SOURCE-LOAD INTERACTIONS IN MULTI-UNIT POWER SYSTEMS

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Source-Load Interactions In Multi-Unit Power Systems

Abstract:

Switching regulators have negative input impedance, at least at low frequency. This is evidenced by the fact that as the input voltage increases, the input current decreases, and vice-versa. Before switching regulators came into such widespread use, they were mainly stand-alone units operating from a low source impedance and driving a passive load. Oscillation was rarely a problem. Now, as switching regulators become lower in cost and more widespread in application, there are many instances where switching regulators serve as both source and load. This is happening both at low power, as in distributed power systems, and at high power, in utility systems with DC links. The space station power system is a good example of switching regulators driving other switching regulators. If a source has a resonant output characteristic, connecting a negative resistance to that source can allow the resonant output to oscillate. This characteristic is shared by both L-C input filters and switching power supply outputs. This paper discusses various criteria that have been developed recently to assure stability under these source-load conditions and gives practical suggestions of design and testing criteria to assure stability under all operating conditions.

Introduction

There is increasing interest in calculating or measuring the stability of complex power systems. This interest originally took the form of input filter considerations in power supplies. Papers on this subject extend back at least to 1971 with a paper by Yuan Yu and John Biess1. In 1976 and 1978 Dr. David Middlebrook wrote papers on the subject 2, 3, the former of which has become a classic in the field. Middlebrook pointed out that oscillation could be positively prevented by assuring that the magnitude of the output impedance of the source (filter) remained lower than the magnitude of the input impedance of the power supply (measured beyond the filter).

In 1992, Dr. Fred Lee proposed a less stringent criterion based on consideration of the phase of the impedance as well as the magnitude. It is true that while switching power supplies have negative input impedance at DC, both the phase and amplitude of this impedance are functions of frequency. At frequencies above the bandwidth of the open loop gain of the voltage feedback loop, the phase and amplitude both shift dramatically. It is possible to have the magnitude of the filter output impedance exceed the magnitude of the power supply input impedance and still have a stable system, but from a practical standpoint the stability margins will probably not be satisfactory for high-reliability applications.

This paper starts with a simple L-C filter, then progresses through damping to stability criteria and gives practical examples of real-life impedance plots in both magnitude and phase. Both magnitude and phase guidelines are discussed as the examples are presented.

Simple L-C Filter

Figure 1 shows a simple L-C filter. L-C filters are common in power supplies. They are used to attenuate AC signals generated inside the power supply and prevent these signals from being conducted either back into the source or out into the load. One characteristic of a simple L-C filter such as the one shown is that, if perfect L's and C's could be constructed, the output impedance of the filter approaches infinity at the resonance of the filter. (Resonance is the frequency where the inductive reactance of the inductor is equal to the capacitive reactance of the capacitor.)

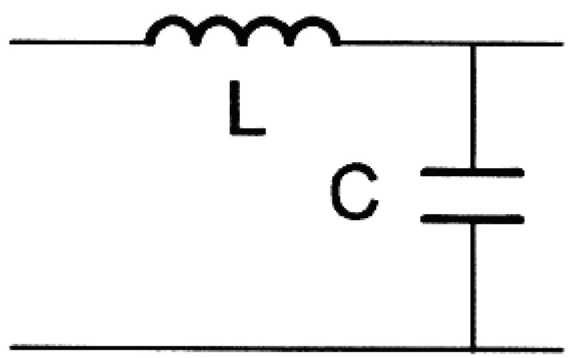

Figure 1. Simple L-C filter

L-C Filter With Damping

To control the impedance peaking, damping is normally employed in L-C filters. There is some natural damping because of series losses in the inductor and the capacitor. It is frequently the case that additional damping is necessary. Figure 2 shows an L-C filter with damping. Parallel damping of the capacitor is shown in lieu of series damping of the inductor because, in general, series losses are to be avoided. Parallel damping of the sort shown would cause excessive losses also, but the next section of the paper discusses a method of controlling the losses from parallel damping.

