, attach the current probe tc a loop of wire attached to the Wave Analyzer input terminals. E_g will then be read directly in volts on the Wave Analyzer. I_{out}, I_{in}, and I_g can also be used to determine the phase angle of Aβ.

Using the method of loop gain measurement explained in this article, Figure B-6 shows the measurement of Aβ versus frequency for the circuit shown in Figure B-7. The probe was clipped around point A and the voltage ratio on each side of the probe was measured to determine the loop gain at the desired frequencies. The time to take this data was only a matter of minutes. The convenience and simplicity of this method should make it popular with anyone who has had to measure loop gain in the old, conventional way.

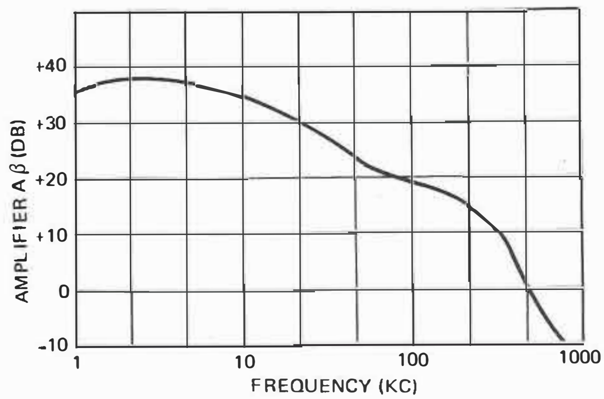


FIGURE B-6

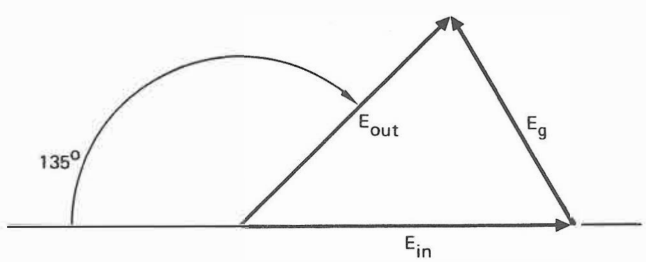


FIGURE B-5

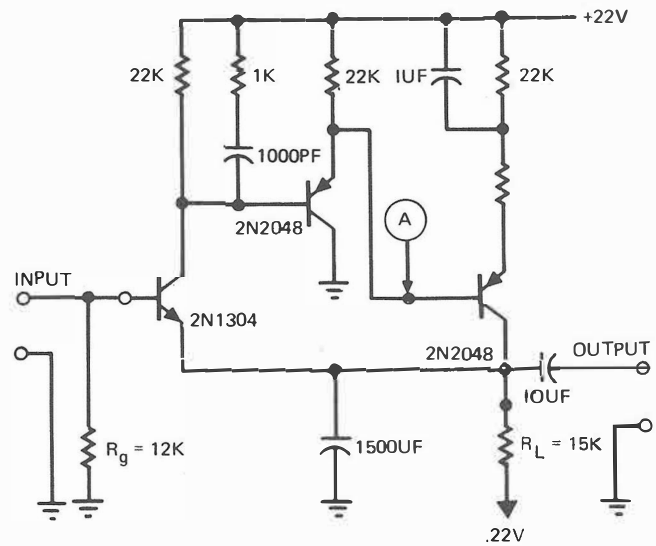


FIGURE B-7