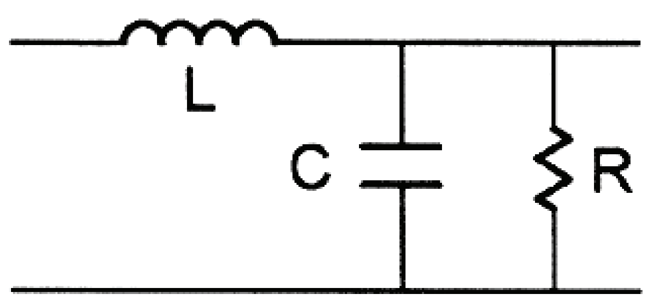

Figure 2. L-C filter with damping

Figure 3 shows a plot of impedance versus frequency of the inductor and the capacitor together with plots of the filter output impedance in the under-damped and over-damped cases. The characteristic impedance of the L-C circuit is given by Zo= the square root of (L/C). A value of damping resistance roughly equal to this value is normally

L-C Filter with Practical Damping

Just as it is not practical to place a resistor in series with the inductor (series damping) because of the conduction losses, it is not practical to simply place a resistor in parallel with the capacitor. This is because the value of this resistor is usually low, on the order of 1 ohm, and the shunt loss would be excessive. Figure 4 shows an L-C filter with a more practical damping scheme. A capacitor is placed in series with the damping resistor to block the DC voltage and prevent DC current from flowing through the damping resistor. The value of this series capacitor has to be larger than the value of the filter output capacitor, a factor of 4 being a common rule of thumb.

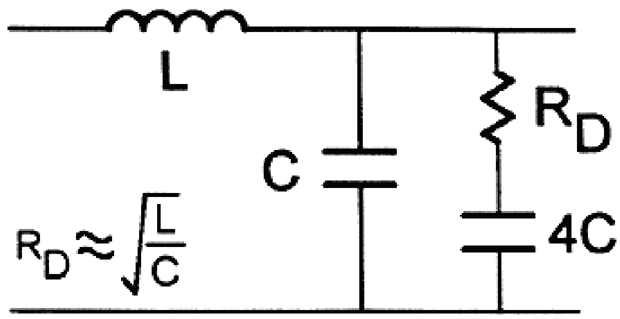

Figure 4. L-C filter with practical damping

Figure 5 shows an impedance versus frequency plot of the three branches that make up the output impedance of the damped filter. Remember in calculating output impedance the input (input side of the inductor) is grounded. The output impedance is the lower of the curves on the graph. The impedance still follows the inductor impedance up to resonance, the impedance at resonance is still controlled by the resistor, and the curve follows the capacitor impedance down at frequencies above resonance. The damping capacitor impedance is always higher than the inductor impedance and its effect is relatively insignificant.

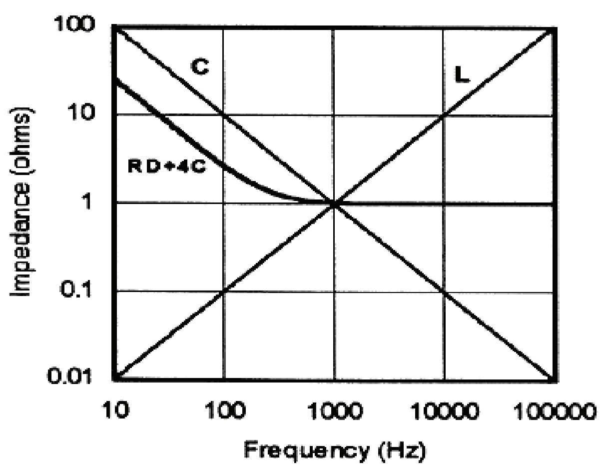

Figure 5. Damped filter impedances

The use of a damping network including a large capacitor may seem like an excessive burden in size, weight, and cost but the benefits are many and the burden is not as great as it first appears. The output capacitor needs to be of good quality with low internal resistance (internal resistance is called Equivalent Series Resistance or ESR). The damping capacitor does not need to be of this high quality, since a resistor will be placed in series with it anyway in the application. For example, in a space application, the filter output capacitor may be a low ESR solid tantalum while the series damping capacitor could be a lossy but much smaller wet-slug tantalum.

L-C Filter With Damping And Negative Resistance Load

When an L-C filter is driving a switching power supply, a different and fundamentally dangerous situation occurs. Switching power supplies are for all practical purposes constant power devices. If the output power is constant, the input power will be also. If the input voltage increases, the input current will decrease so that the volt-amp product will remain the same. This characteristic of decreasing current with increasing voltage is effectively a negative resistance. Figure 6 shows schematically what happens when a negative load is connected to a damped L-C filter. If the magnitude of the effective negative resistance is lower than the magnitude of the effective damping resistor, the L-C filter will have a net negative damping resistance and the filter will oscillate.

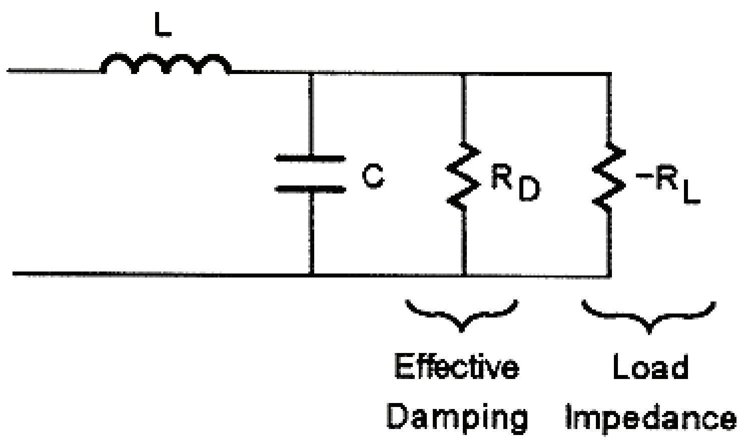

Figure 6. Filter with negative resistance load

This phenomenon is obvious at DC, but what may not be so obvious is that the negative resistance is relatively constant up to the bandwidth of the internal voltage feedback loop of the power supply regulation circuitry. If the filter resonance frequency is lower than the power supply voltage feedback bandwidth, then it is necessary to control the peaking of the output impedance of the input filter so that the peak value remains below the magnitude of the negative resistance of the power supply input.

Relative Placement of Filter Resonance and Power Supply Bandwidth

Problems of filter resonance occur primarily when the resonance of the filter is lower than the bandwidth of the power supply voltage feedback loop. This is rare in commercial power supplies since the input filter is usually placed there for prevention of electromagnetic interference (EMI) and has a resonant frequency on the order of 300 kHz, well above the feedback loop bandwidth.
In military and aerospace power supplies, constraints of conducted emissions and conducted susceptibility testing require that the filter resonance always be below the bandwidth of the power

supply voltage feedback loop. This situation comes about because the conducted susceptibility requirement does not allow signals to pass through the power supply from input to output at any frequency. At low frequency, noise on the input is rejected by the regulating action of the feedback loop. At high frequency, noise on the input is rejected by the input filter. There is necessarily an overlap between these two frequency ranges, since to leave a gap would create a band of frequencies where noise would pass through the power supply unimpeded by either the loop or the input filter. This means that in military and aerospace power supplies there is always the potential of an input filter oscillation and tests and/or analysis should be used to verify sufficient margin between the filter output impedance at resonance and the power supply input impedance. Watch out for cables. The resonance and oscillation problem is not restricted to overt filters. Long cables terminated by capacitors can create the same conditions if the load is a switching regulator.

Power Supply With Input Filter

Figure 7 shows a power supply with input filter. Notice that there is not only an input L-C filter, but an output L-C filter as well. The power supply is partitioned in a way that is perhaps unusual but is more representative of the actual functional blocks at work. The portion labeled "Power Supply" is the actual switching regulator stripped of the input and output filters. The power supply draws pulses of current from the input filter and puts out pulses of voltage to the output filter.

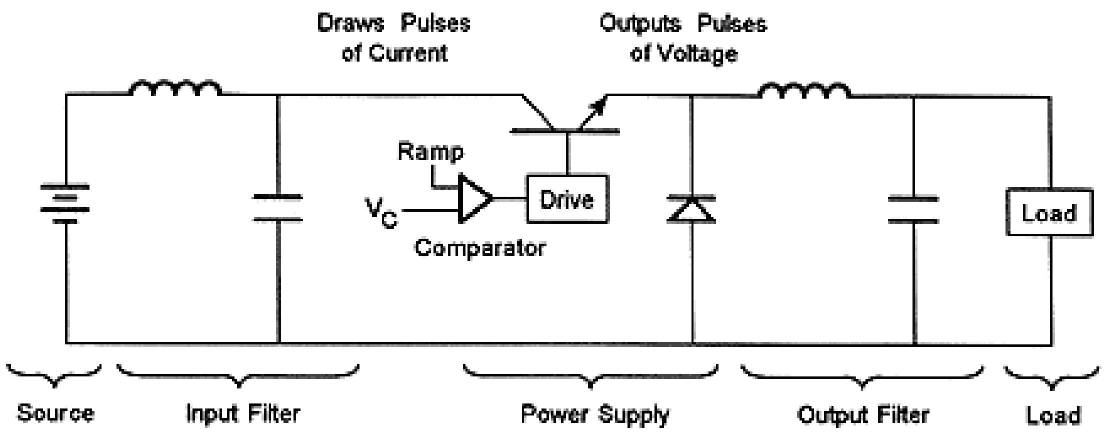

Figure 7. 'Plant' portion of power supply with input filter

Dual Nature of Filters

One interesting characteristic of all passive filters is a duality whereby the voltage transfer function in the forward direction is identical to the current transfer function in the reverse direction. Filters which are designed to attenuate voltage in the forward direction are equally adept at filtering current in the reverse direction.

A major purpose of the input filter is to attenuate the AC components of the pulses of current drawn by the power supply so that only the DC average value flows from the source. A major purpose of the output filter is to attenuate the AC portion of the pulses of voltage at the output of the power supply so that only the DC average value of the voltage is applied to the load.

In the circuit of Figure 7, a control voltage Vc is compared to a ramp to generate a pulse width modulated drive signal for the transistor. This circuit is not a complete power supply, since the compensation amplifier is not shown. This portion of the circuit is called the "plant" or "modulator". The transfer function from control voltage Vc to the output normally has constant gain and negligible phase shift at low frequency, and then falls at a 2 slope (-40 dB/decade) beyond the corner frequency of the output filter. A typical plant transfer function is shown in Figure 8.

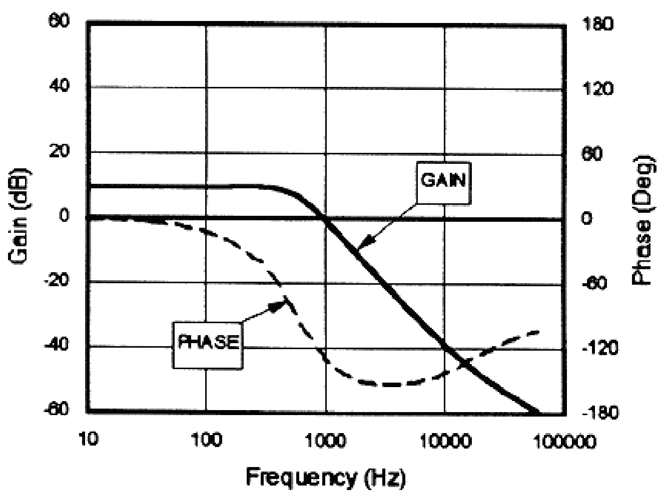

Figure 8. Control to output transfer function without effects of input filter

Figure 8 does not include any effects of the input filter. If the input filter is well damped and the output impedance peak of the input filter is well below the negative input impedance of the power supply, this is what a typical plant transfer function looks like.

Effect of Input Filter on Transfer Function

In the case of a power supply with poorly damped input filter, the gain portion of the plant transfer function dips significantly at the resonant frequency of the input filter. Figure 9 shows the plant transfer function of a circuit with little damping. The input filter resonant frequency in this particular example is 225 Hz and the output filter resonates at 503 Hz. Note the large perturbations in both gain and phase at approximately 200 Hz. These are due to the input filter and are characteristic of systems with a potential input filter problem.

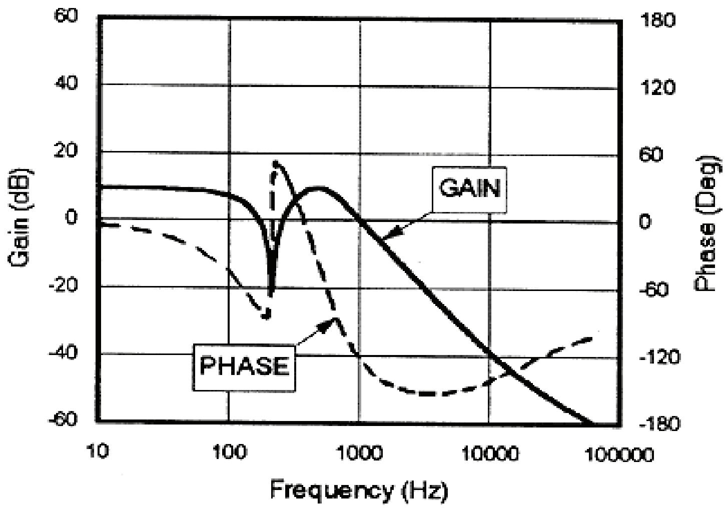

Figure 9. Control to output transfer function with input filter effects included

Effect Of Filter On Feedback Loop Transfer Function

In order to complete the feedback loop, a compensation amplifier is needed to sense the power supply output voltage, compare it to a reference, and output an appropriate control voltage to the Vc input of the plant. Figure 10 shows a complete power supply with plant and compensation amplifier interconnected. Gain of the compensation amplifier is set by the ratio of the feedback impedance Zf to the input impedance Zin. This ratio can be freely adjusted to achieve a wide range of overall loop bandwidth and phase margin. When selecting amplifier gain, it is usually best to design for loop bandwidth greater than the resonant frequency of the output L-C filter. The reason is that the loop provides active damping for the output filter if the loop bandwidth is greater than the filter resonant frequency. If the loop bandwidth is less than the output filter resonant frequency, then the loop cannot damp the filter and the filter Q determines the amount of output impedance peaking with no help from the loop. There is no essential difference between an input filter and power supply interface and the interface between one switching power supply driving another. As in the examples just discussed, it is necessary to keep the output impedance of a filter or an entire power supply lower than the negative impedance of the next unit in series.

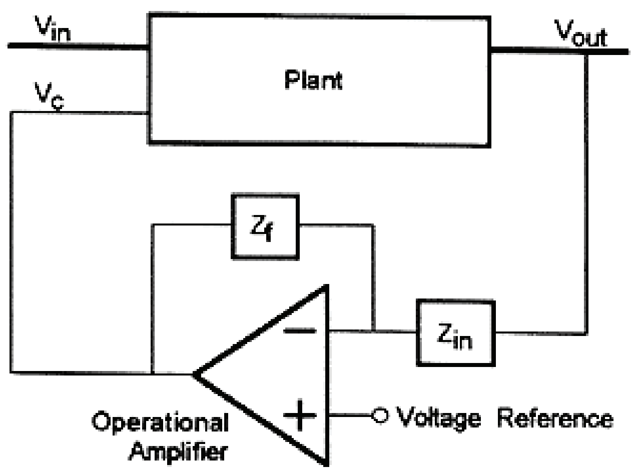

Figure 10. Complete power supply schematic

Figure 11 shows the open loop transfer function of the complete power supply including compensation amplifier and under-damped filter. This loop was designed for a bandwidth of 3 kHz and phase margin of 60 degrees. The effect of the input filter resonance at 200 Hz can be clearly seen. Loop gain actually dips below unity (0 dB) at the filter resonance, but the loop is stable because the phase dip did not quite reach zero degrees. (When discussing a loop, zero phase is really 360 degrees, and the condition to avoid is the one where the gain is unity (0 dB) and the total phase shift is 360 degrees or one whole cycle. This is the condition for oscillation.) In this circuit, the loop crosses over well above the output filter resonance (3 kHz vs. 500 Hz) and the phase margin near optimum at 60 degrees. The output impedance peaking will be well controlled by the feedback loop, but the input filter resonance is severe and will have a significant effect on the output impedance near the 200 Hz resonant frequency.

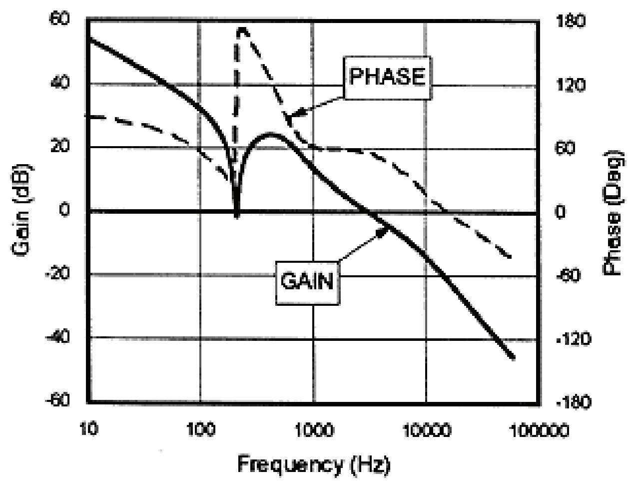

Figure 11. Overall loop transfer function including effects of input filter

Testing Impedance

Figure 12 shows a test set-up for measuring both output impedance and input impedance of the next unit using a Venable Model 350 System or equivalent frequency response analysis system. Other equipment also required is a power amplifier, injection transformer capable of carrying high DC current, and a current probe for measuring the current. The injection transformer should be located near the source and the current probe should be near the injection transformer. The idea is to make the measurement at a "slice" in the power transfer cables so that the output impedance can be measured as well as the input impedance of the next unit in conjunction with the connecting cables. It is of course possible to measure the input impedance of the load without the cable effects simply by moving the Channel 2 voltage measurement point to the input terminals of the load, but that measurement in itself would not be sufficient to assure system stability since the cables are in fact a part of the input impedance of the load from the standpoint of the source. Source output impedance is CH3/CH1. Load input impedance is CH2/CH1. The scale factor of the current probe and the direction of the current probe have to account for in scaling the measurement data. The measurement system chosen must be capable of plotting user-selected channels, scaling the data to compensate for the scale factor of the current probe, and inverting the data on one measurement to compensate for the direction of the current probe. The data for the unit the current probe is pointing to will be correct. The data for the unit the current probe is pointing away from needs to be inverted to be accurate.

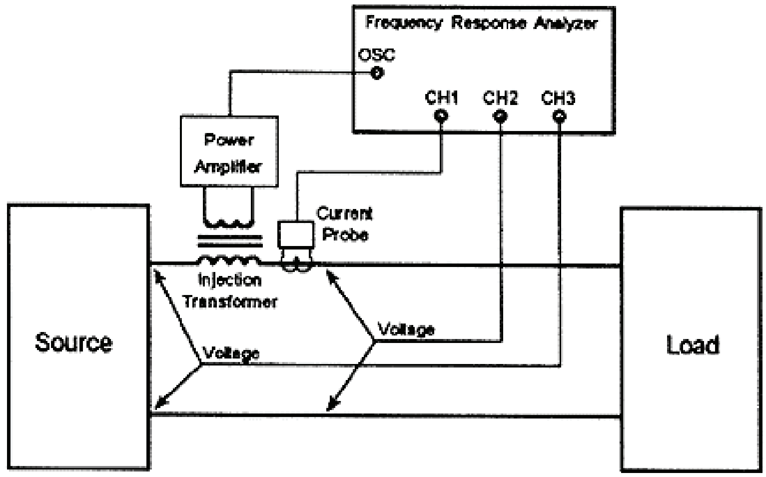

Figure 12. Impedance measurement technique

Relative Source and Load Impedance

Figure 13 shows a plot of impedance versus frequency of the input filter output impedance over-plotted with input impedance of the power supply in the circuit of Figure 7 and for the under-damped filter case. In this example the impedance magnitude of the input filter output impedance does actually touch the magnitude of the power supply input impedance, but the phase of the power supply input impedance is a few degrees from the 180° required to have a pure negative resistance and the circuit does not oscillate. This circuit is on the verge of oscillation and would be unacceptable in any high-reliability application. In a transient loading situation, this circuit will ring at 200 Hz because of the input filter resonance. There will be no appreciable ringing at 3 kHz, the true loop bandwidth and the frequency that would normally be associated with a potential oscillation.

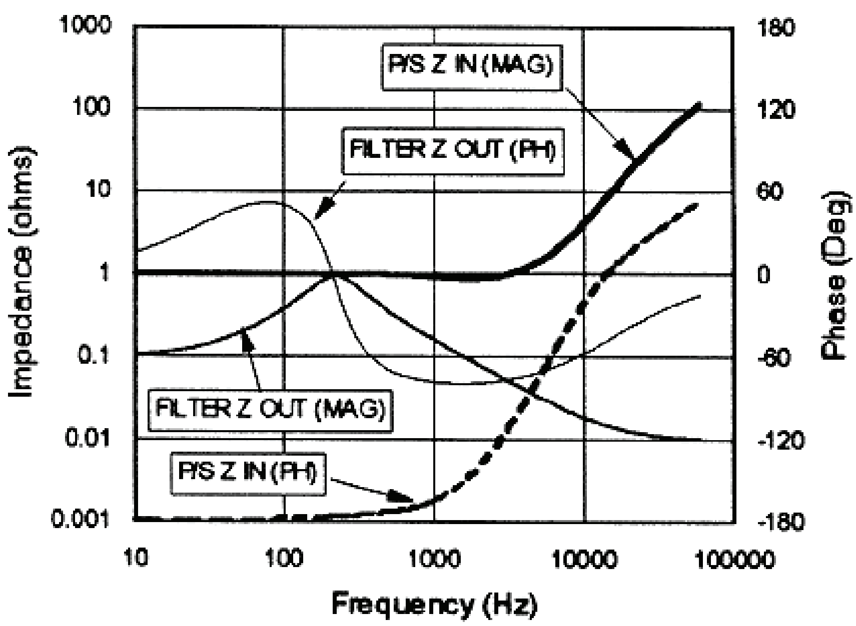

Figure 13. Output and input impedances

Effect of Feedback Loop on Power Supply Output Impedance

In addition to the design and damping of the input filter, the voltage feedback loop bandwidth and phase margin have dramatic effects on the overall output impedance of the power supply. This impedance is important because it becomes the source impedance for the next switching regulator down the line. If the feedback loop bandwidth is less than the output filter resonant frequency, then the peaking will come mostly from the filter and the loop will have relatively little effect. If, as should be the case, the voltage feedback loop bandwidth is higher than the output filter resonance, then the loop characteristics dominate. The peaking frequency is determined by the loop bandwidth and the amplitude of peaking is determined by the phase margin.

Crossover Below Output L-C Corner

Figure 14 is a plot of the overall power supply output impedance when the loop bandwidth is less than the filter resonance. Note that the absolute magnitude of the impedance is higher than in the following example, even though the circuit was identical except for the loop bandwidth and the phase margin was actually higher than in the following case. The input filter was disabled for the purposes of this test.

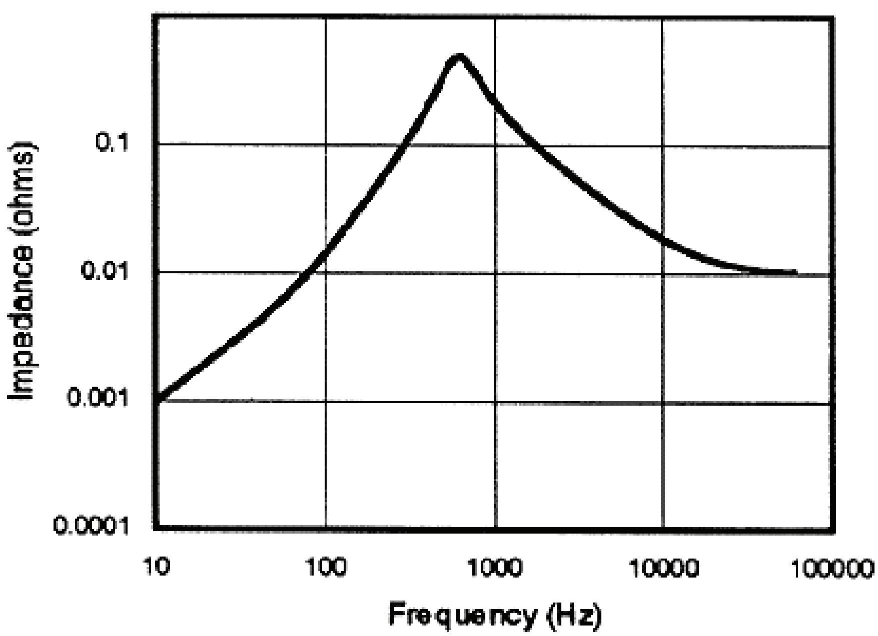

Figure 14. P/S output impedance, low B/W

Crossover Above Output L-C Corner

Figure 15 shows a plot of overall power supply output impedance with 3 kHz loop bandwidth and 60 degrees of phase margin. The input filter was disabled for this test. Note that the peaking occurs at the loop bandwidth, not at the resonant frequency of the output filter.

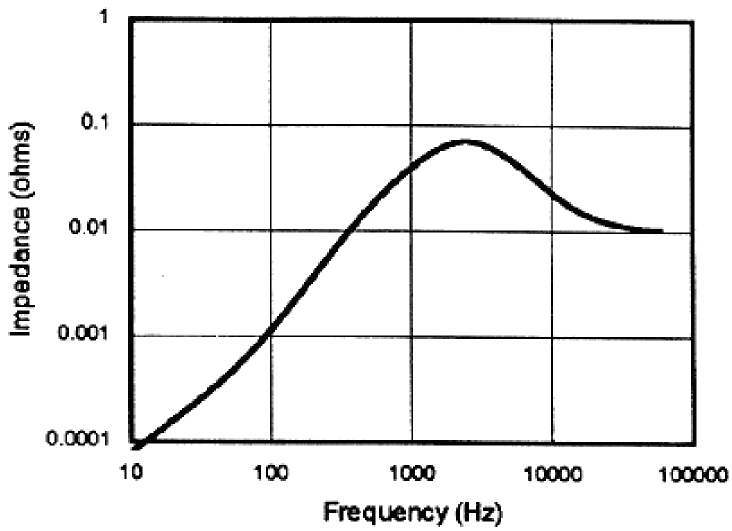

Figure 15. P/S Zout with 3 kHz loop, no filter

Crossover Above Output L-C Corner with Under-Damped Filter

Figure 16 is a plot of the overall output impedance of the power supply with 3 kHz bandwidth but also with the under-damped input filter. Note the impedance peak at 200 Hz due to the filter. It is actually higher than the principal peaking at the loop bandwidth.

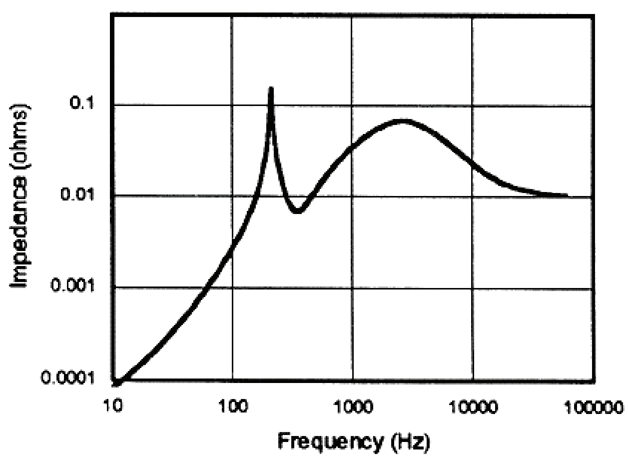

Figure 16. Output impedance of the power supply with 3 kHz bandwidth and input filter

One Supply Driving Another

When one power supply is driving another, exactly the same principles apply as in the case of a power supply driven by an input filter. If the power supply feedback loop does not have sufficient bandwidth or phase margin, the output impedance of the source supply can be excessive. If this impedance exceeds the input impedance of the supply, which is acting as a load, and the phase of that impedance is close to 180 degrees, the system will oscillate or be dangerously close to doing so.

Also, the cables have to be taken into account. If the circuit is "cut" for measurement purposes near the source, then the cables count as part of the load. If, on the other hand, the circuit is "cut" near the load, the cables become part of the source impedance and again have to be accounted for. I have personally experienced a case where an off-line supply had a large electrolytic filter capacitor as a line frequency filter, then cables over to a bank of high frequency bypass capacitors near the input to a switching power supply. The cable from the electrolytic capacitor resonated with the bank of bypass capacitors and caused a system oscillation, even though there was no overt input filter.

One Power Supply Driving Several Others

In the case of a distributed power system where one power supply is driving many others, the same principles still apply although the analysis becomes very difficult. Measurement remains easy and is the recommended way to assure system stability and reliability.

If analysis is absolutely required, the easiest way to do it is to convert each impedance into an admittance for each branch of interest at the "cut" where the system is to be analyzed. At any one frequency, each admittance can be converted to an equivalent inductive or capacitive admittance and a parallel resistor. The resistor may be positive or negative. All of these quantities will be functions of frequency. At the frequencies where the inductive and capacitive admittances add to zero (inductive and capacitive admittances cancel each other out if the amplitudes are equal), if the sum of the resistive admittances is negative then the system has a stability problem. This calculation is not trivial since in many cases loads are combinations of series and parallel elements, which have to be accurately combined, and calculation of essential elements such a cable impedance is no trivial task in itself. Fortunately, measurement remains easy and the measurement equipment automatically gives the results of these combinations without having to calculate and combine each one independently.

Conclusions

Criteria for stable combinations of sources and loads were developed starting with simple examples of L-C filters. Easily understood principles were derived and expanded to cover increasingly complicated situations, which occur in real-life power and power distribution systems. The culprit was shown to be the negative input impedance of switching power supplies and the ability of this negative impedance to reduce damping of L-C filters and even feed energy into them to cause oscillation. An easy and simple measurement technique was presented for measuring these difficult to model quantities and assuring system stability and reliability.

References

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2. R. D. Middlebrook, "Input Filter Considerations in Design and Application of Switching Regulators," IEEE Industry Applications Society Annual Meeting, 1976 Record, pp. 336-382 (IEEE Publication 76CH1122-1-IA).

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